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Dynamic stabilization of the ablative Rayleigh–Taylor instability for heavy ion fusion ☆

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ABSTRACT

Dynamic stabilization of the ablative Rayleigh–Taylor instability of a heavy ion fusion target induced by a beam wobbling system is studied. Using a sharp-boundary model and Courant–Synder theory, it is shown, with an appropriately chosen modulation waveform, that the instability can be stabilized in certain parameter regimes. It is found that the stabilization effect has a strong dependence on the modulation frequency and the waveform. Modulation with frequency comparable to the instability growth rate is most effective in terms of stabilizing the instability. A modulation with two frequency components can result in a reduction of the growth rate larger than the sum of that due to the two components when applied separately.

1. Introduction

In heavy ion fusion, the compression dynamics of the target is subject to the well-known Rayleigh–Taylor (RT) instability. To reduce the deleterious effects of the RT instability on target performance and increase the coupling efficiency, it is necessary to reduce the initial seed for instability growth by making the target illumination by ion beams as symmetric and smooth as possible. In laser-driven inertial fusion research, a sophisticated smoothing system using distributed phase-plate technology has been developed [1]. Recently, a similar technology using oscillating wobbler fields has been proposed for ion-beam-driven inertial fusion energy [2–10] to achieve the desired uniform illumination over an annular region (see Fig. 1). The improvement of stability properties can be attributed to two factors. First, uniform illumination reduces the initial seeding amplitude of the RT instability [4,11–13]. Second, at a given location on the target, the energy/momentum input is pulsating rapidly with time, which results in a dynamic stabilization effect on the instability.

The dynamical stabilization of the Rayleigh–Taylor instability was first studied by Wolf [14] and by Troyon [15]. For applications to inertial confinement fusion, the concept has been investigated by Boris [16] and Betti et al. [17]. In particular, Betti et al. [17] derived an ordinary differential equation for the interface oscillation associated with the ablative RT instability with time-dependent acceleration and ablation [see Eq. (1)]. For heavy ion fusion application, Kawata et al. [11,18,19] showed that time-dependent acceleration effectively reduces the growth of the RT instability. On the other hand, Piriz et al. [20] concluded that time-modulation of the acceleration is ineffective using a model of time-modulation consisting of a sequence of pulsed accelerations with the shape of δ-functions.

In this paper, we show that the time-modulated acceleration rendered by the wobbler system for heavy ion fusion drivers can significantly reduce the growth rate of the ablative Rayleigh–Taylor instability with an appropriate choice of the time-modulation waveform.

Before our discussion of dynamic stabilization using a wobbler, we note that a beam wobbling system might increase the transverse dimension of the beam, and thus brings in a trade-off between beam brightness and smoothing afforded by the wobbler. Such a trade-off needs to be carefully studied for specific heavy ion fusion driver designs.

To study the dynamic stabilization theoretically, we adopt a sharp-boundary model with an ablative front [17,21], and start from the differential equation derived by Betti et al. [17]. The difficulty in correctly describing the dynamical behavior of the instability in this case is the time-dependence of the acceleration, the driving force of the instability. It turns out that Courant–Snyder theory [22] for a second-order ordinary differential equation with general time-dependent coefficient is an ideal theoretical tool to tackle this problem, even though the original Courant–Snyder theory was

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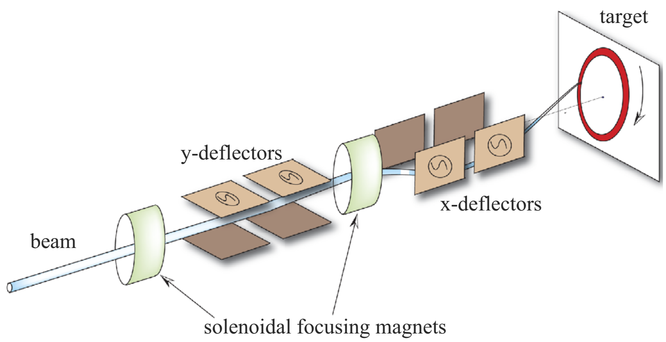

Fig. 1. Schematic of wobbler system and solenoidal focusing lattice for heavy ion fusion.

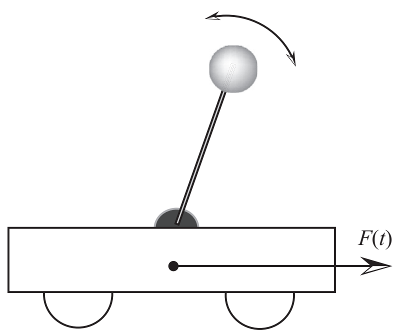

Fig. 2. Inverted pendulum on a moving platform.

intended only for stable cases. Using this method, we find that the stabilization effect has a strong dependence on the modulation frequency. In particular, modulation with frequency comparable to the growth rate is most effective in terms of stabilizing the instability. It is also found that the reduction in growth rate has a complicated dependence on the modulation waveform. For example, a modulation with two frequency components can result in a reduction of the growth rate larger than the sum of the reductions due to the two components when applied separately. With a properly chosen modulation waveform, the instability can be completely stabilized in certain parameter regimes.

The basic idea of dynamic stabilization can be amply illustrated by the example of the inverted pendulum on a moving platform as shown in Fig. 2. If the platform is fixed, the pendulum is obviously unstable. However when a time-dependent force $F(t)$ is applied, the platform will move accordingly, and with an appropriate choice of the functional form of $F(t)$ it is possible to stabilize the dynamics of the inverted pendulum. In general, the types of time-dependent force $F(t)$ can be divided into three categories: feedback controlled, pre-described, and random. For the first type, the driving force is generated dynamically according to the position of the pendulum. This is how an acrobat stabilizes an upside-down wine bottle on one finger. An acrobat can train his motor system and visual system into an excellent feedback control system for the upside-down wine bottle, but it is not possible to design a feedback control system for the RT instability in a heavy ion fusion target. This is because the timescale of the instability is several nanoseconds, which is too fast for any possible beam feedback control. The third type, random modulation, is probably the easiest to implement while also being the most ineffective with the same modulation amplitude. The wobbler system for heavy ion fusion fits into the second category. Needless to say that the challenge is to find a systematic method to determine the optimal time-modulation waveform for the driving beam.

Note that in Refs. [11,19], the stabilizing effects due to the second type of modulation are referred as "dynamic mitigation". In this paper, we do not adopt this terminology, and use the general phrase "dynamic stabilization" for the stabilizing effect due to any type of time-modulation.

The paper is organized as follows. In Section 2, we introduce the sharp-boundary model for the ablative Rayleigh–Taylor instability. The Courant–Snyder theory for unstable solutions of second-order ordinary differential equations with time-dependent coefficients is described in Section 3, and the dynamic stabilization of the ablative RT instability with wobbling beams for heavy ion fusion applications is studied in Section 4.

2. Sharp-boundary model for the ablative Rayleigh–Taylor instability

In this section, we describe the sharp-boundary model for the ablative Rayleigh–Taylor instability and the corresponding governing differential equation that will be used in the study of the dynamic stabilization of the instability in Section 4. In this model, the heavy medium and the light medium are separated by a sharp-boundary interface (see Fig. 3). The density is constant on both sides of the interface, but discontinuous across the interface, which is accelerated in the $\mathbf{e}_x$ direction with an acceleration $g(t)$ by the ablative force. In the frame moving with the interface, an object with mass m is subject to an inertial force $mg(t)$ in the $-\mathbf{e}_x$ direction. The density and ablative velocity in the moving frame in the two regions are denoted by (ρ_1, v_1) and (ρ_2, v_2), respectively. The values of $g(t)$, v_1 and v_2 are positive.

The ablative Rayleigh–Taylor instability can be characterized by the unstable perturbation of the interface, $\eta(y,t){\sim}\eta(t)\exp(iky-i\omega t)$, between the heavy and light media. It is assumed that $k>0$ without loss of generality. In the limit of $A\equiv(\rho_2-\rho_1)/(\rho_2+\rho_1)\rightarrow 1$, the ordinary differential equation for $\eta(t)$ derived by Betti et al. [17] is

$$\frac{d^2\eta}{dt^2}+kv_A\frac{d\eta}{dt}-kAg\eta=0 \tag{1}$$

where g is the acceleration, and $v_A\equiv v_2>0$ is the ablative velocity of the heavy medium. Both g and v_A are time-dependent, determined by the time-dependent energy deposition by the driver at the ablative front. In the present study, we treat $g(t)$ and $v_A(t)$ as prescribed functions. The first-order derivative term in Eq. (1) can be transformed away by the following transformation from η to ξ:

$$\eta=\xi\exp\left(-\frac{1}{2}\int_0^t kv_A(t')\,dt'\right). \tag{2}$$

In terms of ξ, the differential equation is

$$\frac{d^2\xi}{dt^2}-\left[kAg+\frac{1}{4}k^2v_A^2+\frac{k}{2}\frac{dv_A}{dt}\right]\xi=0. \tag{3}$$

From Eq. (2), it is evident that η is more stable than ξ due to the factor $\exp(-\frac{1}{2}\int_0^t kv_A(t')\,dt')$, which is the well-known effect of ablative stabilization. Once the ablative velocity $v_A(t)$ is prescribed, this stabilization effect is determined, and we only need to focus on the dynamics of ξ.

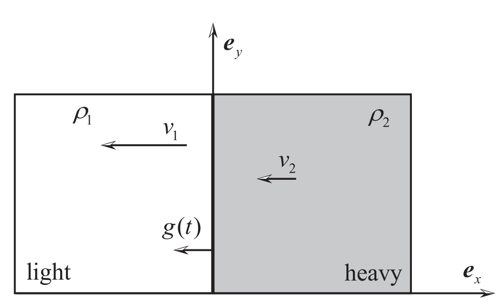

Fig. 3. Sharp-boundary model for the ablative Rayleigh–Taylor instability.

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The coefficient of ξ in Eq. (3) can be viewed as a time-dependent drive for ξ. To separate the time-dependent part of the drive from the time-independent part, we write

$$g(t)=g_0+\delta g(t), \quad v_A(t)=v_{A0}+\delta v_A(t). \tag{4}$$

Then, it can be shown that

$$kAg+\frac{1}{4}k^2v_A^2+\frac{k}{2}\frac{dv_A}{dt}=\gamma_0^2+\delta\gamma^2 \tag{5}$$

$$\gamma_0\equiv\sqrt{kAg_0+\tfrac{1}{4}k^2v_{A0}^2} \tag{6}$$

where γ_0 is the growth rate when there is no time-modulation, and $\delta\gamma^2$ is the time-dependent part of the drive. If we normalize the time t by the $1/\gamma_0$, then by using the normalized time $s\equiv t\gamma_0$, Eq. (3) can be simplified to give

$$\frac{d^2\xi}{ds^2}-h(s)\xi=0 \tag{7}$$

$$h(s)\equiv 1+\delta h(s), \quad \delta h(s)\equiv\delta\gamma^2/\gamma_0^2. \tag{8}$$

We assume here that $\delta h(s)$ has a prescribed functional form determined by the time variation of the beam energy. According to the study by Betti et al. [17] and Takabe et al. [23], the typical size of $\delta h(t)$ is in the range of $3.5\le\delta h(t)\le 5.5$. We will use Eq. (7) to study the dynamic stabilization of the ablative RT instability with a time-dependent drive in the next two sections.

3. Courant–Snyder theory

Eq. (7) is a second-order ordinary differential equation with a time-dependent coefficient. It describes a harmonic oscillator with time-dependent spring constant, which can be viewed as the second simplest physics problem and has many important applications [24,25]. The well-studied Matthew's equation is a special case of Eq. (7). If $\delta h(s)$ is piece-wise constant or a series of δ-functions, then the solution of Eq. (7) can be constructed piece-wise [20]. However, it is not desirable to restrict to a specific class of functions, since our goal is to find the most optimal functional form of the modulation such that the dynamic stabilization effect is maximized.

It turns out that the Courant–Synder theory for a second-order ordinary differential equation with a time-dependent coefficient is an effective tool to tackle Eq. (7), even though the Courant–Synder theory [22] was first developed for stable charged particle dynamics in a focusing lattice. It applies to the unstable case studied here with only little modification. Here we list the main result of the Courant–Synder theory without a detailed derivation, which can be found in Refs. [22,26].

The solution of Eq. (7) can be expressed as a linear map $M(s)$ of the initial conditions $(\xi_0,\dot\xi_0)$ at $s=s_0$ [22,26], *i.e.*,

$$\begin{pmatrix}\xi\\ \dot\xi\end{pmatrix}=M(s)\begin{pmatrix}\xi_0\\ \dot\xi_0\end{pmatrix}. \tag{9}$$

The linear map is given by

$$M(s)=\begin{pmatrix}w & 0\\ \dot w & \frac{1}{w}\end{pmatrix}\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}w_0^{-1} & 0\\ -\dot w_0 & w_0\end{pmatrix} \tag{10}$$

where $w(s)$ is a solution of the envelope equation

$$\frac{d^2w}{ds^2}-h(s)w=w^{-3} \tag{11}$$

with initial conditions $(w_0,\dot w_0)$ at $s=s_0$, and $\phi(s)$ is the phase advanced associated with $w(s)$:

$$\phi(s)=\int_{s_0}^{s}\frac{1}{w^2(s')}\,ds'. \tag{12}$$

In general, we can choose $h(s)$ to be a periodic function of $s\equiv\gamma_0t$ with normalized period $S=\gamma_0T$, where T is the unnormalized period. Then the one-period map $M(S)$ completely determines the dynamic behavior of ξ. The eigenvalues of $M(S)$ determines the eigenfrequencies of the dynamics of ξ. In particular, let μ denote the eigenvalue of $M(S)$ with the largest absolute value, then the growth rate of ξ is given by $\ln|\mu|$. Using the symmetry properties of the envelope equation [24], it can be proven that the eigenvalues of $M(S)$ are independent of the choice of the initial time s_0 and initial conditions. Therefore, any one-period solution of Eq. (11) from $s=s_0$ to $s=s_0+S$ for any initial condition $(w_0,\dot w_0)$ can be used to calculate the growth rate $\ln|\mu|$ of the ξ dynamics.

4. Dynamic stabilization of the ablative Rayleigh–Taylor instability

In this section, we apply the Courant–Snyder theory outlined in Section 3 to calculate the growth rate of the transformed interface displacement ξ for different choices of the modulation function $\delta h(s)$ with the form

$$h(s)=1+\delta h(s)=1+q\ \sin(2\pi s/S) \tag{13}$$

where $s\equiv\gamma_0t$ is the normalized time, q is the modulation amplitude, and $S\equiv\gamma_0T$ is the normalized period. The modulation amplitude is selected to be in the range of $0<q<6$.

Shown in Fig. 4 is the growth rate plotted as a function of the modulation amplitude q for two different periodicities corresponding to $S=1$ and $S=2$. It is clear that larger modulation amplitude results in a larger reduction in growth rate, as expected. For a modulation with a period twice the unmodulated e-folding time, $S\equiv\gamma_0T=2$, the instability can be completely suppressed when the modulation amplitude q reaches 4.6. Comparing the two curves in Fig. 4, we note that a slower modulation generates a larger reduction of growth rate. This fact is further demonstrated in Fig. 5, where the growth rate is plotted as a function of the periodicity S for two different modulation amplitudes. For the case of $q=6$, the instability can be stabilized when $S=1.5$. We note that the slope of the curve near $S=1.5$ is steep, indicating a sensitive functional dependence of the growth rate on the periodicity $S\equiv\gamma_0T$. The complex functional dependence is further illustrated in Fig. 6, where two modulations with different amplitudes and periodicities are applied simultaneously. An interesting synergy is observed. For the first modulation

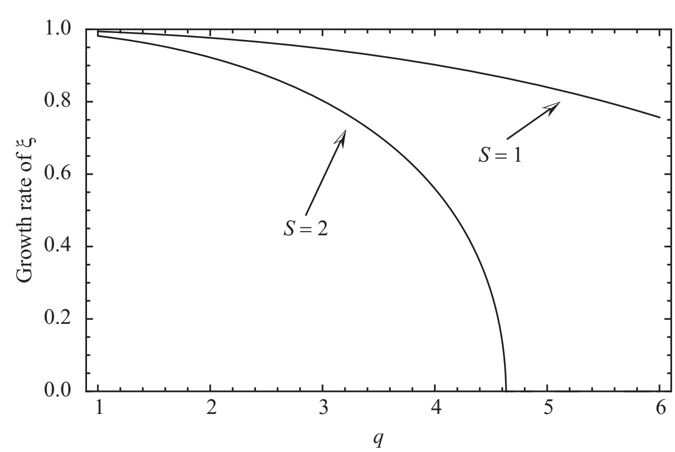

Fig. 4. Growth rate plotted as a function of the modulation amplitude q for normalized period $S=1$ and $S=2$.

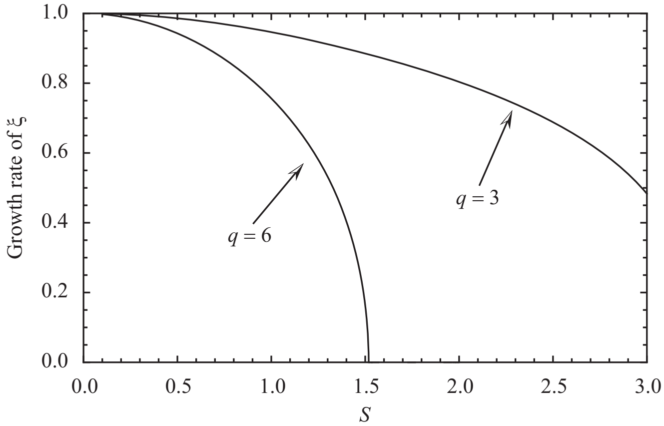

Fig. 5. Plots of growth rate as a function of the periodicity $S \equiv \gamma_0 T$ for $q=6$ and $q=3$.

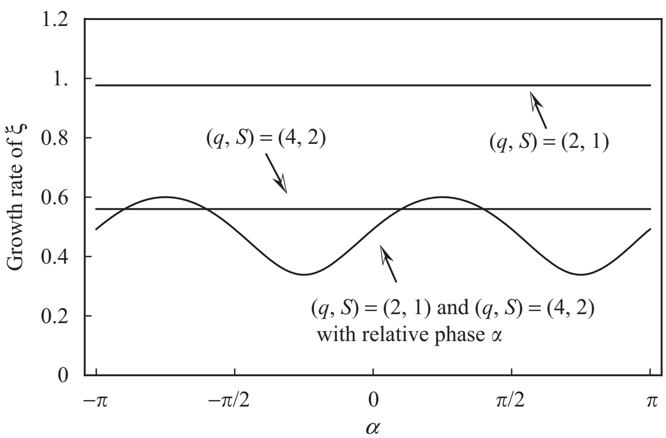

Fig. 6. Plots showing the synergy between two modulations with different periodicities and amplitudes. The wavy curve is the growth rate as a function of the relative phase α for the modulation $h(s)=1+2\,\sin(2\pi s)+4\,\sin(\pi s+\alpha)$.

with $(q,S)=(2,1)$ there is almost no reduction in growth rate. For the second modulation with $(q,S)=(4,2)$, the reduction is about 44%. However, when the two modulations are applied together with a relative phase α, *i.e.*, $h(s)=1+2\,\sin(2\pi s)+4\,\sin(\pi s+\alpha)$, the reduction reaches 67% provided the relative phase α is chosen correctly. This reduction in growth rate is much larger than the sum of the reductions due to the two modulations when applied separately. Furthermore, when the relative phase α between the two modulations is not selected correctly, the reduction can be even smaller than that when the second modulation is applied alone. These results imply that when a wobbler system for heavy ion fusion drivers is designed, it is necessary to carry out a thorough optimization of the modulation waveform, such that the dynamic stabilization effect can be maximized for a given modulation amplitude. For heavy ion fusion targets, the expected growth rate of the ablative RT instability is $\gamma_0 \sim 2\times 10^9$ Hz. The case of $S=1$ corresponds to a modulation frequency of 13 GHz. If the wobbler plates are placed at the beam upstream before the drift compression, then the frequency of the wobbler field required is in the range of 130 MHz, if we assume a typical longitudinal compression ratio of 100.

5. Conclusions and future work

To conclude, we have studied the dynamic stabilization of the ablative Rayleigh–Taylor instability induced by a beam wobbler system that can deliver a time-modulated energy deposition on the ablation front. Using a sharp-boundary model for the ablative Rayleigh–Taylor instability and Courant–Synder theory, we have shown, with an appropriately chosen modulation waveform, that the instability can be completely stabilized in certain parameter regimes. It is found that the stabilization effect has a strong dependence on the modulation frequency. Modulation with frequency comparable to the growth rate is most effective in terms of stabilizing the instability. It is also found that the reduction of the growth rate has a complex dependence on the modulation waveform. For example, a modulation with two frequency components can result in a reduction of the growth rate larger than the sum of the reductions due to the two components when applied separately.

The sharp-boundary model reduces the collective dynamics to a second-order ordinary differential equation for the displacement of the interface with a time-dependent coefficient. Because it is a system with one degree of freedom, the analysis of the dynamics is greatly simplified. In principle, the dynamic stabilization mechanism should also be applicable when more degrees of freedom are allowed. Generalization of the analysis to higher dimensions [27–29] can include more physical effects, such as compression and heat conductivity, in the system, and thus increases the fidelity of the model. It is also possible to develop numerical simulation methods for the dynamic stabilization process in a more realistic geometry with smooth density gradient, which corresponds to a dynamic system with infinite degrees of freedom. Progress in these directions will be reported in the future.

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