CHARGE AND CURRENT NEUTRALIZATION OF AN ION BEAM PULSE BY BACKGROUND PLASMA IN THE PRESENCE OF APPLIED MAGNETIC FIELD*

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Abstract

Plasma can be used as a convenient medium for manipulating intense charged particle beams, e.g., for ballistic focusing and steering, because the plasma can effectively reduce the self-space charge potential and self-magnetic field of the beam pulse. We previously developed a reduced analytical model of beam charge and current neutralization for an ion beam pulse propagating in a cold background plasma. The reduced-fluid description provides an important benchmark for numerical codes and yields useful scaling relations for different beam and plasma parameters. This model has been extended to include the additional effects of an applied solenoidal magnetic field. Simulations show that the self-magnetic field structure of the ion beam pulse propagating through background plasma can be complex and non-stationary. The linear system of Maxwell's equations for the self-electromagnetic fields can be solved analytically in Fourier space. For a strong enough applied magnetic field, poles emerge in Fourier space. These poles are an indication that whistler and low-hybrid waves can be excited by the beam pulse.

INTRODUCTION

Background plasma can be used as an effective neutralization scheme to transport and compress intense charged particle beam pulses. To neutralize the large repulsive space-charge force of the beam particles, the beam pulses can be transported through a background plasma. The plasma electrons can effectively neutralize the beam charge, and the background plasma can provide an ideal medium for beam transport and focusing. Neutralization of the beam charge and current by a background plasma is an important issue for many applications involving the transport of fast particles in plasmas, including astrophysics, accelerators, and inertial fusion, in particular heavy ion fusion [1] and fast ignition [2], etc.

The application of a solenoidal magnetic field allows additional control and focusing of beam pulses. A strong magnetic lens with a magnetic field up to a few Tesla can effectively focus beams in short distances order of a few tens of centimeters. However, due to the very strong magnetic field in the solenoid, the magnetic field leaking outside the solenoid can affect the degree of charge and current neutralization. In this paper, we show that even a small solenoidal magnetic field, typically less than 100G, strongly changes the self-magnetic and self-electric fields in the beam pulse propagating in a background plasma. Such values of magnetic field can be present over distances of a few meters from the strong solenoid, and thereby can affect the focusing of the beam pulse. Moreover, an additional small solenoidal magnetic field can be applied to optimize propagation of a beam pulse through a background plasma over long distances [3].

Because, the detailed parameter values for heavy ion fusion drivers are not well prescribed at the present time, an extensive study is necessary for a wide range of beam and plasma parameters to determine the conditions for optimum beam propagation [4]. To complement the numerical simulation studies [5,6], a number of reduced models have been developed. Based on well-verified assumptions, reduced models can yield robust analytical and numerical descriptions and provide important scaling laws for the degrees of charge and current neutralization [7]. In this paper an analytical model is developed to describe the self-electromagnetic fields of a finite-length beam pulse propagating in a cold background plasma in a solenoidal magnetic field. The applied magnetic field is directed along the ion beam velocity. In Ref. [3] a slice model is developed for the self-magnetic field of an ion beam pulse. Simulations [6] and analytical models [3] show that when the magnitude of the applied magnetic field exceeds a certain limit, whistler and lower-hybrid waves are excited by the beam pulse.

BASIC EQUATIONS AND RESULTS

To investigate the onset of wave generation, a two-dimensional analytical model has been developed in the linear approximation for slab geometry, assuming that the beam density, n_b, is small compared with the plasma density, n_p ($n_b << n_p$). This study allows us to verify the assumptions in the slice model of Ref. [3] for a cylindrical beam. We consider a configuration where the beam pulse propagates along a uniform magnetic field with $\mathbf{B} = B_{z0}\hat{z}$. We choose coordinates, without loss of generality, so that $\mathbf{k} = k_z\hat{z} + k_x\hat{x}$, i.e, $k_y = 0$. We look for a stationary solution in the beam frame, so that $\omega = \beta_b c k_z$ is the effective frequency of the beam pulse, c is the speed of light, and $\beta_b c$ is the beam velocity. In the linear regime, Maxwell's equations can be combined to produce a set of equations that describes the electromagnetic fields produced by the ion beam pulse. Applying a Fourier transform to this system gives

$$k^2\mathbf{E} - \mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - \frac{\omega^2}{c^2}\varepsilon\mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{j}_b , \qquad (1)$$

where $\mathbf{j}_b = eZ_b\beta_b c n_b\hat{z}$ is the beam current density, and ε is the plasma dielectric tensor given by

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$$\varepsilon = \begin{pmatrix} \varepsilon_{\perp} & i\varepsilon_{\angle} & 0 \\ -i\varepsilon_{\angle} & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, \quad \begin{aligned} \varepsilon_{\perp} &= 1-\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} \\ \varepsilon &= 1-\frac{\omega_{pe}^2}{\omega^2} \\ \varepsilon_{\angle} &= \frac{\omega_{ce}\omega_{pe}^2}{\omega\left(\omega^2-\omega_{ce}^2\right)} \end{aligned}, \tag{2}$$

where, $\omega_{pe} = \left(4\pi e^2 n_p / m\right)^{1/2}$ is the plasma frequency, and $\omega_{ce} = eB_{z0}/mc$ is the electron cyclotron frequency. The beam density profile is decomposed in Fourier space characterized by wavevector $\mathbf{k}$. These equations can be solved to obtain the three components of the electric field E_x, E_y, E_z. Faraday's Law then provides the components of the self-magnetic field, B_y and the perturbation in the solenoidal magnetic field δB_z. The radial force acting on the beam ions, $F_x = e\left(E_x - \beta_b B_y\right)$ depends on the self-magnetic and electric fields, which are given by

$$B_y(\mathbf{k}) = -\frac{4\pi i e n_b(\mathbf{k})\beta_b k_x}{k_x^2 - \beta_b^2 k_z^2 \varepsilon / G}, \tag{3}$$

where

$$G = 1-\frac{1}{1-\beta_b^2\varepsilon_{\perp} - H}, \quad H = \frac{\left(\beta_b^2 k_z \varepsilon_{\angle}\right)^2}{k^2 - \beta_b^2 k_z^2 \varepsilon_{\perp}}, \tag{4}$$

$$E_x(\mathbf{k}) = -\frac{4\pi i e k_x n_b(\mathbf{k})}{k_x^2(\varepsilon_{\perp} + H/\beta_b^2) + k_z^2\varepsilon\left(1-\beta_b^2\varepsilon_{\perp} - H\right)}, \tag{5}$$

If a magnetic field is not applied, then $G = 1$ and $B_y(\mathbf{k}) = -\frac{4\pi i e n_b(\mathbf{k})\beta_b k_x}{k_x^2 + \omega_{pe}^2/c^2}$, which describes the screening of the magnetic field over a skin depth c/ω_{pe}. Analysis of Eqs. (3)-(5) shows that the application of a solenoidal magnetic field affects the self-electric and self-magnetic fields when (see also Ref. 3)

$$\omega_{ce} > \frac{\beta_b}{\sqrt{1-\beta_b^2}}\omega_{pe}. \tag{6}$$

Comparison of analytical theory and particle-in-cell simulation results

The analytical model has been verified by comparison with numerical simulations. Figure 1 shows a comparison between analytical theory and particle-in-cell simulations for the self-magnetic field, the perturbation in solenoidal magnetic field, and the radial electric field in the ion beam pulse. The beam velocity is $V_b = 0.33c$, and the beam density profile is gaussian, $n_{b0}\, exp(-r^2/r_b^2 - z^2/l_b^2)$, with $r_b = 1cm$, $l_b = 17cm$.

Onset of wave generation by the ion beam pulse

As the strength of the applied magnetic field increases, a wave pattern begins to appear in the self-magnetic field profile. The existence of wave excitations corresponds to the effect of poles in the denominator of Eq.(3) for the Fourier representation of the magnetic field. In the limit $\omega \ll \omega_{ce}$ and $k_z \ll k_x$, the zeros of the denominator are determined from:

$$\frac{\omega_{pe}^2+\omega_{ce}^2}{\omega_{ce}^2}k_x^4 + \left(\frac{2\omega_{pe}^4}{c^2\omega_{ce}^2} - \frac{\omega_{pe}^2(1-\beta_b^2)}{c^2\beta_b^2}\right)k_x^2 + \frac{\omega_{pe}^6}{c^4\omega_{ce}^2} = 0, \tag{7}$$

and poles appear when

$$\omega_{ce} > \frac{2\beta_b}{1-\beta_b^2}\omega_{pe}. \tag{8}$$

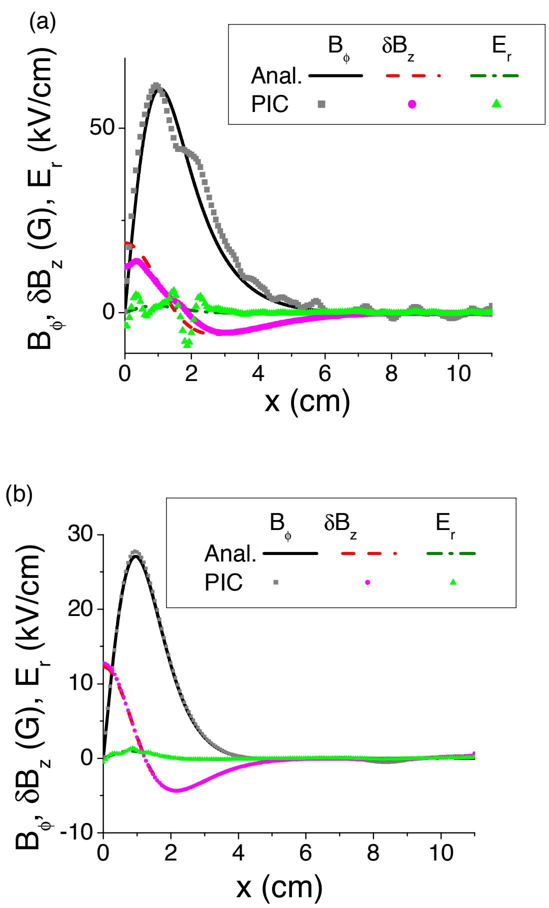
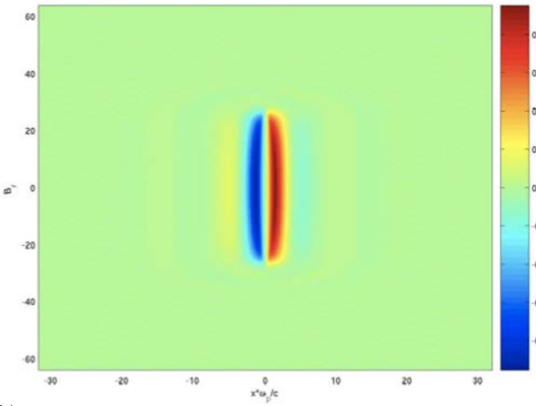


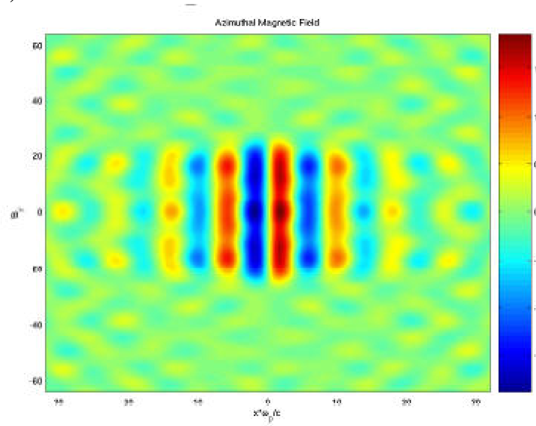
Figure 1 Comparison of analytical theory and particle-in-cell simulation results for the self-magnetic field; perturbation in the solenoidal magnetic field; and the radial electric field in a perpendicular slice of the beam pulse. The beam parameters are (a) $n_{b0} = n_p/2 = 1.2 \times 10^{11} cm^{-3}$; and (b) $n_{b0} = n_p/8 = 0.6 \times 10^{11} cm^{-3}$. The values of the applied solenoidal magnetic field, B_{z0} are: (a) $B_{z0} = 300G$; and (b) $B_{z0} = 600G$.

Figure 2 shows the self-magnetic field of a beam pulse for different values of applied magnetic field. Because $\beta_b = 0.5$, **t**he waves should appear for $\omega_{ce} > (4/3)\omega_{pe}$ according to Eq.(8). Indeed, the onset of wave generation is evident for $\omega_{ce} > (4/3)\omega_{pe}$.

(a)



(b)



(c)

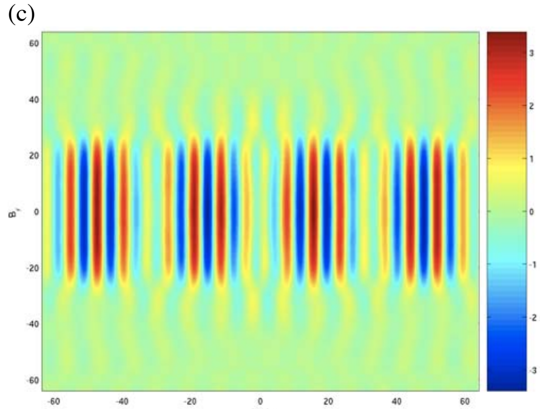


Figure 2 The two-dimensional profile of the self-magnetic field ($eB_y / 2mc\omega_{pe}$) calculated according to Eq.(3) for a beam pulse with a flat-top profile and $r_b = c/\omega_{pe}$, l_b=20 c/ω_{pe}, and $\beta_b = 0.5$ for different values of the applied solenoidal magnetic field corresponding to (a) $\omega_{ce} = \omega_{pe}$, (b) $\omega_{ce} = 1.32\omega_{pe}$, and (c) $\omega_{ce} = 1.35\omega_{pe}$.

CONCLUSIONS

In summary, an analytical model has been developed to describe the self-electromagnetic fields of a finite-length beam pulse propagating in a cold background plasma in a solenoidal magnetic field. Application of a solenoidal magnetic field strongly affects the degree of current and charge neutralization whenever

$$\omega_{ce} > \frac{\beta_b}{\sqrt{1-\beta_b^2}}\omega_{pe}. \qquad (9)$$

The threshold value of the applied magnetic field given in Eq.(9) is relatively small for nonrelativistic beams, of the order of 100G. For larger values of the solenoidal magnetic field, corresponding to

$$\omega_{ce} > \frac{2\beta_b}{1-\beta_b^2}\omega_{pe}, \qquad (10)$$

poles in the Fourier representation of the magnetic field appear, which confirm that the beam pulse generates whistler and lower-hybrid waves. When the value of the applied solenoidal magnetic field is below this threshold, the results of the two-dimensional model agree with the slab approximation for long beam pulses developed in Ref. 3. However, for higher values of the magnetic field, there is a nontrivial z-dependence. There are, for example, signatures of collective excitations in the longitudinal direction.

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