Anisotropy-Driven Instability in Intense Charged Particle Beams*

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Collective Instabilities Can Lead to Temperature Equilibration

- Temperature anisotropies develop naturally in accelerators.

- This provides free energy to drive classical electrostatic Harris instability.

- Instability may lead to a deterioration of the beam quality.

- The instability leads to an increase in the longitudinal velocity spread.

- The instability acts much faster then collisions.

Harris Instability in Eclectically Neutral Plasma with Uniform Magnetic Field

- Anisotropic electron distribution is required

$$T_{||b}/T_{\perp b} < 1/2.$$

- Plasma must be sufficiently dense that

$$\omega_{pe} > \omega_{ce},$$

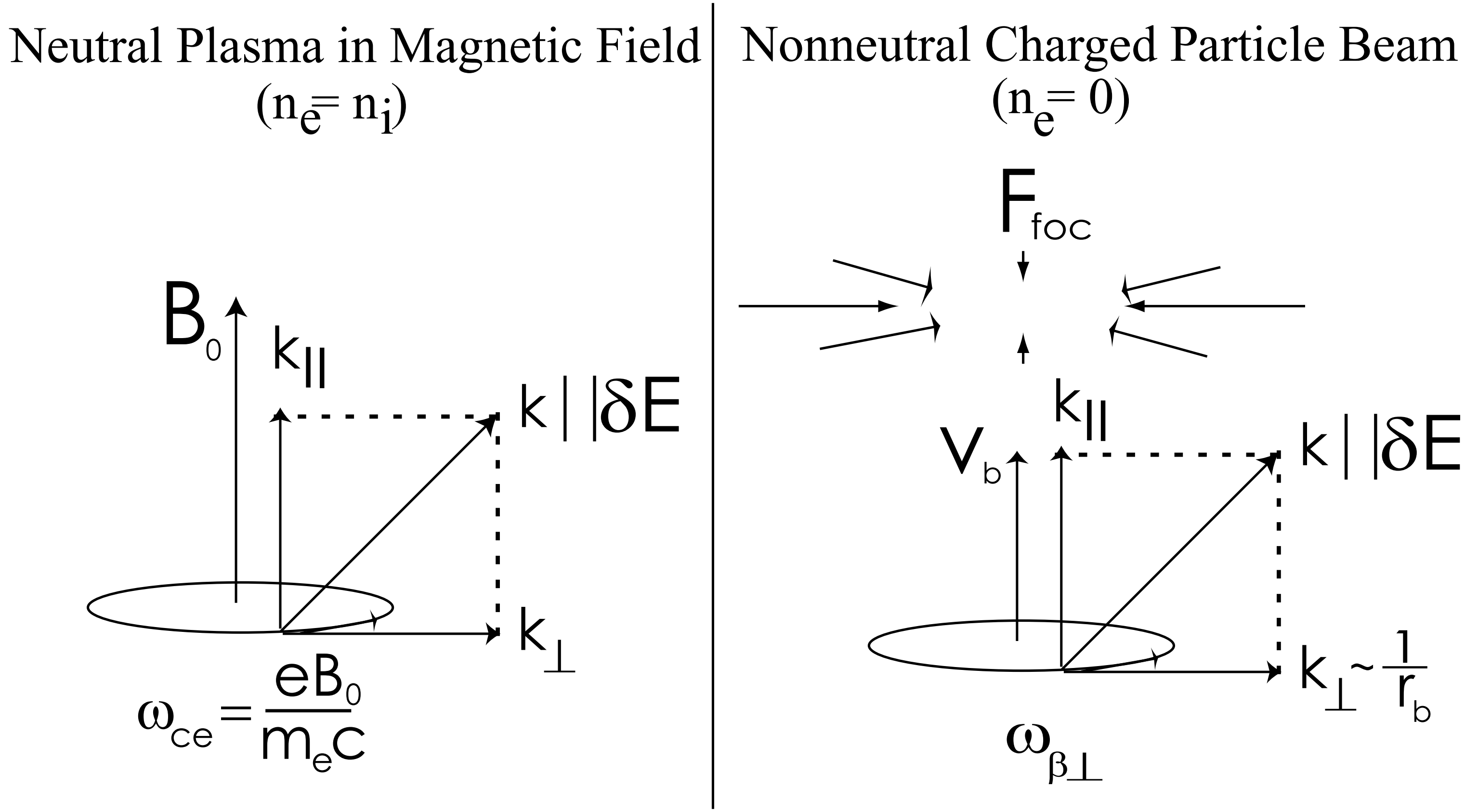
where $\omega_{pe} = (4\pi e^2 n/m)^{1/2}$ is the electron plasma frequency and $\omega_{ce} = eB/mc$ is the electron cyclotron frequency.

- Instability is very fast

$$\gamma \sim \omega_{pe}.$$

*E. G. Harris, Phys. Rev. Lett. **2**, 34 (1959).

Harris Instability in Intense One-Component Beams



$$\omega_{\mathbf{ce}} \longrightarrow \omega_{\beta\perp}.$$

- For good coupling need

$$\mathbf{k_z \sim k_\perp \sim 1/r_b}.$$

- For instability need $\omega_{\beta\perp} < \omega_{\mathbf{pb}}$.

Temperature Anisotropies ($T_{||b} << T_{\perp b}$) Develop Naturally in Accelerators

- For particles accelerated by a voltage V

$$m_b \Delta v_{bi}^2/2 = m_b \left(\frac{2e_b V}{m_b}\right)^{1/2} \Delta v_{bf},$$

- Temperature is proportional to velocity-squared

$$T_{||bf} = T_{||bi}^2/2e_b V.$$

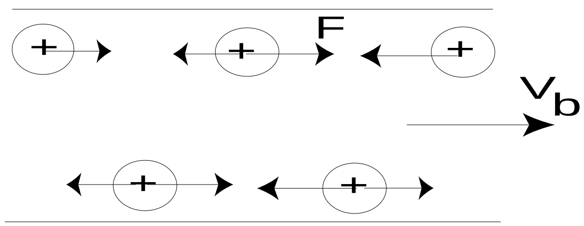
- For example for $T_{||i} = 1eV$, $e_b V = 1MeV$, $T_{||f} = 5 \times 10^{-7} eV$

Previous Studies of Temperature Anisotropy Instability in Intense One-Component Beams

- Analytical linear theory by Wang and Smith (1982)
  - axisymmetric perturbations
  - Kapchinskij – Vladimirskij (KV) distribution
  - $T_{||b}/T_{\perp b} = 0$.
- 3D PIC simulations with WARP code
  - Friedman, et. al.(1990) observed a rapid temperature 'equilibration' process of KV beam with $T_{||b}/T_{\perp b} \ll 1$.
  - Lund, et. al.(1998) looked at growth rates, mode frequencies and instability thresholds. Used a semi-gaussian as well as KV distributions.
- Drawbacks:
  - WARP PIC code is noisy.
  - KV distribution has a highly unphysical (inverted) population in transverse phase-space variables.
  - Semi-gaussian distribution is not an equilibrium.

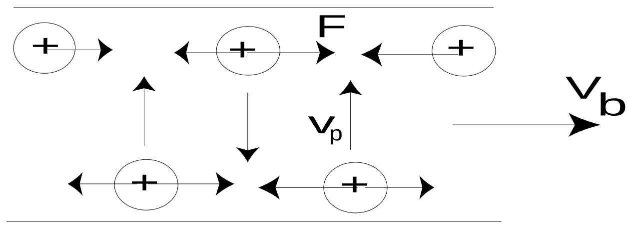
Instability Mechanism

$\omega_p >> \omega_{\beta\perp}$



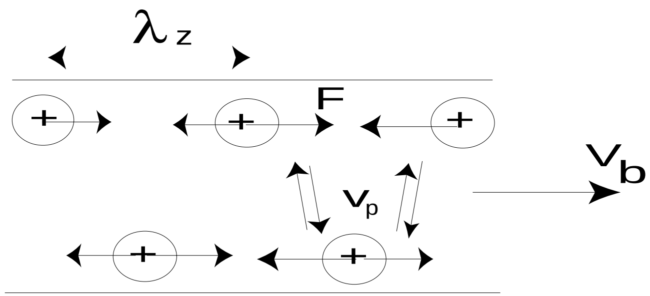
$\omega = \omega_p$

$\omega_p << \omega_{\beta\perp}$



$\omega = \omega_{\beta\perp}$

$\omega_p = \omega_{\beta\perp}$



Re $\omega = 0$ (unstable)

$$\lambda_z > v_{||}^{th} \frac{2\pi}{\omega_{\beta\perp}} \qquad \Rightarrow \qquad \frac{T_{||}}{T_{\perp}} < \frac{1}{k_z^2 r_b^2}$$

Simplified Model

- Use equivalent KV beam to illustrate instability mechanism.
- Dipole mode with $k_z^2 r_b^2 \gg 1$ has the highest growth rate.

$$\delta\phi(\mathbf{x}, t) = \hat{\phi}\frac{x_\perp}{r_b}\,\mathrm{exp}(ik_z z - i\omega t)$$

- Electric field is mostly longitudinal

$$E_z = -ik_z\delta\phi \sim x_\perp\,\mathrm{exp}(ik_z z - i\omega t)$$

- Transverse betatron oscillation

$$x_\perp(t) = \hat{x}\cos(\omega_{\beta\perp} t + \alpha_0)$$

Perimetrically couples through electric field to drive longitudinal motion

Longitudinal Oscillations are Modulated by Betatron Oscillations

- Longitudinal equation of motion for a beam particle becomes

$$\ddot{z} = -ik_z \frac{e_b}{m_b} \widehat{\phi} \frac{\widehat{x}}{r_b} \cos(\omega_{\beta\perp} t + \alpha_0) e^{ik_z z_0 - i\omega t}$$

- Integrating with respect to time, we obtain

$$z_\alpha = ik_z \frac{e_b}{m_b} \widehat{\phi} \frac{\widehat{x}}{2r_b} \left[\frac{e^{i\alpha}}{(\omega - \omega_{\beta\perp})^2} + \frac{e^{-i\alpha}}{(\omega + \omega_{\beta\perp})^2} \right] e^{ik_z z_0 - i\omega t},$$

$$\alpha = \alpha_0 + \omega_{\beta\perp} t.$$

- Individual particle motion has two characteristic frequencies,

$$\omega - \omega_{\beta\perp} \text{ and } \omega + \omega_{\beta\perp}.$$

Dispersion relation

- Average displacement $\langle z\rangle(x_\perp, z, t) = (z_\alpha + z_{-\alpha})/2$ is

$$\langle z\rangle(x,z,t) = -\frac{e_b \delta E_z}{2m_b}\Big[\frac{1}{(\omega-\omega_{\beta\perp})^2} + \frac{1}{(\omega+\omega_{\beta\perp})^2}\Big].$$

- Displacement creates restoring electric field

$$\delta E_z = -4\pi e_b n_b \langle z\rangle.$$

- Combining, we obtain the dispersion relation

$$1 = \frac{\bar{\omega}_{pb}^2}{2}\left[\frac{1}{(\omega-\omega_{\beta\perp})^2} + \frac{1}{(\omega+\omega_{\beta\perp})^2}\right].$$

- Here, the beam plasma frequency-squared is defined as

$$\bar{\omega}_{pb}^2 = \frac{4\pi e_b^2 n_b}{m_b}.$$

Intensity Threshold for the Instability

- Solution of the dispersion equation is

$$\left(\frac{\omega}{\omega_f}\right)^2 = 1 \pm \sqrt{\left[1 - \left(\frac{\bar{\nu}}{\nu_0}\right)^2\right]\left[1 + 3\left(\frac{\bar{\nu}}{\nu_0}\right)^2\right]}.$$

where we have introduced the normalized depressed tune

$$\frac{\bar{\nu}^2}{\nu_0^2} \equiv \frac{\omega_{\beta\perp}^2}{\omega_f^2} = 1 - \frac{\bar{\omega}_{pb}^2}{2\omega_f^2}.$$

- The mode with lower sign is unstable and purely growing for

$$\frac{\bar{\nu}}{\nu_0} < \frac{\bar{\nu}}{\nu_0}^{th} = \sqrt{\frac{2}{3}} \approx 0.82.$$

- Maximum growth rate

$$\frac{(Im\omega)^{max}}{\omega_f} = \sqrt{\frac{2}{\sqrt{3}} - 1} \approx 0.3 \quad \text{for} \quad \frac{\bar{\nu}}{\nu_0}^{max} = \sqrt{\frac{1}{3}} \approx 0.58.$$

Numerical Codes are Required for Detailed Investigation

- Regular PIC codes (such as WARP) are too noisy.
- Need codes capable of simulating the linear as well as nonlinear stages.
- Linear codes:
  - Eigenvalue code, Beam Eigenmode And Spectra (bEASt) Code.
  - linearized δf PIC, Beam Equilibrium Stability and Transport (BEST) Code.
- Nonlinear code:
  - nonlinear δf PIC (BEST) Code.

Description of Beam Eigenmode And Spectra (bEASt) Code

- Electrostatic perturbations of the form

$$\delta\phi(\mathbf{x}, t) = \widehat{\delta\phi}(r)\exp(im\theta + ik_z z - i\omega t)$$

- Equilibrium distribution

$$f_b^0(r, \mathbf{p}) = \frac{\widehat{n}_b}{(2\pi m_b)^{3/2} T_{\perp b} T_{||b}^{1/2}} \exp\left(-\frac{H_\perp}{T_{\perp b}} - \frac{p_z^2}{2m_b T_{||b}}\right).$$

- Perturbation is expanded into the complete set of vacuum eigenfunctions

$$\widehat{\delta\phi}(r) = \sum_n \alpha_n J_m\left(\frac{\lambda_n r}{r_w}\right),$$

  where $J_m(\lambda_n) = 0$.

- Using the method of characteristics, analysis of the linearized Vlasov-Maxwell equations leads to an infinite dimension matrix dispersion equation

$$\sum \alpha_n D_{n,m}(\omega) = 0$$

Description of Beam Equilibrium Stability and Transport (BEST) Code

- The solutions to the nonlinear Vlasov-Maxwell equations are expressed as

$$f_b = f_b^0 + \delta f_b, \qquad \phi = \phi^0 + \delta\phi,$$

where (f_b^0, ϕ^0, A_z^0) are known equilibrium solutions $(\partial/\partial t = 0)$.

- Use particles (markers) to represent only $\delta f_b(\mathbf{x}, \mathbf{p}, t)$ part of the distribution

$$\delta f_b = \frac{N_b}{N_{sb}} \sum_{i=1}^{N_{sb}} w_{bi} \delta(\mathbf{x} - \mathbf{x}_{bi}) \delta(\mathbf{p} - \mathbf{p}_{bi}),$$

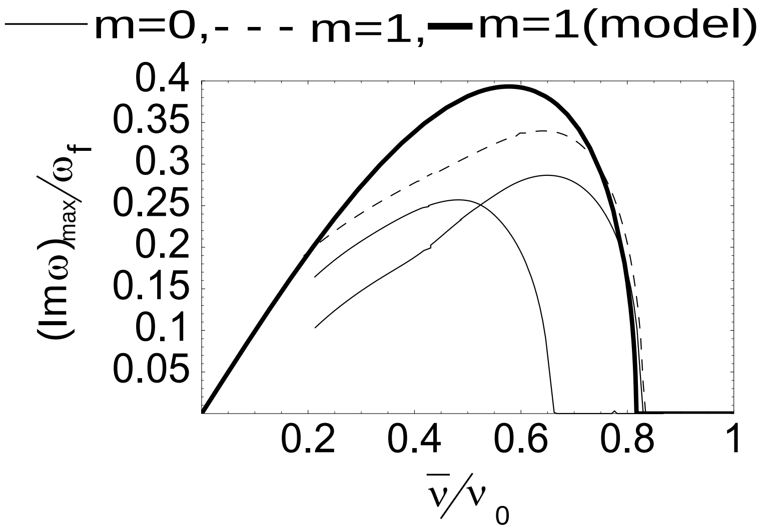
$$w_b \equiv \delta f_b / f_b$$

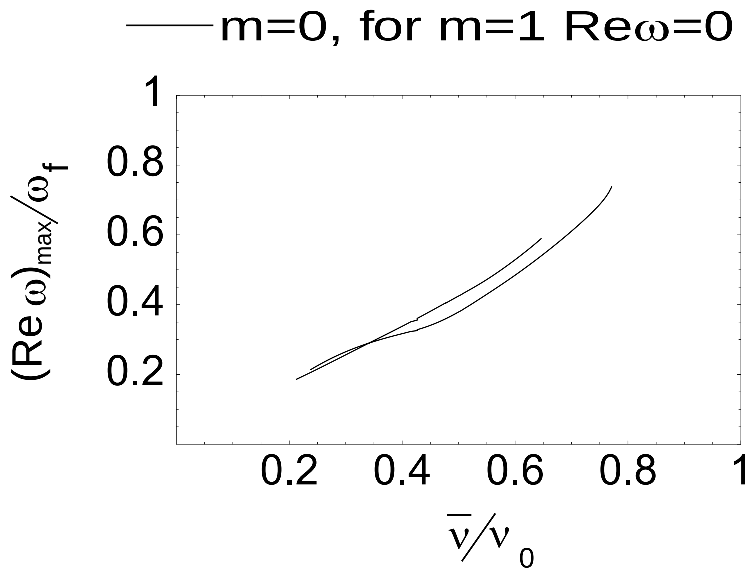
- Noise associated with f_b^0 is removed.

- The typical gain in accuracy is

$$\epsilon_{\delta f} / \epsilon_f = \bar{w}_{bi}.$$

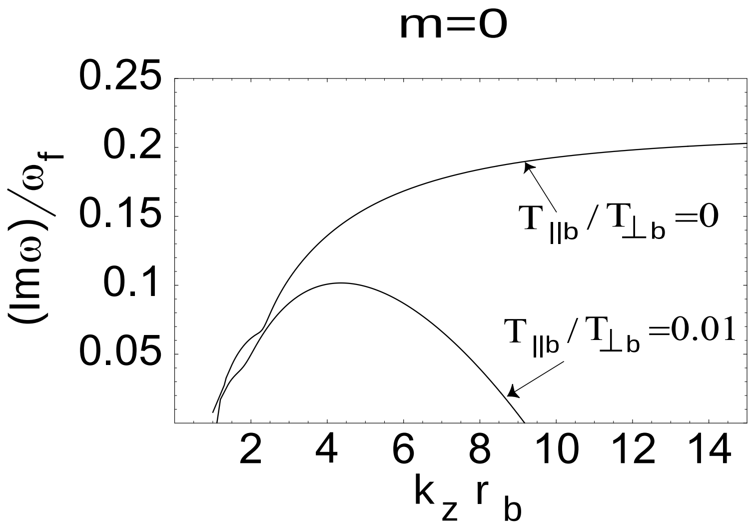
The $m = 1$ Dipole Mode has the Highest Growth Rate

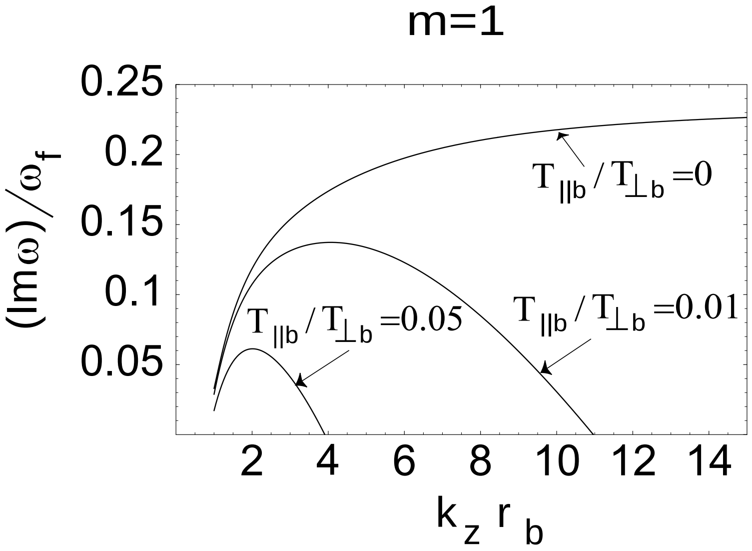




- $r_w = 3r_b$, $T_{||b}/T_{\perp b} = 0$
- m=1 mode is purely growing $Re\omega = 0$ and $(Im\omega)_{max}/\omega_f \simeq 0.34$ for $\bar{\nu}/\nu_0 \simeq 0.62$.
- The instability is absent for $\bar{\nu}/\nu_0 > 0.82$.

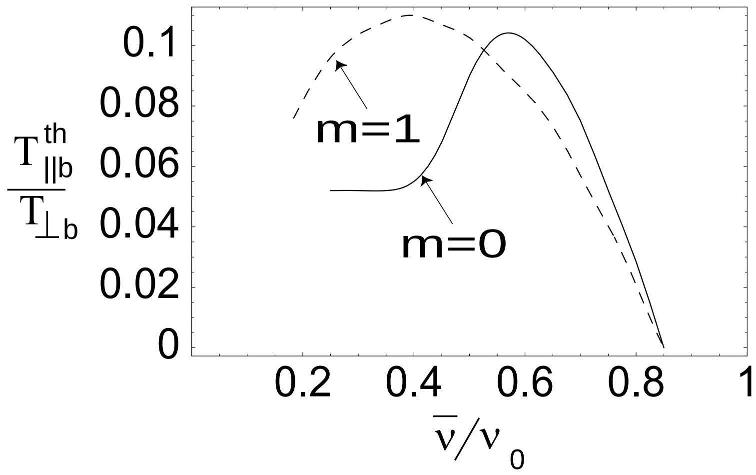
Short Wavelength Modes $k_z^2 r_b^2 \gg 1$ are Landau Damped





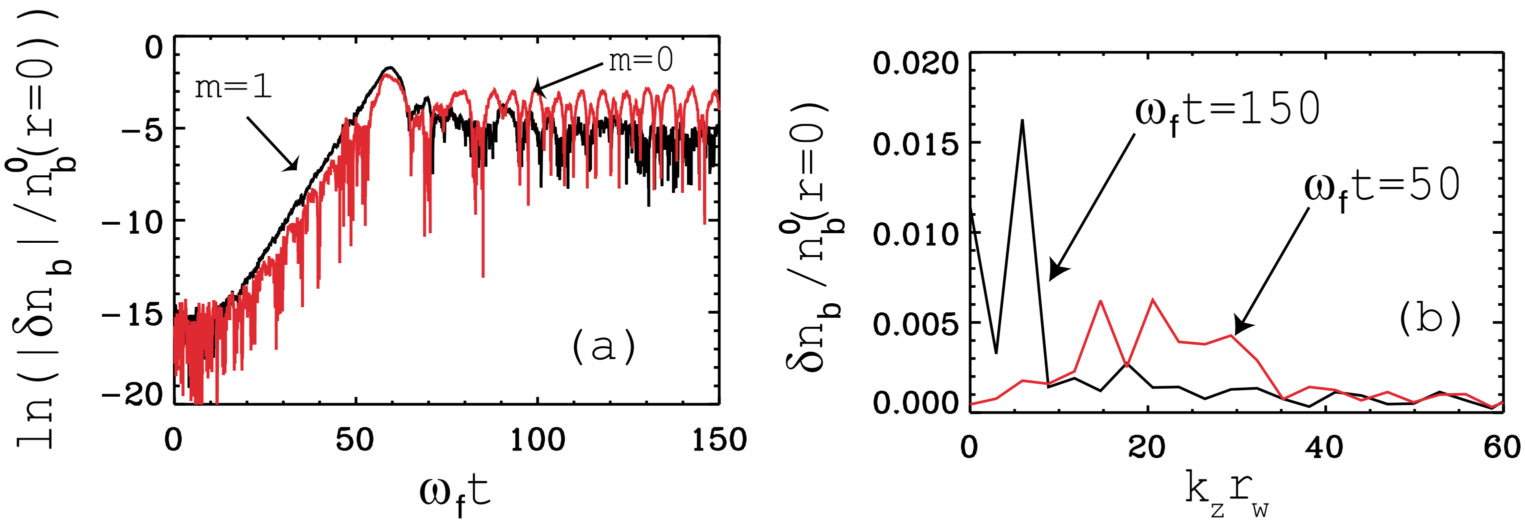
- $\bar{\nu}/\nu_0 = 0.3$
- Instability is present only for short-wavelength perturbations $k_z^2 r_b^2 > 1$.

Longitudinal Threshold Temperature $T_{\|b}^{th}$ Versus Normalized Tune Depression $\bar{\nu}/\nu_0$



$$\left(\frac{T_{\|b}^{th}}{T_{\perp b}}\right)_{max} = 0.11$$

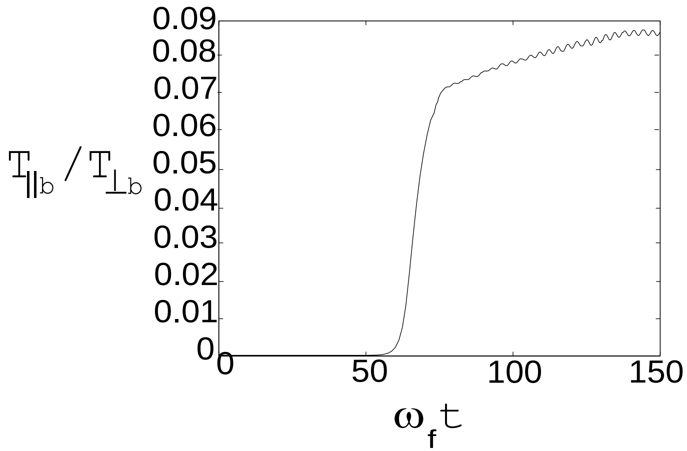
Nonlinear Stage of the Instability is Dominated by Long-Wavelength m=0 Mode

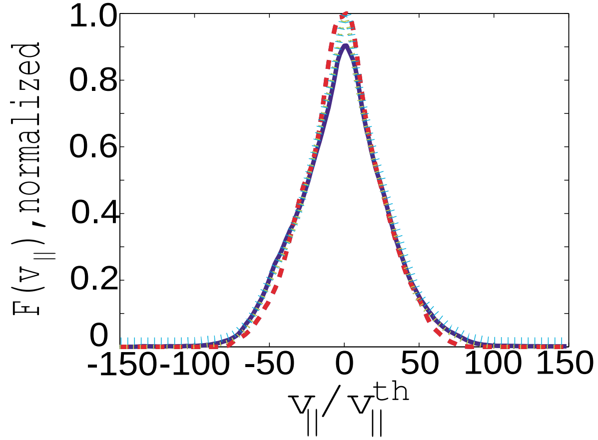


Results obtained using the BEST nonlinear δf simulation code, with $\bar{\nu}/\nu_0 = 0.6$ and initial temperature ratio $T_{||b}/T_{\perp b} = 10^{-4}$.

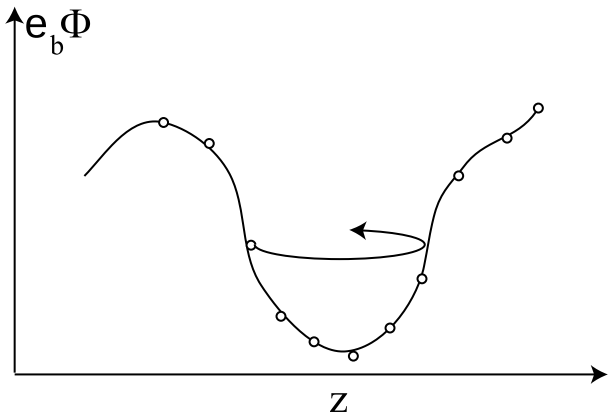
Instability Saturates Nonlinearly by Particle Trapping and Quasilinear Relaxation

$T_{\parallel b} = m_b < v_{\parallel}^2 >$

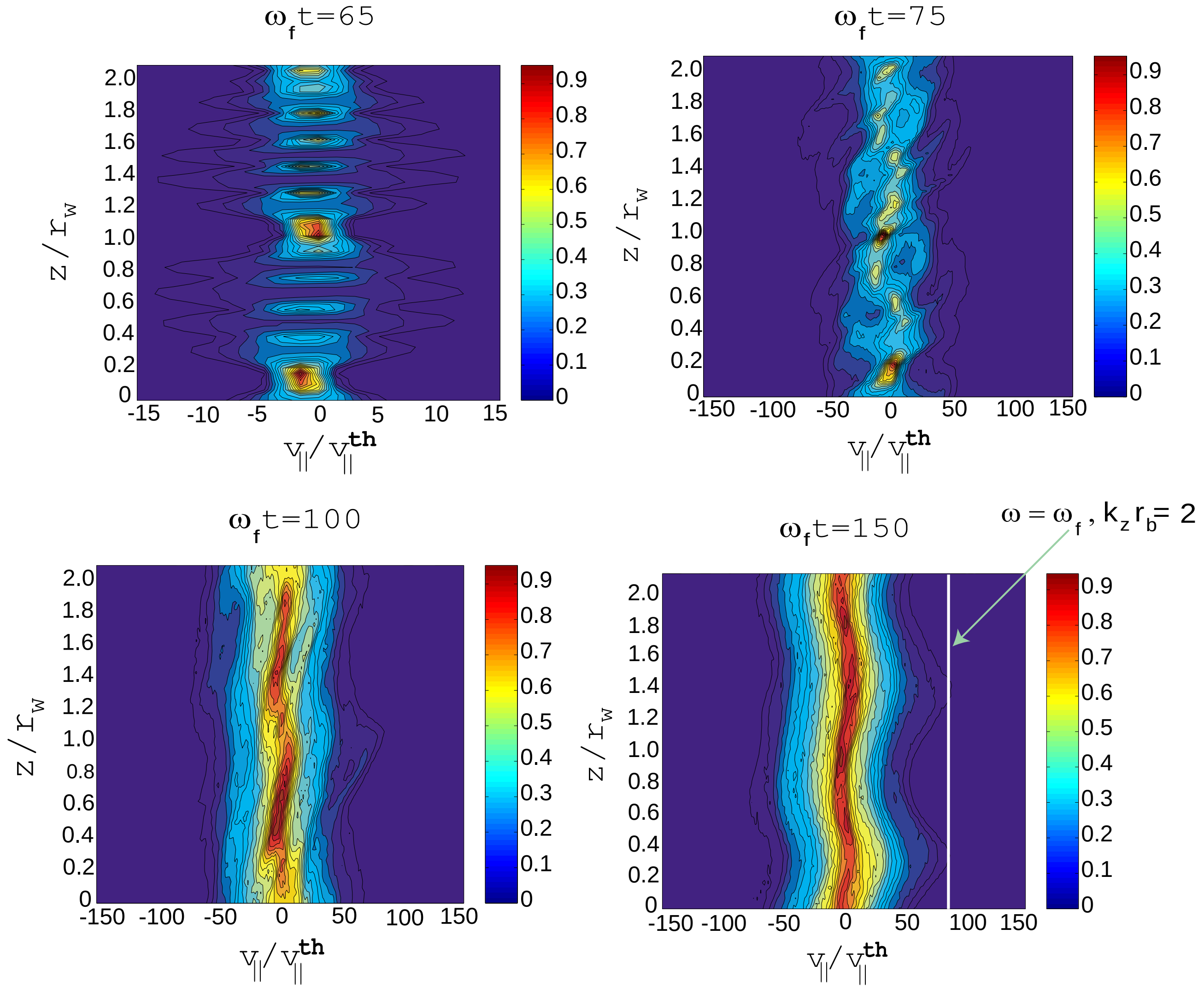




$$T_{\parallel} \sim e_b \Phi_{max} \sim m_b \left(\frac{\gamma^2}{k_z^2} \right)_{max}$$



Formation of a Stable, Longitudinal, Nonlinear BGK-like Wave Structure



Conclusions

- We have generalized the analysis of the classical Harris instability to the case of a one-component intense charged particle beam with anisotropic temperature.

- For a long, coasting beam, the delta-f particle-in-cell code BEST and the eighenmode code bEASt have been used to determine detailed 3D stability properties over a wide range of temperature anisotropy and beam intensity.

- Intense beams with $\bar{\nu}/\nu_0 < 0.82$ and $T_{||b}/T_{\perp b} < 0.11$ are linearly unstable to short-wavelength perturbations with $k_z^2 r_b^2 \geq 1$.

- The instability is kinetic in nature and is due to the coupling of the particles' transverse betatron motion with the longitudinal plasma oscillations excited by the perturbation.

- The nonlinear saturation is governed by longitudinal particle trapping.

- The final longitudinal velocity distribution is not Maxwellian and can be characterized by a remnant temperature anisotropy ($T_{||b}/T_{\perp b} \simeq 0.09$), where $T_{||b} \equiv m_b < v_{||}^2 >$.