Drift Compression and Final Focus of Intense Heavy Ion Beams

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Motivation

- In the currently envisioned configurations for heavy ion fusion (HIF), it is necessary to longitudinally compress the beam bunches by a large factor after the acceleration phase and before the beam particles are focused onto the fusion target.
  - In order to obtain enough fusion energy gain, the peak current for each beam is required to be order 10^3A, and the bunch length to be as short as 0.5m.
  - To deliver the beam particles at the required energy, it is both expensive and technically difficult to accelerate short bunches at high current.
- The objective of drift compression is to compress a long beam bunch by imposing a negative longitudinal velocity tilt over the length of the beam in the beam frame.

Drift Compression and Final Focus of Heavy Ion Beam

- Assume a Cs^+ beam for HIF driver with $A = 132.9,\ q = 1,\ (\gamma - 1)mc^2 = 2.43GeV,\ z_{bf} = 0.27$m, and $< I >= 2254$A.

- The goal of drift compression is:
  - Length $z_b \longrightarrow \times \frac{1}{21.8}$. Perveance $K \longrightarrow \times 21.8$.

- Allowable changes of other system parameters:
  - Velocity tilt $|v_{zb}| \longrightarrow \leq 0.01$.
  - Beam radius $a \longrightarrow \times 2.33$.
  - Half lattice period $L \longrightarrow \times \frac{1}{2}$.
  - Filling factor $\eta \longrightarrow \times 4$. $\eta B' \longrightarrow \times 4$.

- The beam pulse need to focused onto a target with 2mm characterisitic size.

Important Questions

- Longitudinal Dynamics:
  - What is the dynamics of $z_b(s)$?
  - How long is the beam line? ($s_f = 516\text{m}$)
  - How large initial velocity tilt can we afford? ($v_{zb0} = -0.0143$)
  - Space charge effect?
  - Stability? (stable without longitudinal focusing by envelope equation)
- Transverse Dynamics:
  - Non-periodic lattice design, $L(s)$, $B'(s)$, $\eta(s)$, $\kappa(s)$, $K(s)$.
  - Non-periodic envelope, matched solutions? adiabatically-matched solutions?
- Final Foucus:
  - How to focus the entire beam onto the target.

Outline

- Longitudinal Dynamics:
  - 1D fluid model.
  - Self-similar solutions.
  - Longitudinal envelope equation.
  - Drift compression design.
  - Pulse shaping
- Transverse Dynamics and Final Focus:
  - Non-periodic lattice design.
  - Time-dependent lattice design.

Longitudinal Dynamics – 1D fluid model

- One dimensional fluid model in the beam frame for
  - $\lambda(t,z)$: line density,
  - $v_z(t,z)$: longitudinal velocity,
  - $p_z(t,z)$: longitudinal pressure.
- g-factor model for electric field.

$$eE_z = -\frac{ge^2}{\gamma^2}\frac{\partial\lambda}{\partial z}, \tag{1}$$

$$g = 2\ln\frac{r_w}{r_b}. \tag{2}$$

- Take g and r_b as constants for present purpose.
- External focusing: $-\kappa_z z$.

Longitudinal Dynamics – 1D fluid model

- In the beam frame:

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \quad \text{(continuity)}, \tag{3}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + \frac{e^2 g}{m\gamma^5}\frac{\partial \lambda}{\partial z} + \frac{\kappa_z z}{m\gamma^3} + \frac{r_b^2}{m\gamma^3\lambda}\frac{\partial p_z}{\partial z} = 0 \text{ (momentum)}, \tag{4}$$

$$\frac{\partial p_z}{\partial t} + v_z \frac{\partial p_z}{\partial z} + 3p_z \frac{\partial v_z}{\partial z} = 0 \text{ (energy)}. \tag{5}$$

- Eqs. (3) , (4), and (5) form a nonlinear hyperbolic PDE system. If neglecting κ_z and p_z, Eqs. (3) and (4) have the same form as the shallow-water equations.

- Eq. (5) is equivalent to

$$\frac{d}{dt}\left(\frac{p_z}{\lambda^3}\right) = 0. \tag{6}$$

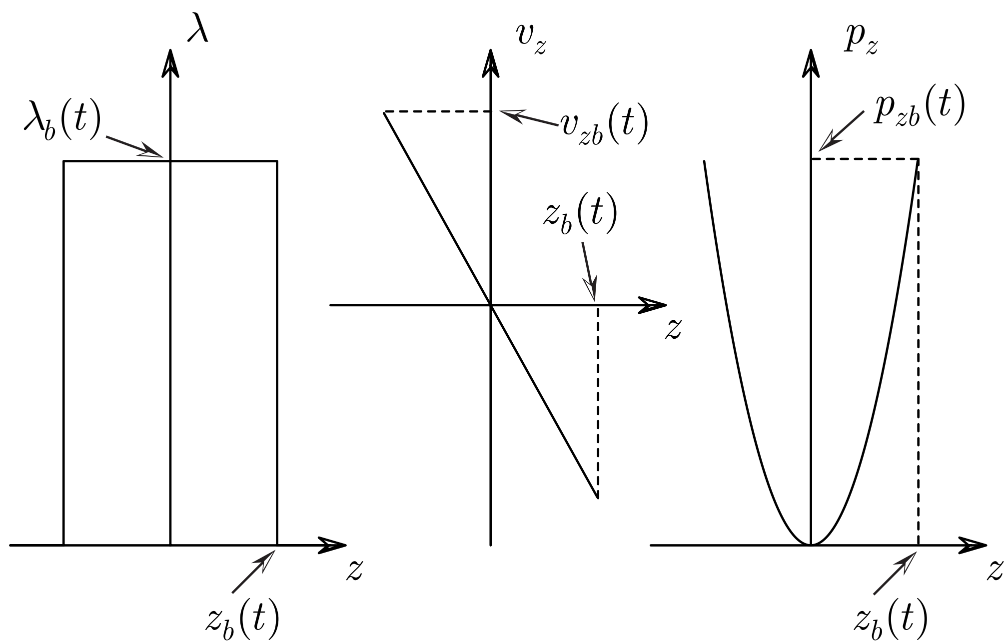
- Self-similar drift compression schemes preserve the geometric shape of the bunched beam, as well as the density profile, the pressure profile, and the velocity distribution. The nonlinear PDE system, Eqs. (3), (4), and (5), admits at least two self-similar drift compression solutions.

Linear self-similar drift compression solution

⇨

$$\lambda(t,z) = \lambda_b(t),\ v_z(t,z) = -v_{zb}(t)\frac{z}{z_b(t)}, \tag{7}$$

$$p_z(t,z) = p_{zb}(t)\frac{z^2}{z_b^2(t)},\ \frac{dz_b(t)}{dt} = -v_{zb}(t). \tag{8}$$



Linear self-similar drift compression solution:

⇨ From the continuity equation (3), we obtain

$$\frac{1}{\lambda_b}\frac{d\lambda_b}{dt} + \frac{1}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b\lambda_b = const. = N_b/2\,, \tag{9}$$

⇨ From the energy equation (5) , we obtain

$$z_b^3 p_{zb} = const. = W\,. \tag{10}$$

⇨ Similarly, for the momentum equation (4), the z-dependence drops out as well, giving

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2c^2} z_b + \frac{\varepsilon_l^2}{z_b^3} = 0\,, \tag{11}$$

where $\varepsilon_l \equiv \left(2r_b^2 W/m\gamma^3\beta^2c^2N_b\right)^{1/2}$.

⇨ Equations (9), (10) and (11) describe the dynamics of the time-dependent variables $\lambda_b(t)$, $z_b(t)$, and $p_{zb}(t)$.

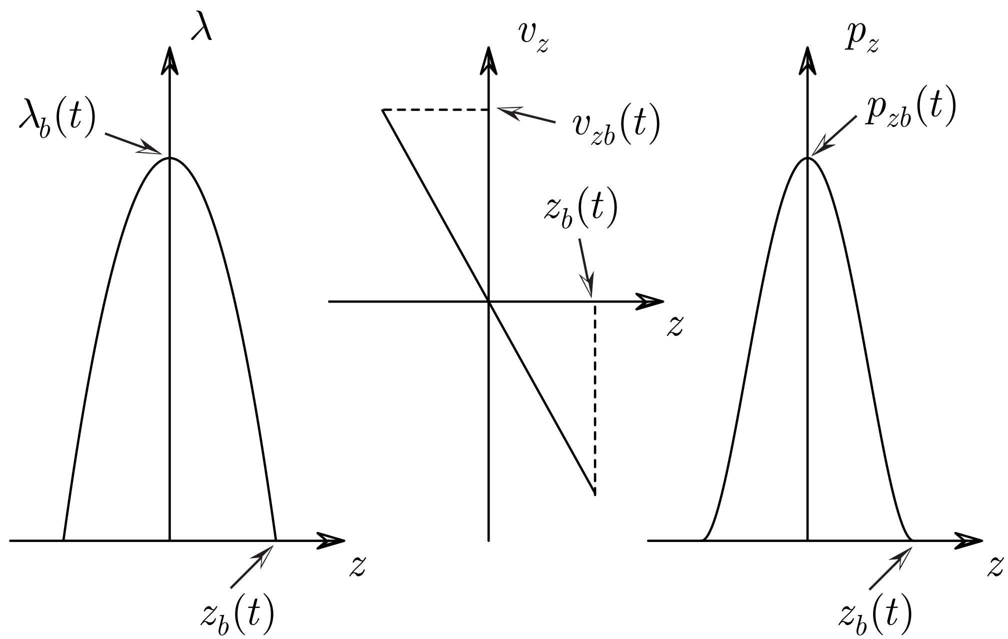
⇨ Equation (11) predicts a dramatic compression scenario where the beam longitudinally "implodes" because of the singularity of the (focusing) pressure term in Eq. (11) as $z_b \to 0$.

Parabolic Self-Similar Solution

⇨

$$\lambda(t,z) = \lambda_b(t)\left(1 - \frac{z^2}{z_b^2(t)}\right), \; v_z(t,z) = -v_{zb}(t)\frac{z}{z_b(t)}, \tag{12}$$

$$p_z(t,z) = p_{zb}(t)\left(1 - \frac{z^2}{z_b^2(t)}\right)^2, \; \frac{dz_b(t)}{dt} = -v_{zb}(t). \tag{13}$$



Parabolic Self-Similar Solution

⇨ Substituting Eqs. (12) and (13) into Eqs. (3) and (5), we find that the z-dependence drops out, and

$$\frac{d\lambda_b}{dt} - \frac{v_{zb}}{z_b}\lambda_b = 0, \tag{14}$$

$$\frac{dp_{zb}}{dt} - 3\frac{v_{zb}}{z_b}p_{zb} = 0. \tag{15}$$

⇨ Remarkably, but not surprisingly, for the momentum equation (4), the z-dependence also drops out, giving

$$-\frac{dv_{zb}}{dt} - \frac{e^2 g}{m\gamma^5}\frac{2\lambda_b}{z_b} + \frac{\kappa_z z_b}{m\gamma^3} - \frac{4r_b^2 p_{zb}}{m\gamma^3\lambda_b z_b} = 0 \tag{16}$$

⇨ Eqs. (13) – (16) form a coupled ordinary differential equation (ODE) system. Most remarkably, these equations recover the longitudinal envelope equation. From Eqs. (13), (15), and (14), we obtain

$$\frac{1}{\lambda_b}\frac{d\lambda_b}{dt} + \frac{1}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b\lambda_b = const. = \frac{3}{4}N_b\,, \tag{17}$$

$$\frac{1}{p_{zb}}\frac{dp_{zb}}{dt} + \frac{3}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b^3 p_{zb} = const. = W\,. \tag{18}$$

Longitudinal Envelope Equation

⇨ Substituting Eqs. (17), (18) and (13) into Eq. (16), we obtain

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2c^2} z_b - K_l \frac{1}{z_b^2} - \varepsilon_l^2 \frac{1}{z_b^3} = 0, \tag{19}$$

where $s = \beta ct$, $K_l \equiv 3N_b e^2 g / 2m\gamma^5\beta^2c^2$ is the effective longitudinal self-field perveance, and $\varepsilon_l \equiv (4r_b^2 W / m\gamma^3\beta^2c^2)^{1/2}$ is the longitudinal emittance.

⇨ The longitudinal envelope equation can be integrated once to give

$$(z_{b0}'^2 - z_{bf}'^2) = 2K_l(\frac{1}{z_{bf}} - \frac{1}{z_{b0}}) + \varepsilon_l^2(\frac{1}{z_{bf}^2} - \frac{1}{z_{b0}^2}), \tag{20}$$

where $z_{b0} = z_b(s = 0)$, $z_{bf} = z_b(s = s_f)$, $z_{b0}' = dz_b/ds(s = 0)$, and $z_{bf}' = dz_b/ds(s = s_f)$.

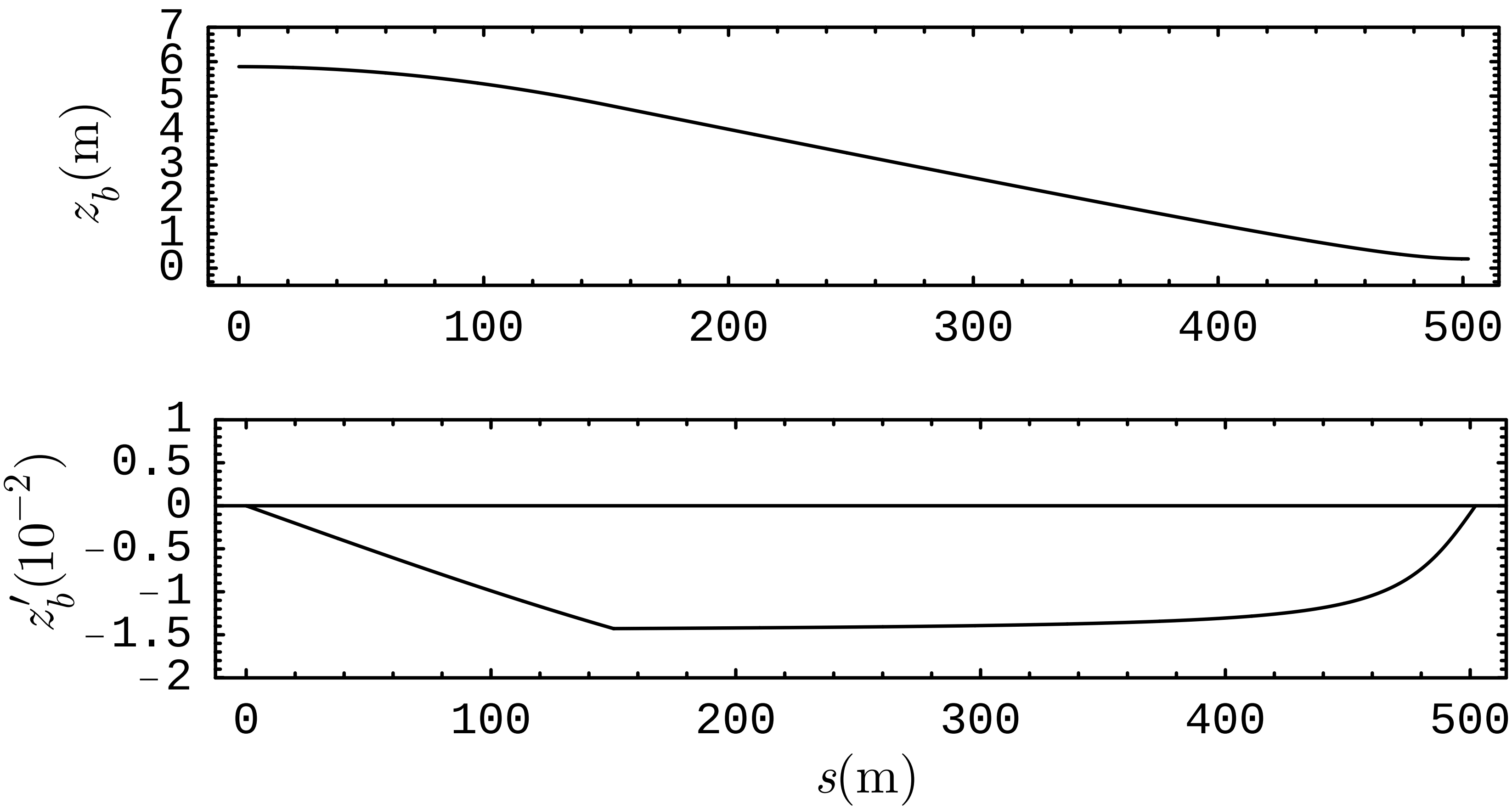
⇨ Given $(z_{bf}, z_{b0}, K_l, \varepsilon_l)$, we want (v_{zb0}, v_{zbf}, s_f) to be as small as possible. But

- Smaller $v_{bz0} \Longleftrightarrow$ Larger s_f.
- Smaller $v_{bzf} \Longleftrightarrow$ Larger s_f.

Need to study the trade-off.

Longitudinal Dynamics

- $\varepsilon_l = 1.0 \times 10^{-5}$ m and $K_z = 2.88 \times 10^{-5}$ m , corresponding to an average final current $\langle I_f \rangle = 2254\,\mathrm{A}$, $z_{bf} = 0.268\,\mathrm{m}$, and $g = 0.81$.
- An initial longitudinal focusing force is imposed for $s < 150\,\mathrm{m}$ so that the beam acquires a velocity tilt $z_b' = -0.0143$ at $s_b = 150\,\mathrm{m}$.



Longitudinal Pulse Shaping

- The parabolic self-similar drift compression solution requires the initial beam pulse shape to be parabolic.
- Need to shape the beam pulse into a parabolic form before imposing a velocity tilt.
- Need to solve the pulse shaping problem in general — finding the initial velocity distribution $V(z) \equiv v_z(t = 0, z)$ such that a given initial pulse shape $\Lambda(z) \equiv \lambda(t = 0, z)$ evolves into a given final pulse shape $\Lambda_T(z) \equiv \lambda(t = T, z)$ at time $t = T$.
- Choose the following normalized variables:

$$\overline{v}_z = \frac{v_z}{\beta c}, \ \overline{z} = \frac{z}{z_{b0}}, \ \overline{\lambda} = \frac{\lambda}{\lambda_{b0}}, \ \overline{t} = \frac{t\beta c}{z_{b0}}, \tag{21}$$

where z_{b0} is the initial beam half-length, and λ_{b0} is the initial beam line density at the beam center ($z = 0$).

Longitudinal Pulse Shaping

⇨ In the normalized variables, the one-dimensional fluid equations, neglecting pressure effects and external focusing, are given by

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \ , \tag{22}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + \overline{K}_l \frac{\partial \lambda}{\partial z} = 0 \ , \tag{23}$$

where $\overline{K}_l \equiv \lambda_{b0} e^2 g / m\gamma^5 \beta^2 c^2$ is the normalized longitudinal perveance.

⇨ $\overline{K}_l$ will be treated as a small parameter.

⇨ To order lowest order,

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \ , \tag{24}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} = 0 \ . \tag{25}$$

Longitudinal Pulse Shaping

⇨ Equations (24) and (25) can solved by integrating along characteristics. On the characteristics defined by

$$C: \quad \frac{dz}{dt} = v_z , \tag{26}$$

Equations (24) and (25) are

$$\frac{d\lambda}{dt} = -\lambda \frac{\partial v_z}{\partial z} , \tag{27}$$

$$\frac{dv_z}{dt} = 0 \; . \tag{28}$$

⇨ Because $dv_z/dt = 0$ on C, the family of characteristics C are straight lines in the (t, z) plan, which can be represented as

$$C: \; z = \xi + V(\xi)t \, , \tag{29}$$

where

$$V(\xi) \equiv v_z(t = 0, \xi) \, . \tag{30}$$

Longitudinal Pulse Shaping

⇨ The solution for $v_z(t,z)$ can be formally written as

$$v_z(t,z) = V(\xi(t,z)) \,, \tag{31}$$

where $\xi(t,z)$ as a function of t and z is determined from Eq. (29).

⇨ From Eqs. (31) and (29), four useful identities can be derived, *i.e.*,

$$\frac{\partial \xi}{\partial z} = \frac{1}{1+V'(\xi)t} \,, \tag{32}$$

$$\frac{\partial \xi}{\partial t} = \frac{-V(\xi)}{1+V'(\xi)t} \,, \tag{33}$$

$$\frac{\partial v_z}{\partial z} = \frac{V'(\xi)}{1+V'(\xi)t} \,, \tag{34}$$

$$\frac{\partial v_z}{\partial t} = \frac{-V(\xi)V'(\xi)}{1+V'(\xi)t} \,. \tag{35}$$

Longitudinal Pulse Shaping

⇨ From Eqs. (27) and (34), we obtain

$$\frac{d\ln\lambda}{dt} = \frac{-V'(\xi)}{1+V'(\xi)t} \quad \text{on } C\ . \tag{36}$$

⇨ Since ξ is a constant on C, Eq. (36) can be integrated to give

$$\begin{aligned}\ln\lambda &= \ln\lambda(t=0,\xi) + \int_0^t \frac{-V'(\xi)}{1+V'(\xi)t}dt \\ &= \ln\Lambda(\xi) + \ln[1+V'(\xi)t]\,,\end{aligned} \tag{37}$$

where $\Lambda(z) \equiv \lambda(t=0,z)$ is the initial line density profile. The solution to Eq. (36) for $\lambda(t,z)$ is

$$\lambda(t,z) = \frac{\Lambda(\xi)}{1+V'(\xi)t}\,. \tag{38}$$

Longitudinal Pulse Shaping

- For the pulse shaping problem, the final line density profile $\Lambda_T(z) \equiv \lambda(t = T, z)$ is specified. We therefore obtain

$$\Lambda_T(z) = \Lambda_T[\xi + V(\xi)T] = \frac{\Lambda(\xi)}{1 + V'(\xi)T}\,, \tag{39}$$

which can be viewed as an ordinary differential equation for $V(\xi)$.

- It can be simplified using the variable $U(\xi)$ defined by

$$U(\xi) \equiv \xi + V(\xi)T\ . \tag{40}$$

In terms of $U(\xi)$, Eq. (39) becomes

$$\Lambda_T(U)dU = \Lambda(\xi)d\xi\,. \tag{41}$$

- Finally, $U(\xi)$ is determined by solving Eq. (41) for the given functional forms $\Lambda_T(z)$ and $\Lambda(z)$. $V(\xi)$ is simply related to $U(\xi)$ by

$$V(\xi) = \frac{U(\xi) - \xi}{T}\,. \tag{42}$$

Example: Pulse Shaping without Compression

⇨ Consider two examples with the following symmetries and boundary conditions,

$$v_z(t,-z) = -v_z(t,z)\,,\ \lambda(t,-z) = \lambda(t,z)\,, \tag{43}$$

$$V(\xi = 0) = 0\,,\ U(\xi = 0) = 0\,. \tag{44}$$

⇨ **Example 1—Pulse Shaping Without Compression:**

$$\Lambda(z) = \begin{cases} 1 - z^m\,, & 0 \le z \le 1\,, \\ 0\,, & 1 < z\,, \\ \Lambda(-z)\,, & z < 0\,, \end{cases} \tag{45}$$

$$\Lambda_T(z) = \begin{cases} (1 - z^n)\dfrac{m(n+1)}{n(m+1)}\,, & 0 \le z \le 1\,, \\ 0\,, & 1 < z\,, \\ \Lambda(-z)\,, & z < 0\,. \end{cases} \tag{46}$$

Example: Pulse Shaping without Compression

⇨ Equation (41) can integrated to give

$$\left[U(\xi) - \frac{U(\xi)^{n+1}}{n+1}\right]\frac{m(n+1)}{n(m+1)} = \xi - \frac{\xi^{m+1}}{m+1} . \tag{47}$$

⇨ The parabolic self-similar drift compression solution corresponds to $n = 2$. In this case, there are three solutions for $U(\xi)$. The solution satisfying the right boundary condition is

$$U(\xi) = -\frac{1 - i\sqrt{3} + \sqrt[3]{-2}p^2}{\sqrt[3]{4}p} , \tag{48}$$

where

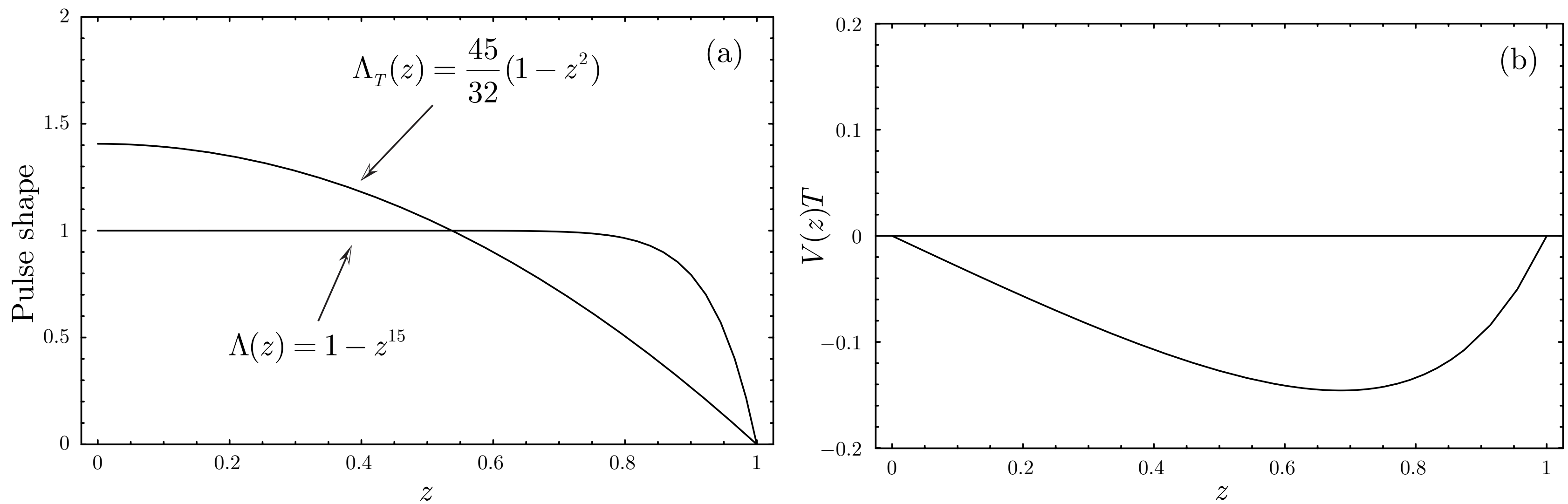
$$p = \sqrt[3]{-3a + \sqrt{-4 + 9a^2}} , \tag{49}$$

$$a = \frac{2(m+1)}{3m}(\xi - \frac{\xi^{m+1}}{m+1}) . \tag{50}$$

⇨ For large value of $m \gg 1$, $\Lambda(z)$ has a flat-top shape with a fast fall-off near the ends of the pulse.

Example: Pulse Shaping without Compression

⇨ Initial pulse shape $\Lambda(z) = 1 - z^{15}$ and final pulse shape $\Lambda_T(z) = (45/32)(1 - z^2)$ are plotted in (a). The initial velocity $V(z)$ given by Eq. (42) is plotted in (b).



- **Example 2—Pulse Shaping With Compression:**

$$\Lambda(z) = \begin{cases} 1 - z^m \,, & 0 \le z \le 1 \,, \\ 0 \,, & 1 < z \,, \\ \Lambda(-z) \,, & z < 0 \,, \end{cases} \tag{51}$$

$$\Lambda_T(z) = \begin{cases} [1 - (\alpha z)^n] \dfrac{\alpha m(n+1)}{n(m+1)} \,, & 0 \le z \le \dfrac{1}{\alpha} \,, \\ 0 \,, & \dfrac{1}{\alpha} < z \,, \\ \Lambda(-z) \,, & z < 0 \,, \end{cases} \tag{52}$$

where $\alpha > 1$ is the compression factor.

- Equation (41) can be integrated to give

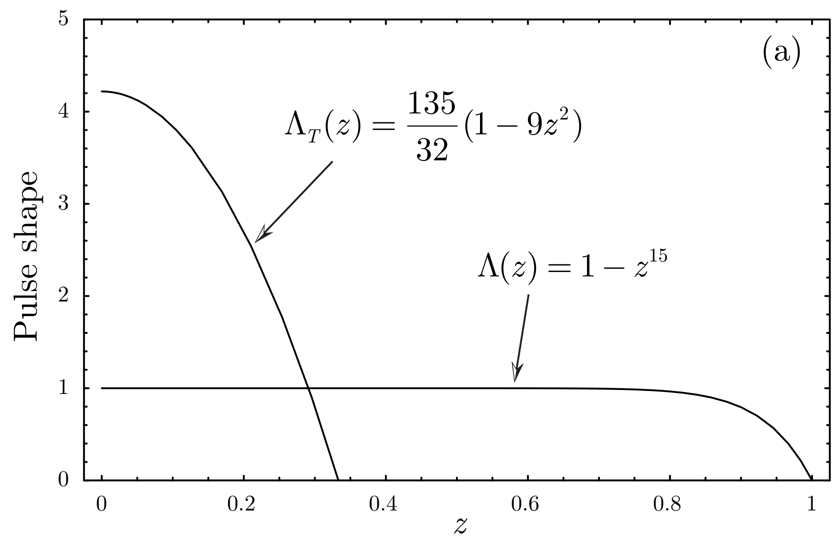
$$\left[\alpha U(\xi) - \frac{(\alpha U(\xi))^{n+1}}{n+1} \right] \frac{m(n+1)}{n(m+1)} = \xi - \frac{\xi^{m+1}}{m+1} \,, \tag{53}$$

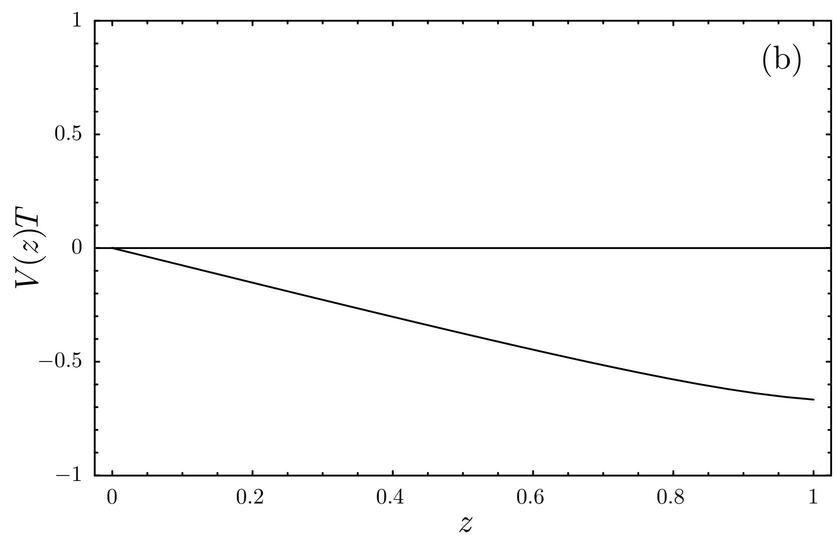
which is identical to Eq. (47) if $\alpha U(\xi)$ is replaced by $U(\xi)$. It is easy to verify that $\alpha U(\xi = 1) = 1$ and therefore

$$V(\xi = 1) = \frac{(1/\alpha - 1)}{T} \,. \tag{54}$$

Example: Pulse Shaping with Compression

- For the case of a beam being shaped but not compressed, $\alpha = 1$ and $V(\xi = 1) = 0$. When $\alpha > 1$, the beam is simultaneously being shaped and compressed, and $V(\xi = 1) < 0$.
- Initial pulse shape $\Lambda(z) = 1 - z^{15}$ and final pulse shape $\Lambda_T(z) = (135/32)(1 - 9z^2)$ are plotted in (a). The initial velocity $V(z)$ given by Eq. (42) is plotted in (b).





1st Order Space-Charge Correction

⇨ We now carry out the analysis to $O(\overline{K}_l)$. Let

$$\lambda(t,z) = \lambda_0(t,z) + \lambda_1(t,z)\,, \tag{55}$$

$$v_z(t,z) = v_{z0}(t,z) + v_{z1}(t,z)\,. \tag{56}$$

⇨ To $O(\overline{K}_l)$, Eqs. (22) and (23) can be expressed as

$$\left(\frac{d}{dt}\right)_0 \lambda_1 = \frac{\partial \lambda_1}{\partial t} + v_{z0}\frac{\partial \lambda_1}{\partial z} = -\lambda_1 \frac{\partial v_{z0}}{\partial z} - \frac{\partial}{\partial z}(\lambda_0 v_{z1})\,, \tag{57}$$

$$\left(\frac{d}{dt}\right)_0 v_{z1} = \frac{\partial v_{z1}}{\partial t} + v_{z0}\frac{\partial v_{z1}}{\partial z} = -v_{z1}\frac{\partial v_{z0}}{\partial z} - \overline{K}_l \frac{\partial \lambda_0}{\partial z}\,. \tag{58}$$

⇨ Using the method of variational coefficients, the solution to Eq. (58) is found to be

$$v_{z1} = \frac{1}{1+V_0'(\xi)t}\left\{V_1(\xi) - \overline{K}_l \frac{\partial}{\partial \xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\ln[1+V_0'(\xi)t]\right]\right\}\,. \tag{59}$$

1st Order Space-Charge Correction

⇨ By the same procedure, Eq. (57) can be integrated to give

$$\lambda_1 = \frac{\Lambda_1(\xi)}{1+V_0'(\xi)t} - \frac{1}{1+V_0'(\xi)t}\frac{\partial}{\partial\xi}\left\{\frac{\Lambda_0(\xi)V_1(\xi)t}{1+V_0'(\xi)t}\right. \tag{60}$$

$$- \overline{K}_l\Lambda_0(\xi)\frac{\partial}{\partial\xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\right]\frac{V_0'(\xi)t - \ln[1+V_0'(\xi)t]}{[1+V_0'(\xi)t]^2}$$

$$\left. - \overline{K}_l\frac{\Lambda_0^2(\xi)}{V_0'(\xi)}V_0''(\xi)\frac{t^2}{[1+V_0'(\xi)t]^2}\right\}.$$

⇨ At time $t = T$, we obtain

$$\Lambda_T(z) = \lambda_0(t=T,z) + \lambda_1(t=T,z). \tag{61}$$

Since $\Lambda_T(z)$ and $\Lambda(z)$ are prescribed in the pulse shaping problem, we take $\Lambda_{T1}(z) = 0$ and $\Lambda_1(z) = 0$. This results in

$$V_1(\xi) = \overline{K}_l\frac{\partial}{\partial\xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\right]\frac{V_0'(\xi) - \ln[1+V_0'(\xi)T]/T}{1+V_0'(\xi)T} \tag{62}$$

$$+ \overline{K}_l\frac{\Lambda_0(\xi)}{V_0'(\xi)}V_0''(\xi)\frac{T}{1+V_0'(\xi)T} + c'.$$

Transverse Dynamics in Non-Periodic Lattice

⇨ Transverse envelope equations:

$$\frac{d^2a(s,z)}{ds^2} + \kappa_q a(s,z) - \frac{2K(s,z)}{a(s,z)+b(s,z)} - \frac{\varepsilon_x^2}{a(s,z)^3} = 0,$$
$$\frac{d^2b(s,z)}{ds^2} - \kappa_q b(s,z) - \frac{2K(s,z)}{a(s,z)+b(s,z)} - \frac{\varepsilon_y^2}{b(s,z)^3} = 0, \qquad (63)$$

⇨ $K(s,z)$ is non-periodic due to the longitudinal compression.

⇨ κ_q need to be non-periodic to reduce the expansion of the beam radius.

⇨ Since the quadrupole lattice is not periodic, the concept of a "matched" beam is not well defined.

⇨ However, if the the non-periodicity is small, that is, if the quadrupole lattice changes slowly along the beam path, we can seek an "adiabatically"-matched beam which, by definition, is locally matched everywhere.

Time-Dependent Focusing Lattice

- ⇨ The drift compression and final focus lattice should apply for all slices in a bunched beam.
- ⇨ Each slice of the beam should be focused onto the same focal point at the target.
- ⇨ A fixed lattice designed for one slice of the beam will not focus other slices onto the same focal point.
- ⇨ Design a lattice for the central slice ($z = 0$), and then replace four quadrupole magnets at the beginning of the drift compression by four time-dependent magnets.
- ⇨ The time-dependent magnets essentially provide a slightly different focusing lattice for different slices.

Non-periodic Lattice Design for Central Slice

- Goal:
  - Constant vacuum phase advance $\sigma_v = \pi/5 \longrightarrow \eta B' L^2 = const.$
  - Length $z_b \longrightarrow \times \dfrac{1}{21.8}$. Perveance $K \longrightarrow \times 21.8$.
  - Beam radius $a \longrightarrow \times 2.33$.
  - Half lattice period $L \longrightarrow \times \dfrac{1}{2}$.
  - Filling factor $\eta \longrightarrow \times 4$. $\eta B' \longrightarrow \times 4$.
- How do $K,\ L, \eta,\ B',\ a$, and b depend on s?
  - $K(s)$ is given by the longitudinal dynamics.
  - $L(s), \eta(s)$, and $B'(s)$ are determined by requirements such as constant vacuum phase advance.
  - $a(s)$ and $b(s)$ are determined by the transverse envelope equations.

Non-periodic Lattice Design for Central Slice

- A lattice which keeps both the vacuum phase advance and depressed phase advance constant is less likely to induce beam mismatch.

- Vacuum phase advance σ_v and depressed phase advance σ are given by

$$2(1-\cos\sigma_v) = (1-\frac{2\eta}{3})\eta^2\left(\frac{B'}{[B\rho]}\right)^2 L^4, \tag{64}$$

$$\sigma^2 = 2(1-\cos\sigma_v) - K\left(\frac{2L}{\langle a\rangle}\right)^2. \tag{65}$$

- Assuming $\eta \ll 1$, we obtain

$$\eta^2(\frac{B'}{[B\rho]})^2L^4 = const., \;\; K(\frac{2L}{\langle a\rangle})^2 = const., \tag{66}$$

for constant vacuum phase advance and constant depressed phase advance.

- It is under-determined. As one possible choice, let

$$L = L_0\exp(-\ln 2\frac{s}{s_f}), \qquad \eta = \eta_0\exp(2\ln 2\frac{s}{s_f}), \qquad B' = const. \tag{67}$$

Non-periodic Lattice Design for Central Slice

⇨ Let the lattice lengths are $L_0,\ L_1,\ ...,\ L_N = L_f$,

$$
\begin{aligned}
L_1 &= L_0 \exp(-\ln 2 \frac{2L_0}{s_f}), \\
L_2 &= L_0 \exp(-\ln 2 \frac{2(L_0 + L_1)}{s_f}), \\
&\ldots\ldots \\
L_i &= L_0 \exp(-\ln 2 \frac{2\sum_0^{i-1} L_i}{s_f}), \\
2(L_1 &+ L_2 + ... + L_N) = S_f.
\end{aligned} \tag{68}
$$

⇨ For $L_f = 3.36$m, $L_0 = 6.72$m, and $s_f = 421.5$m, calculation gives $N = 45$.

Transverse Dynamics for Central Slice

- For an adiabatically-matched solution,
  - The envelope is locally matched and contains no oscillations other than the local envelope oscillations.
  - On the global scale, the beam radius increases monotonically.
- Four final focus magnets will assure that the envelope converge in both directions at the exit of the last focusing magnet
- Then the beam enters the neutralization chamber where the space-charge force is neutralized, and is focused onto a focal point at

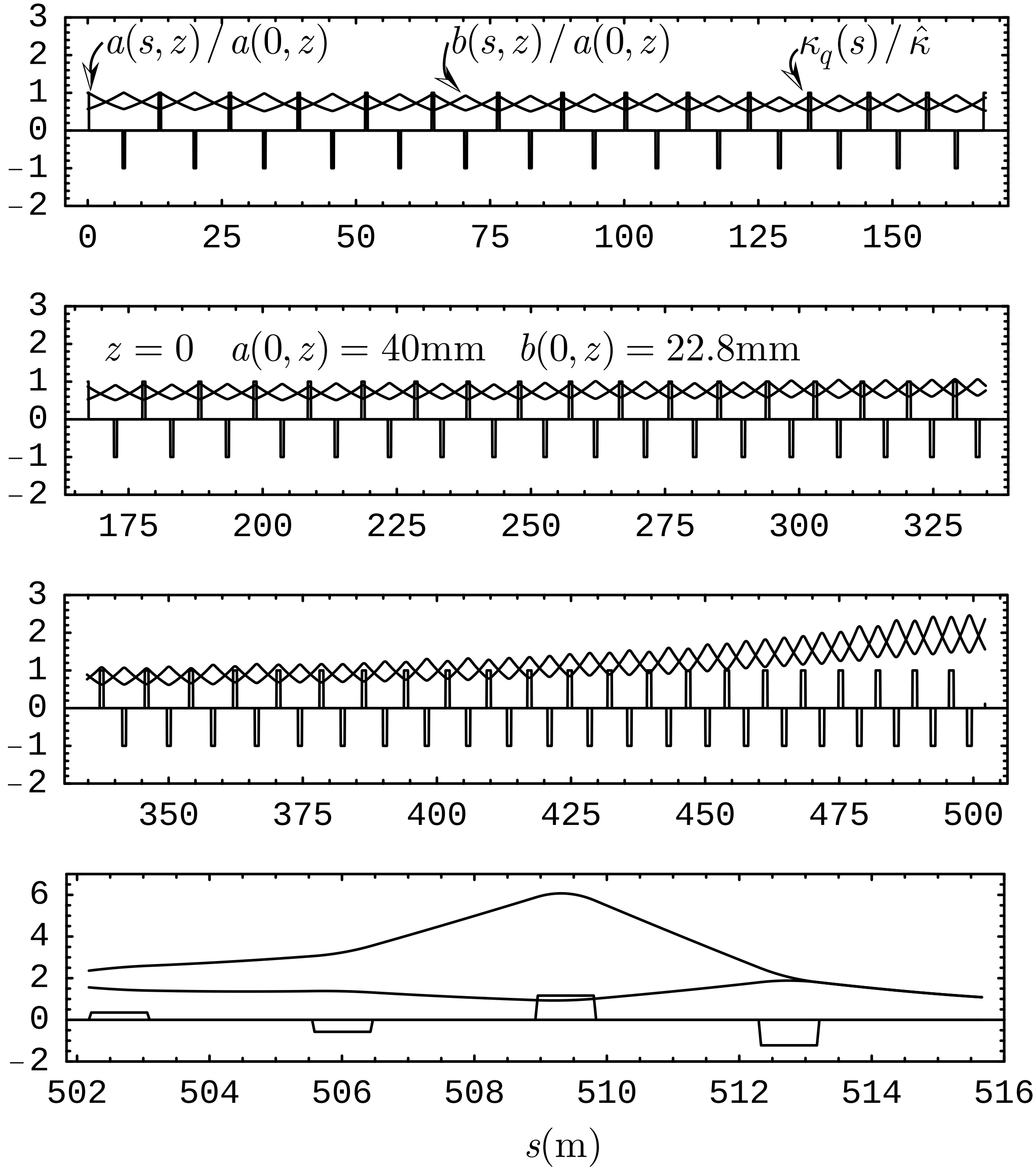
$$z_{fol} = - \left. \frac{a}{\partial a/\partial s} \right|_{s=s_{ff}} = - \left. \frac{b}{\partial b/\partial s} \right|_{s=s_{ff}} , \tag{69}$$

- The transverse spot size is determined by the emittance and incident angle at $s = s_{ff}$,

$$a_{fol} = \left. \frac{\varepsilon_x}{\partial a/\partial s} \right|_{s=s_{ff}} , \; b_{fol} = \left. \frac{\varepsilon_y}{\partial b/\partial s} \right|_{s=s_{ff}} . \tag{70}$$

- For the central slice at $z = 0$, we obtain $z_{fol} = 5.276\,\mathrm{m}$, and $a_{fol} = b_{fol} = 1.22\,\mathrm{mm}$.

Transverse Dynamics for Central Slice



Time-Dependent Lattice for Entire Pulse

- For other slices ($z \neq 0$), we manipulate the beam and magnet configuration so that the beam particles can be focused onto a focal region with the same or smaller spot size,

$$z_{fol} = 5.276\,\mathrm{m}, \quad a_{fol} \approx b_{fol} \lesssim 1.22\,\mathrm{mm}\,. \tag{71}$$

- For the line density profile $\lambda(s,z) = \lambda_b(s)[1 - z^2/z_b^2(s)]$, that the solution for all of the slices can be scaled down from that of the central slice:

$$\begin{pmatrix} a(s,z) \\ b(s,z) \\ \partial a(s,z)/\partial s \\ \partial b(s,z)/\partial s \end{pmatrix} = \sqrt{1 - z^2/z_b^2(s)} \begin{pmatrix} a(s,0) \\ b(s,0) \\ \partial a(s,0)/\partial s \\ \partial b(s,0)/\partial s \end{pmatrix}, \tag{72}$$

if the emittance is

- negligibly small or
- scales with the perveance according to $(\varepsilon_x, \varepsilon_y) \propto 1 - z^2/z_b^2(s)$.

Time-Dependent Lattice for Entire Pulse

⇨ However, the emittance in general is small but not negligible, and does not scale with the perveance.

⇨ In fact, during adiabatic drift compression, the emittance scales with the beam size, i.e., $\varepsilon_x \propto a$ and $\varepsilon_y \propto b$.

⇨ The scaling in Eq. (72) and the requirement in Eq. (71) can't be satisfied.

⇨ Vary the strength of four magnets in the very beginning of the drift compression for different value of z such that the desired scaling in Eq. (72) holds at $s = s_{ff}$.

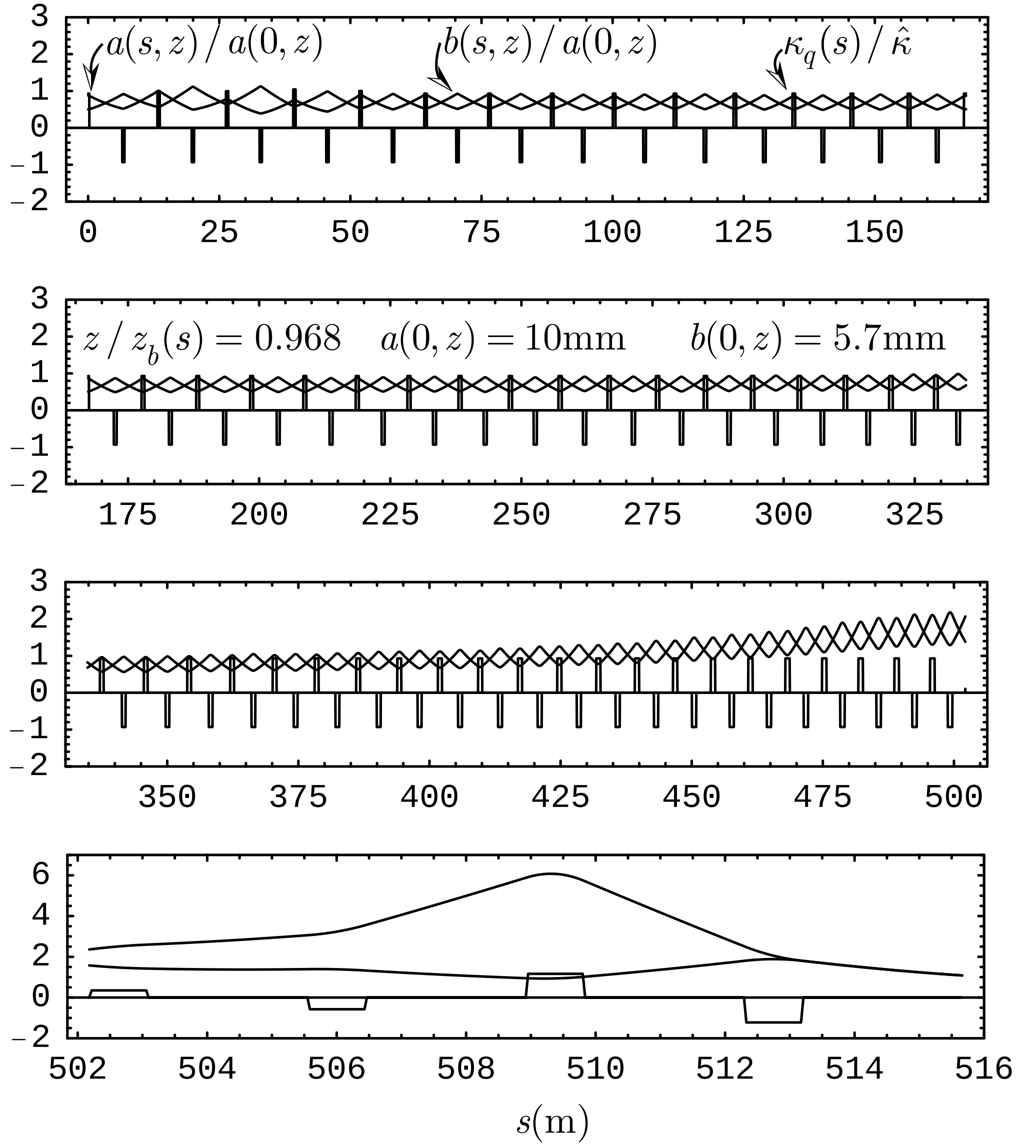
⇨ This will guarantee the satisfaction of the requirement in Eq. (71).

⇨ Numerically, the necessary variation of the strength of the magnets is found by a 4D root-searching algorithm.

⇨ A small perturbation in the strength of the magnets introduces a small envelope mismatch in such a way that Eq. (72) is satisfied at $s = s_{ff}$.

⇨ We note that a similar scaling does not exist for $0 < s < s_{ff}$.

Envelope dynamics for the $z/z_b(s) = 0.968$.



Time-Dependent Lattice for Entire Pulse

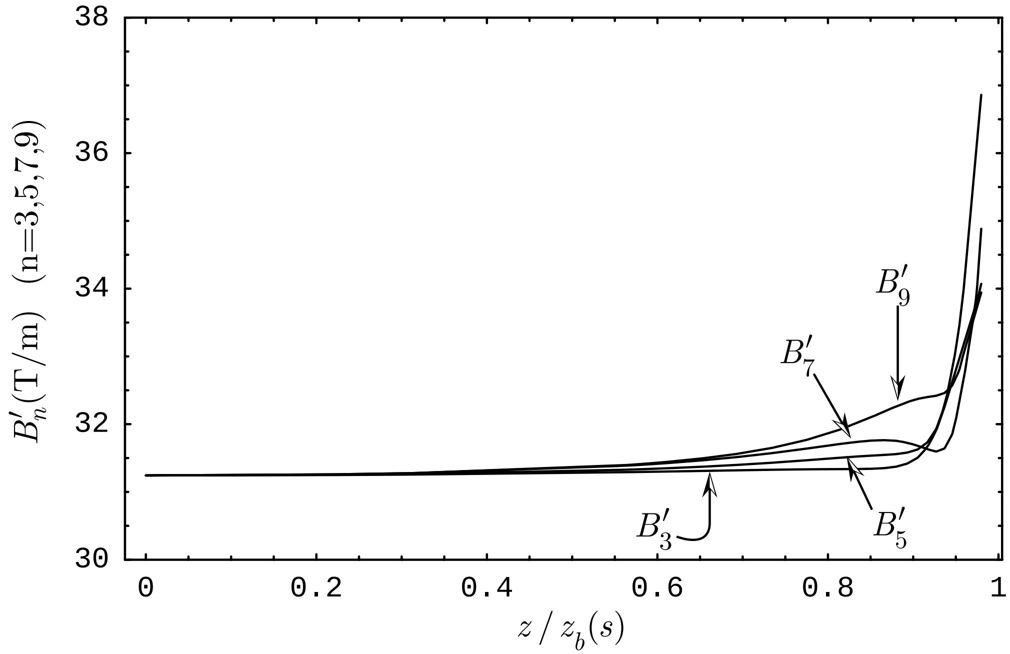


Figure 1: Strengths of the 3rd, 5th, 7th, and 9th magnets as functions of $z/z_b(s)$.

Conclusion

- ⇨ The longitudinal dynamics of drift compression and pulse shaping have been studied using a one-dimensional warm-fluid model.
- ⇨ The pulse shaping problem is solved perturbatively in the weak space-charge limit, such that an arbitrary pulse shape produced after the acceleration phase can be shaped into those required by the self-similar drift compression solutions.
- ⇨ A non-periodic quadrupole lattice for drift compression and four final focusing magnets are designed.
- ⇨ It is demonstrated that the entire pulse can be compressed and focused onto the same focal point on the target by using four time-varying quadrapole magnets at the very beginning of drift compression.