Nonlinear δf Simulations of Collective Effects in Intense Charged Particle Beams

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- High beam intensity is one of the most important features of modern accelerator applications:
  - Spallation Neutron Source (SNS) at ORNL.
  - Proton Storage Ring (PSR) at LANL.
  - Large Hadron Collider (LHC) at CERN.
  - Stanford Linear Collider (SLC) at SLAC.
  - Heavy Ion Fusion drivers (HIF) at Heavy Ion Fusion Virtual National Laboratory (HIF-VNL).
- Beam intensity has increased to regimes where collective processes can have a large influence on the beam dynamics.
- It is increasingly important to develop an improved theoretical understanding of the collective processes and self-field effects.

- A kinetic model based on the nonlinear Vlasov-Maxwell equations has been developed to study self-consistently the collective processes and self-field effects in high intensity charged particle beams.

- The knowledge and methods of one-component nonneutral plasma physics have been proved to be extremely valuable and are being widely accepted in the particle accelerator community.

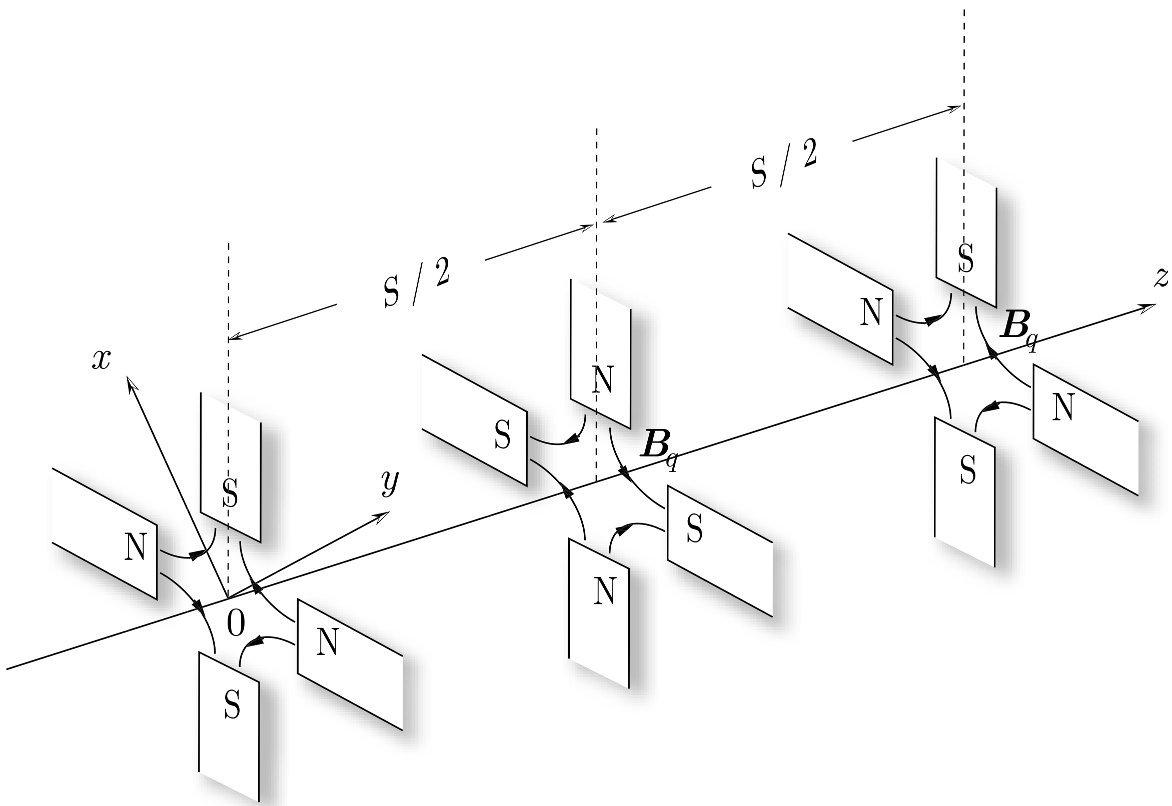
- Recently, the δf formalism, a low-noise, nonlinear perturbative particle simulation technique, has been developed to solve the nonlinear Vlasov-Maxwell equations for intense beam applications.

- Implemented in the Beam Equilibrium, Stability and Transport (BEST) code, it has been used to investigate the electron-ion two-stream instability, temperature anisotropy instability, periodically-focusing beam propagation and other collective processes.

Presentation Outline

- ⇨ Theoretical mode — nonlinear Vlasov Maxwell system.
- ⇨ Nonlinear δf particle simulation method.
- ⇨ Nonlinear properties of stable beam propagation.
- ⇨ Eigenmodes by the BEST code — body modes and surface modes.
- ⇨ Ion-electron two-stream instability for heavy ion fusion beams.
- ⇨ Electron-proton two-stream instability for the Proton Storage Ring experiment.
- ⇨ Conclusions and future work.

⇨ Transverse focusing by an alternating-gradient quadrupole field with axial periodicity length S.



⇨ Longitudinal acceleration by applied electric fields in linacs or synchrotrons.

Theoretical Model — Nonlinear Vlasov-Maxwell System

- Thin, continuous, high-intensity charged particle beam $(j = b)$ propagates in the z-direction through background electron and ion components $(j = e, i)$ described by distribution function $f_j(\boldsymbol{x}, \boldsymbol{p}, t)$.

- Transverse and axial particle velocities in a frame of reference moving with axial velocity $\beta_j c\hat{\boldsymbol{e}}_z$ are assumed to be *nonrelativistic*.

- Adopt a *smooth-focusing* model **[Davidson & Qin, 2001]**

$$\boxed{\boldsymbol{F}_j^{foc} = -\gamma_j m_j \omega_{\beta j}^2 \boldsymbol{x}_\perp .}$$

- Self-electric and self-magnetic fields are expressed as

$$\boxed{\begin{aligned} \boldsymbol{E}^s &= -\nabla\phi(\boldsymbol{x}, t)\,, \\ \boldsymbol{B}^s &= \nabla \times \boldsymbol{A}_z(\boldsymbol{x}, t)\hat{\boldsymbol{e}}_z\,. \end{aligned}}$$

Theoretical Model — Nonlinear Vlasov-Maxwell System

⇨ Distribution functions and electromagnetic fields are described self-consistently by the nonlinear Vlasov-Maxwell equations in the six-dimensional phase space $(\boldsymbol{x}, \boldsymbol{p})$:

$$\boxed{\left\{ \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} - [\gamma_j m_j \omega_{\beta j}^2 \boldsymbol{x}_\perp + e_j (\nabla \phi - \frac{v_z}{c} \nabla_\perp A_z)] \cdot \frac{\partial}{\partial \boldsymbol{p}} \right\} f_j(\boldsymbol{x}, \boldsymbol{p}, t) = 0}$$

and

$$\boxed{\begin{aligned} \nabla^2 \phi &= -4\pi \sum_j e_j \int d^3p f_j(\boldsymbol{x}, \boldsymbol{p}, t) \\ \nabla^2 A_z &= -\frac{4\pi}{c} \sum_j e_j \int d^3p v_z f_j(\boldsymbol{x}, \boldsymbol{p}, t) \end{aligned}}$$

⇨ Complete description at *Physics of Intense Charged Particle Beams in High Energy Accelerators* **(Davidson & Qin, World Scientific, 2001).**

Nonlinear δf Particle Simulation Method

- Divide the distribution function into two parts: $f_j = f_{j0} + \delta f_j$, where f_{j0} is a known solution to the nonlinear Vlasov-Maxwell equations.
- Determine numerically the evolution of the perturbed distribution function $\delta f_j \equiv f_j - f_{j0}$.
- Advance the weight function defined by $w_j \equiv \delta f_j / f_j$, together with the particles' positions and momenta.

$$\boxed{\begin{aligned} \frac{dw_{ji}}{dt} &= -(1 - w_{ji})\frac{1}{f_{j0}}\frac{\partial f_{j0}}{\partial \boldsymbol{p}} \cdot \delta\left(\frac{d\boldsymbol{p}_{ji}}{dt}\right) \\ \delta\left(\frac{d\boldsymbol{p}_{ji}}{dt}\right) &\equiv -e_j(\nabla\delta\phi - \frac{v_{zji}}{c}\nabla_\perp\delta A_z) \end{aligned}}$$

Here, $\delta\phi = \phi - \phi_0$, $\delta A_z = A_z - A_{z0}$, and ($\phi_0$, A_{z0}, f_{j0}) are the equilibrium solutions.

Advantages of the δf method and the BEST code

- Simulation noise is reduced significantly.
  - Statistical noise $\sim 1/\sqrt{N_s}$.
  - To achieve the same accuracy, number of simulation particles required by the δf method is only $(\delta f/f)^2$ times of that required by the conventional PIC method.
- Especially desirable for two-stream modes in high intensity beams because of the small ratio between the growth rate and real frequency.
- Implemented in the Beam Equilibrium Stability and Transport (BEST) code.
- Adiabatic field pusher for light particles (electrons).
- The code has been parallelized using OpenMP and MPI.
- Achieved 2.0×10^{10} ion-steps $+ 4 \times 10^{11}$ electron-steps for instability studies.

What Is Not in the BEST Code?

- ⇨ Simplified electron physics is adopted which neglects:
  - ❍ Secondary electron emission from vacuum chamber.
  - ❍ Electron dynamics in quadrupole field.
- ⇨ With the help of the Hamiltonian averaging techniques **[Davidson & Qin, 2001]**, a smooth focusing model is used without the complication of finding a 6D phase space "equilibrium" in a periodic focusing lattice.
- ⇨ A long coasting beam is considered without bunching effects because the longitudinal wavelength of the unstable modes is generally much shorter than the bunch length.

One-Dimensional Equilibrium

- Equilibrium properties depend on the radial coordinate $r = (x^2 + y^2)^{1/2}$.
- Cylindrical chamber with perfectly conducting wall located at $r = r_w$.
- Thermal equilibrium distribution function for the beam ion is given by

$$
\begin{aligned}
f_{j0}(r, \boldsymbol{p}) &= \frac{\hat{n}_j}{(2\pi m_j)^{3/2}\gamma_j^{5/2} T_{j\perp} T_{j\|}^{1/2}} \exp\left\{ -\frac{(p_z - \gamma_j m_j \beta_j c)^2}{2\gamma_j^3 m_j T_{j\|}} \right\} \exp\left\{ -\frac{H_\perp}{T_{j\perp}} \right\} \\
H_\perp &= p_\perp^2 / 2\gamma_j m_j + \gamma_j m_j \omega_{\beta j}^2 r^2 / 2 + e_j(\phi_0 - \beta_j A_{z0})
\end{aligned}
$$

- Nonlinear Poisson Equations for the equilibrium fields ϕ_0 and A_{z0}

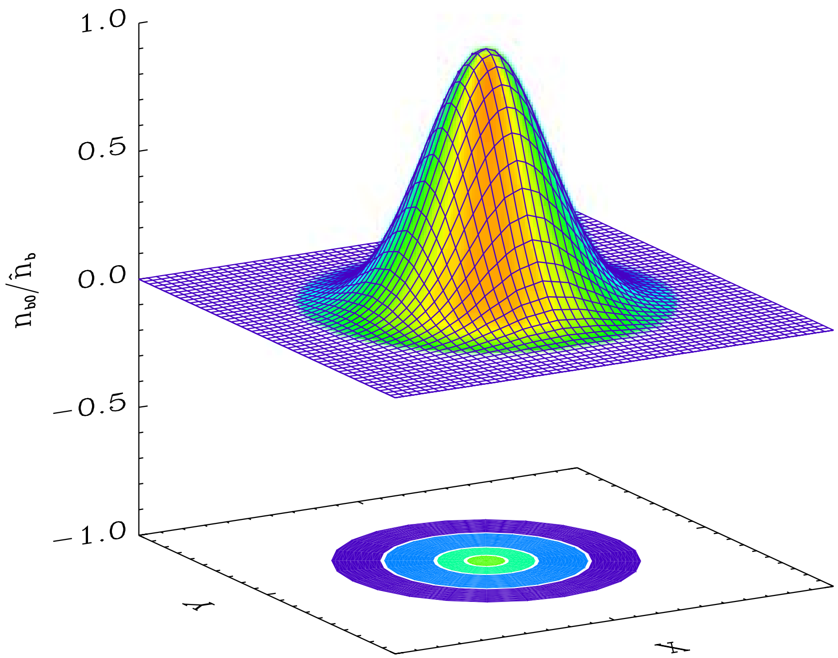
$$
\begin{aligned}
\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \phi_0(r)}{\partial r} &= -4\pi \sum_j e_j \int d^3p f_{j0}(r, \boldsymbol{p}), \\
\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial A_{z0}(r)}{\partial r} &= -\frac{4\pi}{c} \sum_j e_j \int d^3p v_z f_{j0}(r, \boldsymbol{p}).
\end{aligned}
$$

Single Species beam Equilibrium

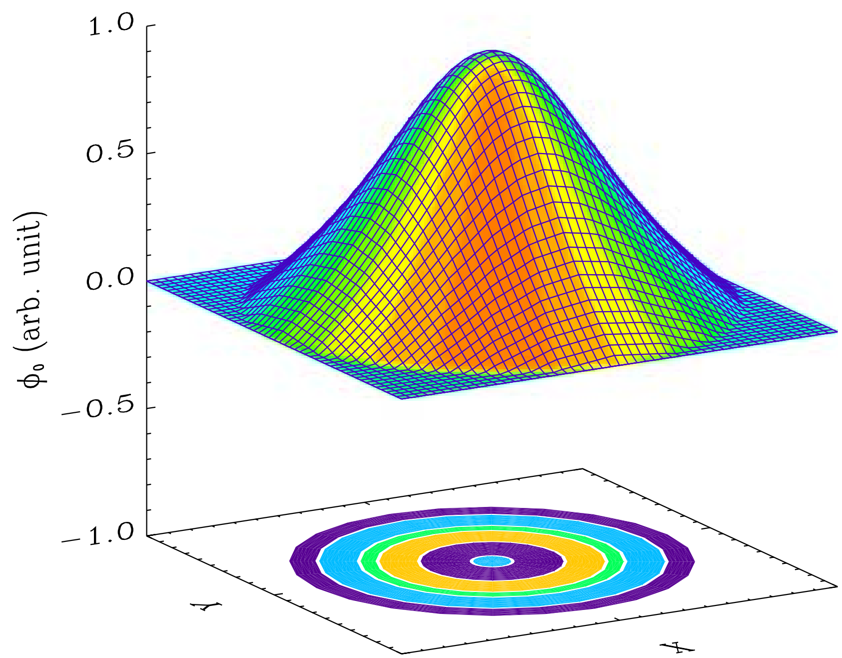
⇨ The space charge intensity is measured by a dimensionless parameter **[Davidson & Qin, 2001]**

$$\boxed{0 \leq s_b \equiv \hat{\omega}_{pb}^2 / 2\gamma_b^2 \omega_{\beta b}^2 \leq 1.}$$

- Heavy Ion Fusion drivers: $s_b = 0.999 \Longrightarrow$ flat-top density profile.
- Proton Storage Ring: $s_b = 0.079 \Longrightarrow$ bell-shape density profile.

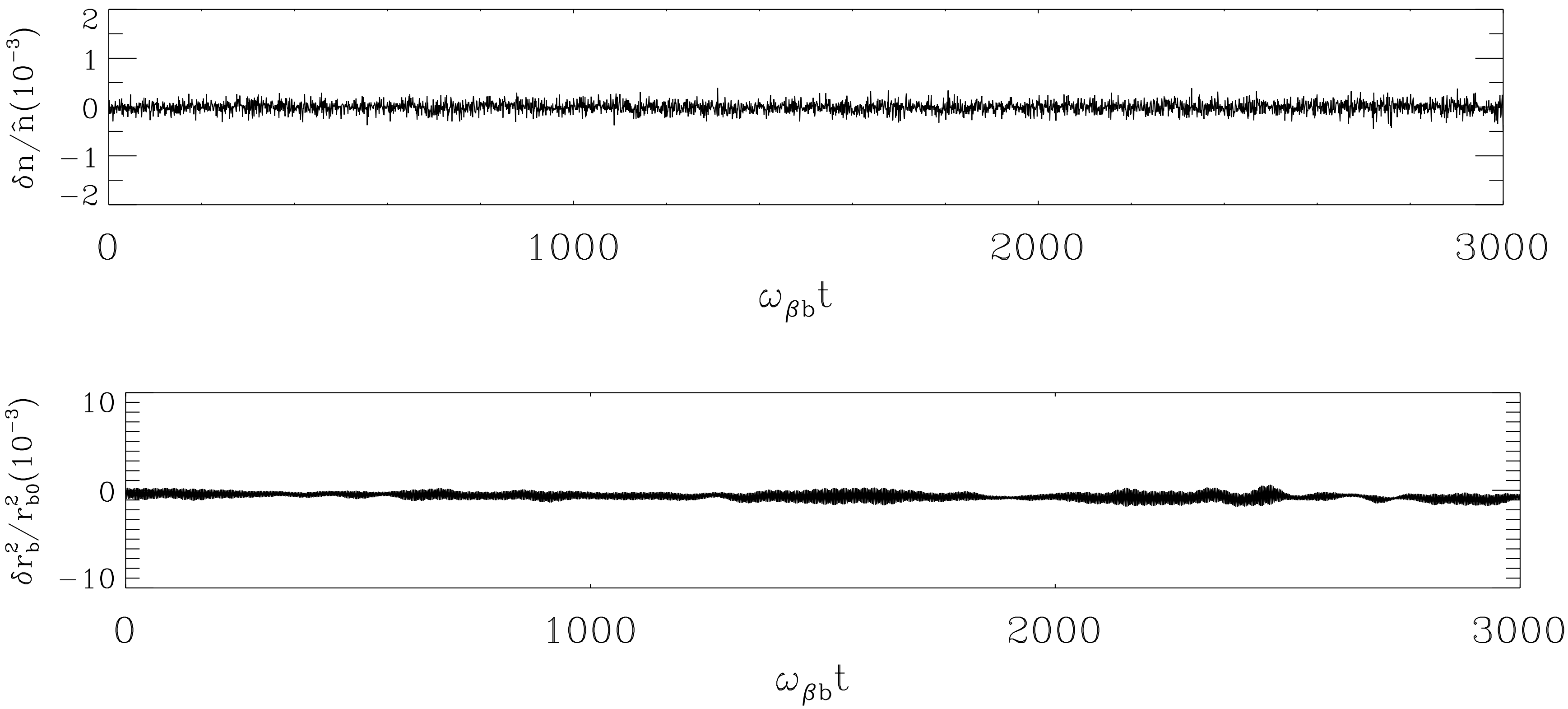


(a) Equilibrium Density



(b) Equilibrium Space-Charge Potential

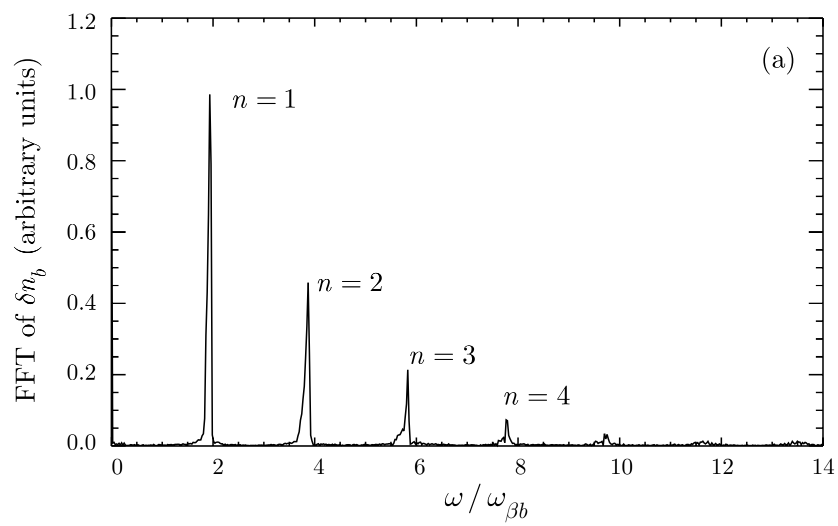
Nonlinear Properties of Stable Beam Propagation



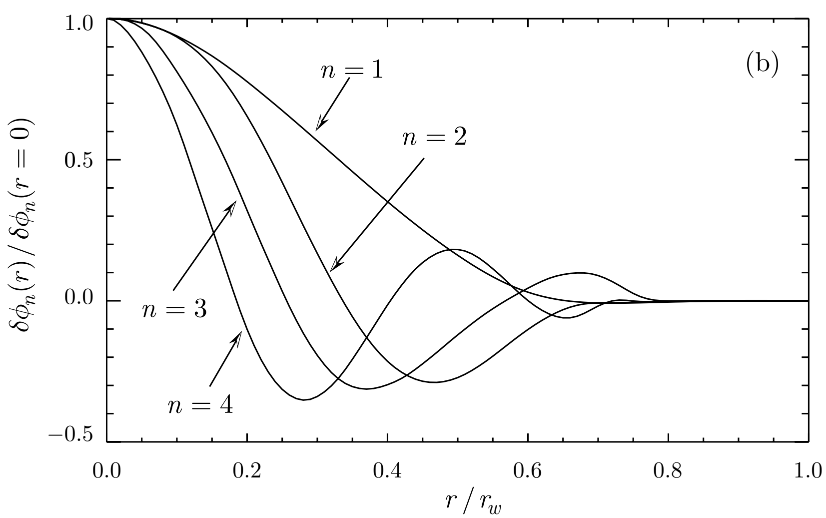
⇨ Simulation results show that the perturbations do not grow and the beam propagates quiescently, which agrees with the nonlinear stability theorem for the choice of thermal equilibrium distribution function **[Davidson, 1998]**.

Axisymmetric Body Modes by the BEST code

- Axisymmetric body modes with $l = 0$ and $k_z = 0$ for a moderate-intensity beam with $s_b \equiv \hat{\omega}_{pb}^2 / 2\gamma_b^2 \omega_{\beta b}^2 = 0.158$.
- First four body eigenmodes of the system at frequencies $\omega_1 = 1.94\ \omega_{\beta b}$, $\omega_2 = 3.87\ \omega_{\beta b}$, $\omega_3 = 5.83\ \omega_{\beta b}$, and $\omega_4 = 7.77\ \omega_{\beta b}$.
- Eigenfunction $\delta\phi_n(r)$ has n zeros when plotted as a function of r.



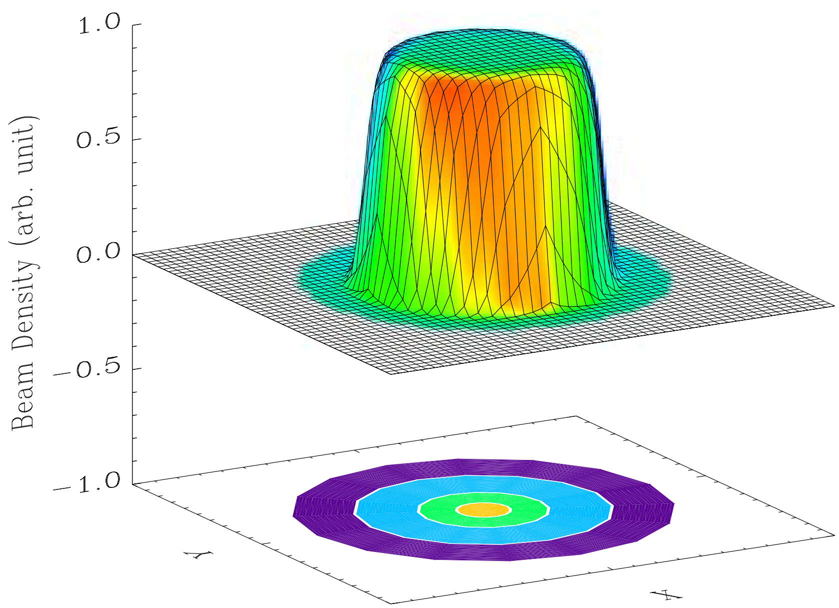
(a) Frequency spectrum



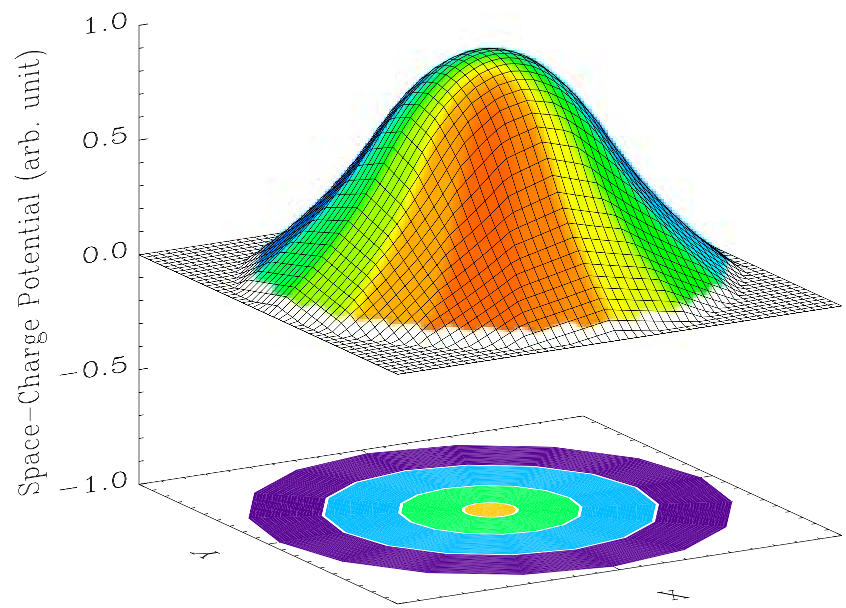
(b) Radial mode structure

Surface Modes

⇨ In the space-charge-dominated regime ($s_b \to 1$), the density profile is flat-top.



(a) Equilibrium Density



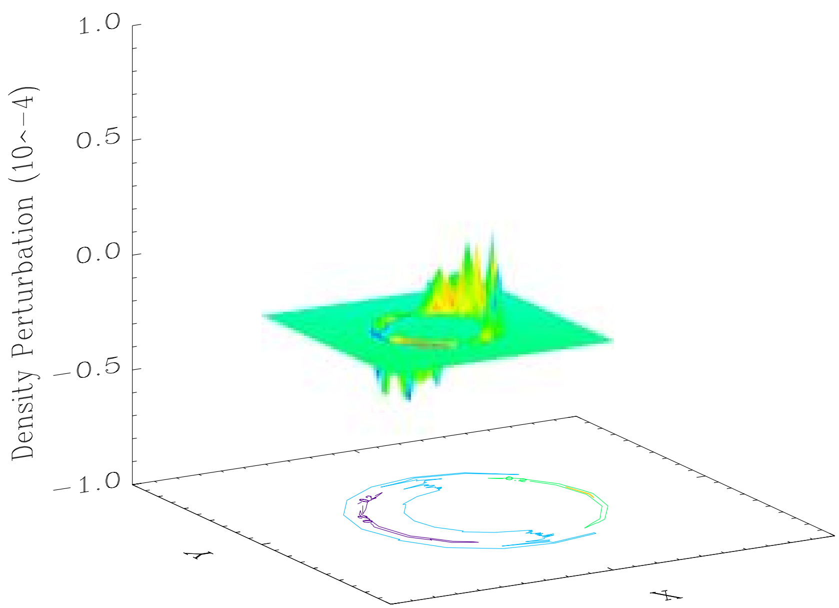
(b) Equilibrium Space-Charge Potential

⇨ There exists a family of linear surface modes.

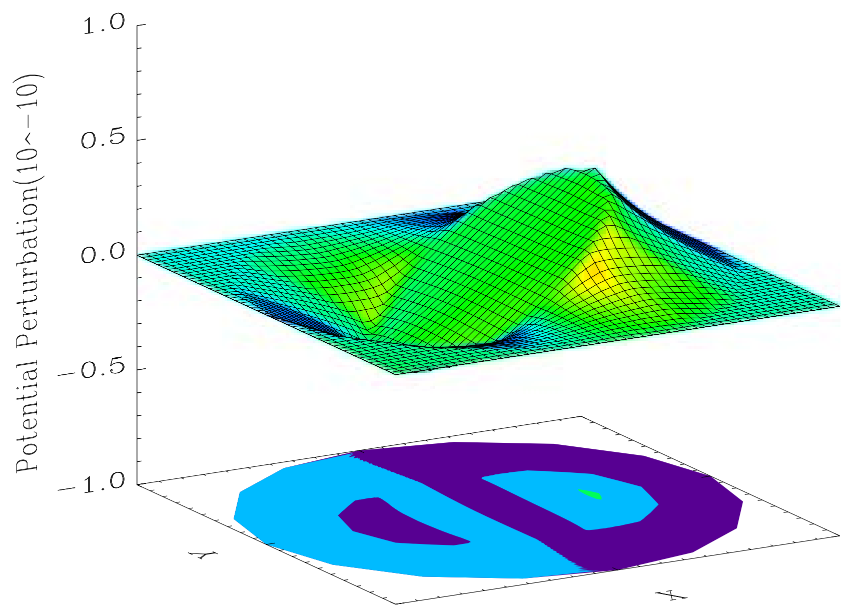
Surface Modes

- The surface modes can be destabilized by the ion-electron two-stream interaction when background electrons are present.

- The BEST code, operating in its linear stability mode, has recovered well-defined eigenmodes which agree with theoretical predications.



(a) Density Perturbation.



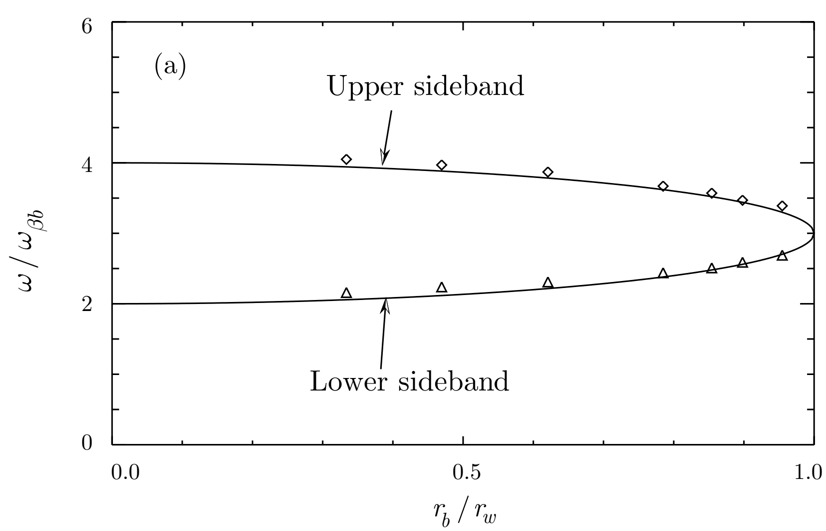
(b) Potential Perturbation.

Surface Modes

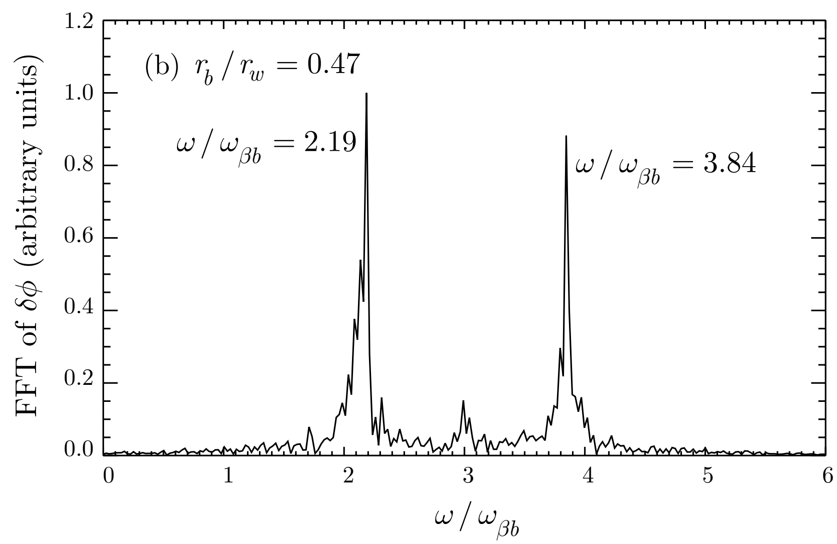
⇨ For azimuthal mode number $l = 1$, the dispersion relation is given by

$$\boxed{\omega = k_z V_b \pm \frac{\hat{\omega}_{pb}}{\sqrt{2}\gamma_b}\sqrt{1 - \frac{r_b^2}{r_w^2}}}$$

- r_b — radius of the beam edge. r_w — radius of the conducting wall.
- $\hat{\omega}_{pb} = 4\pi\hat{n}_b e_b^2/\gamma_b m_b$ — ion plasma frequency.



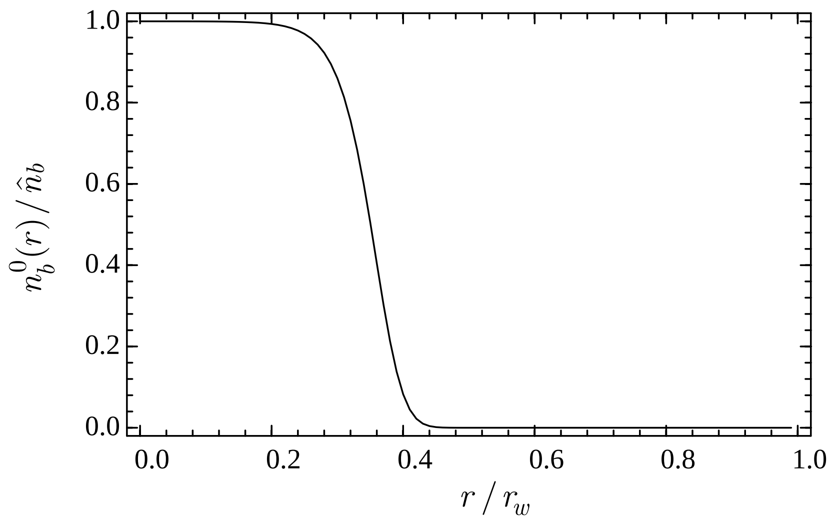
(a) $\omega/\omega_{\beta b}$ versus r_b/r_w



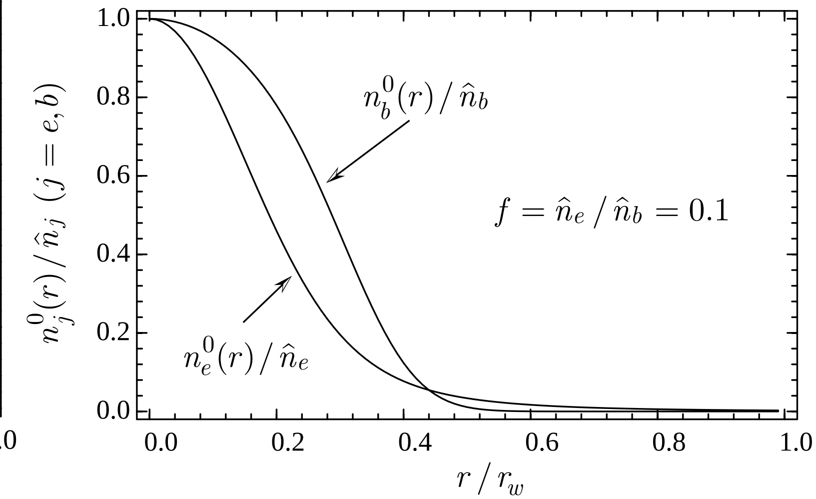
(b) Spectrum for $r_b/r_w = 0.47$

Equilibrium of Ion-Electron Two-Stream System

- Electron population offsets part of the space-charge force.
- Produces bell-shape beam density profile even in the space-charge limit, $s_b = \hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta b}^2 \to 1$.



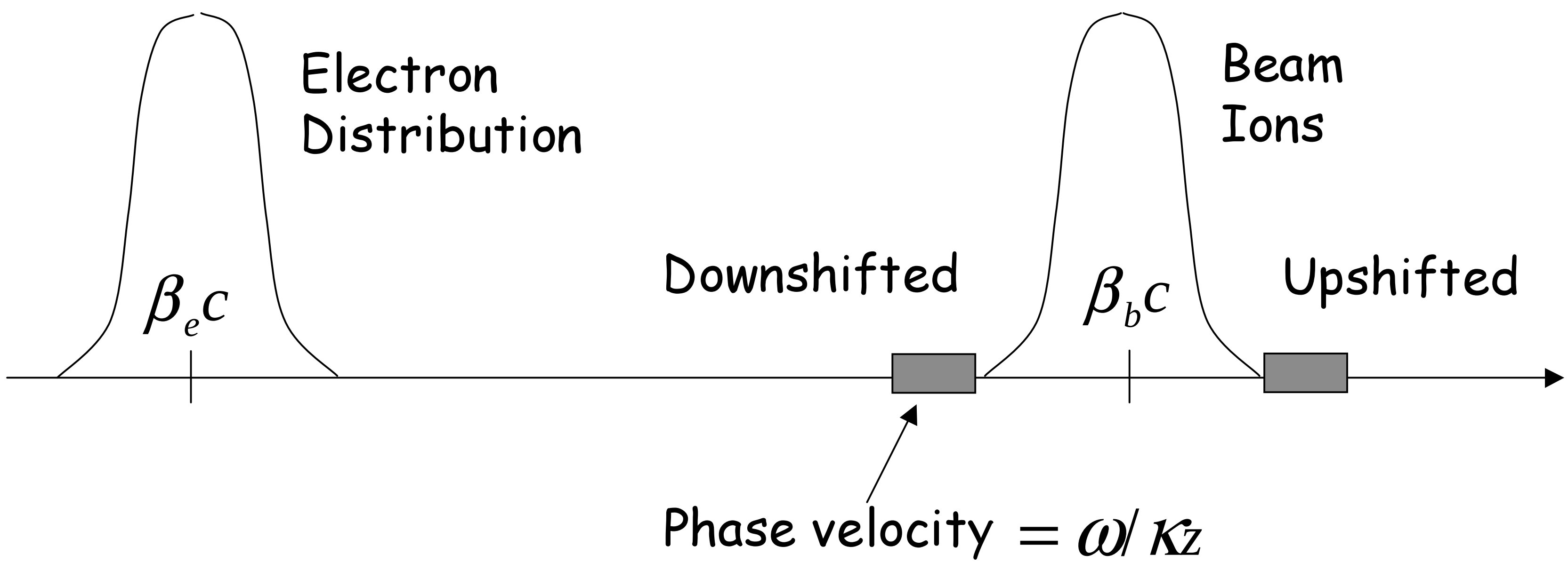
(a) $f = \hat{n}_e/\hat{n}_b = 0$



(b) $f = \hat{n}_e/\hat{n}_b = 0.1$

Two-Stream Instability for Intense Ion Beams

- In the absence of background electrons, an intense nonneutral ion beam supports collective oscillations (sideband oscillations) with phase velocity ω/k_z upshifted and downshifted relative to the average beam velocity $\beta_b c$.
- Introduction of an (unwanted) electron component (produced, for example, by secondary emission of electrons due to the interaction of halo ions with the chamber wall) provides the free energy to drive the classical two-stream instability.

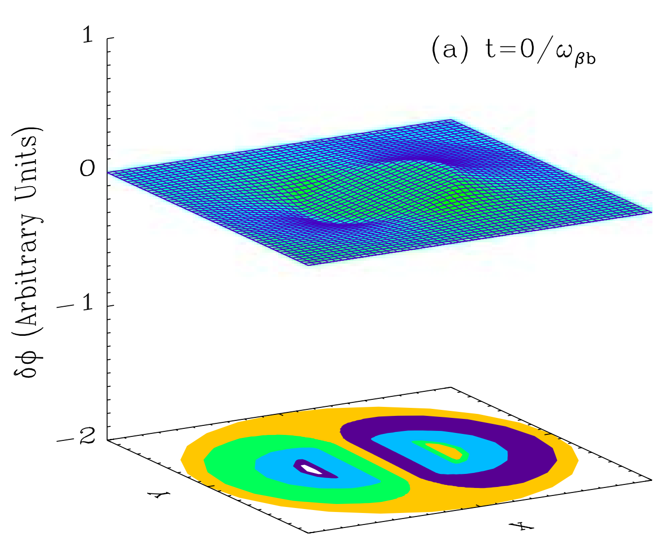


Two-Stream Instability for Intense Ion Beam

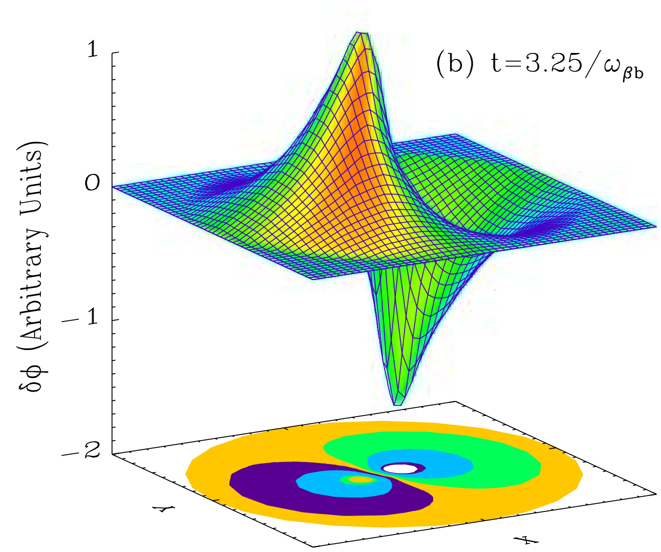
- Unlike the two-stream instability in a homogeneous neutral plasma, the two-stream instability for an intense, thin ion beam depends strongly on:
  - Transverse dynamics and geometry (r_b/r_w, $k_z r_b$).
  - Degree of charge neutralization ($f = \hat{n}_e/\hat{n}_b$).
  - Spread in transverse betatron frequencies due to nonlinear space-charge potential as well as chromaticity.
  - Axial momentum spread.
- Strong experimental evidence for two-species instabilities:
  - Proton Storage Ring (PSR) at Los Alamos National Laboratory.
  - Beam-ion instability in electron machines.
  - Electron cloud instability in hadron machines.

Ion-Electron Two-Stream Instability for Illustrative HIF Parameters

- When a background electron component is introduced with $\beta_e = V_e/c \simeq 0$, the $l = 1$ "surface dipole-mode" can be destabilized for a certain range of axial wavenumber and a certain range of electron temperature T_e.



(a) $t = 0$



(b) $t = 3.25/\omega_{\beta b}$

Ion-Electron Two-Stream Instability for Illustrative HIF Parameters

⇨ The strongest instability occurs for azimuthal mode number $l = 1$ with dispersion relation **[Davidson et al, 2000]**

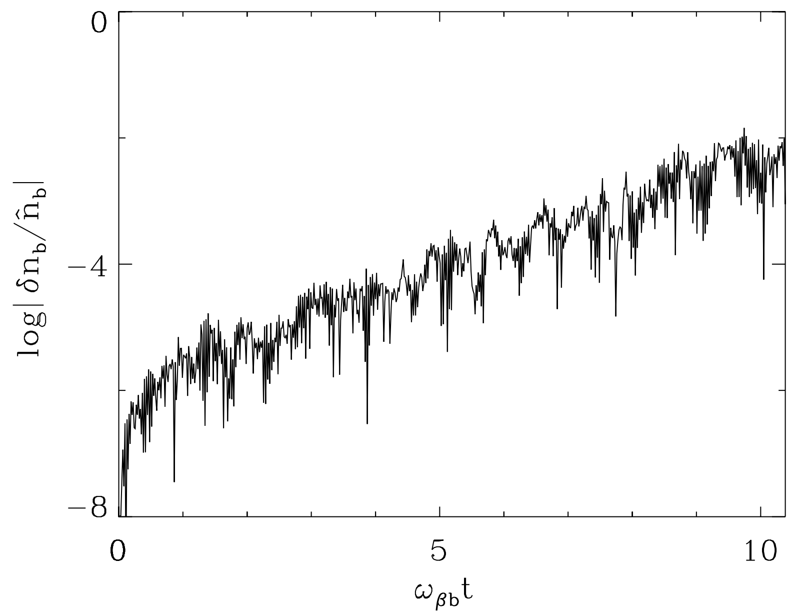
$$[(\omega - k_z V_b + i|k_z|v_{T\|b})^2 - \omega_b^2][(\omega + i|k_z|v_{T\|e})^2 - \omega_e^2] = \omega_f^4 \ .$$

⇨ For $f \neq 0$, the ion and electron modes are coupled by the ω_f^4 term, leading to one unstable mode with $\mathrm{Im}\,\omega > 0$ for a certain range of k_z.

⇨ The $l = 1$ dipole-mode instability has features similar to the hose instability in the collisionless limit **[Lee, 1978]**. Other $l = 1$ modes include electron cloud instability and fast transverse instability.

⇨ For azimuthal mode number $l = 0$ $(\partial/\partial\theta = 0)$, the two-stream dispersion relations have also been derived by Uhm and Davidson for the so-called sausage $(n = 1)$ and hollowing $(n = 2)$ instabilities in the collisionless regime.

Growth Rate for Illustrative HIF Parameters



⇨ Illustrative parameters for a heavy ion fusion driver:

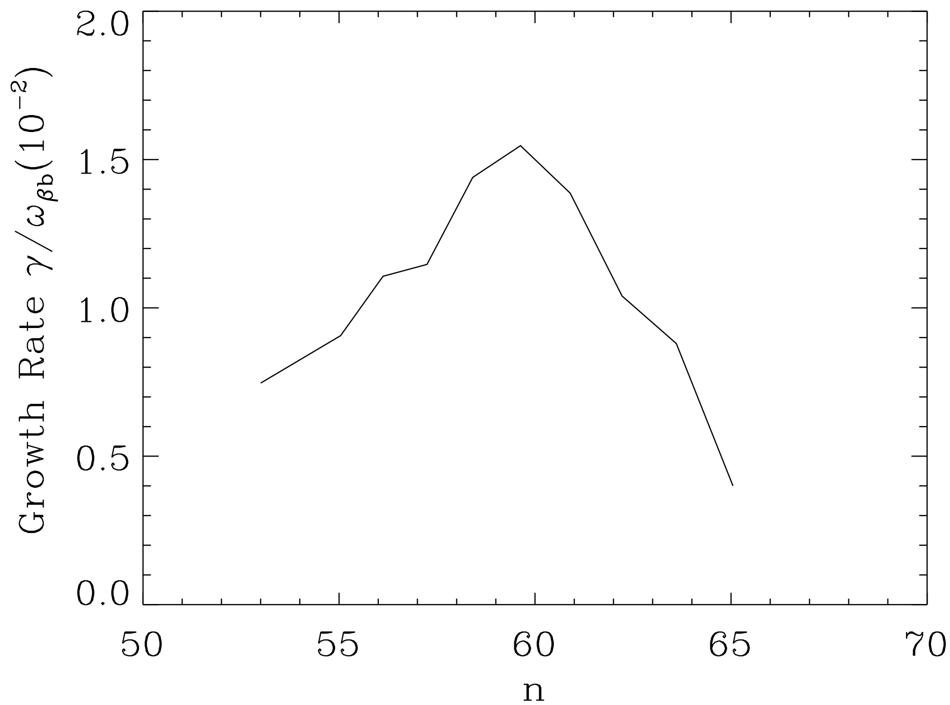
- Cs^+ beam with rest mass $m_b = 133m_p$ and kinetic energy $(\gamma_b - 1)mc^2 =$ 2.5 GeV, $m_e/m_b = 1/(1836 \times 133) = 4.1 \times 10^{-6}$, $V_e = 0$, and $\omega_{\beta e} = 0$.
- $s_b \equiv \hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta b}^2 \to 1$. $f \equiv \hat{n}_e/\hat{n}_b = 0.1$, where $\hat{n}_e$ and $\hat{n}_b$ are the electron and beam ion densities on axis $(r = 0)$.
- $T_{b\perp}/\gamma_b m_b V_b^2 = 1.1 \times 10^{-6}$ and $T_{e\perp}/\gamma_b m_b V_b^2 = 2.47 \times 10^{-6}$.
- The linear growth rate is measured to be $\mathrm{Im}\,\omega = 0.78\omega_{\beta b}$

Electron-Proton Two-stream Instability in PSR

- Proton Storage Ring experiment at Los Alamos is a prototype of the Spallation Neutron Source.
- Electron-proton (e-p) two-stream instability was first observed and experimentally studied at the PSR.
- Illustrative PSR parameters:
  - Coasting or bunched proton beam with $\gamma_b = 1.85$ in a storage ring of 90m circumference.
  - Moderate space-charge intensity corresponding to $\lambda_b = 9.13 \times 10^8$cm^{-1} or $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta b}^2 = 0.079$.
  - 10% fractional neutralization, $\lambda_e = 9.25 \times 10^7$cm^{-1}, $T_{b\perp} = 4.41$keV, $T_{e\perp} = 0.73$keV, $\phi_0(r_w) - \phi_0(0) = -3.08 \times 10^3$Volts, and nonlinear space-charge induced tune shift $\delta\nu/\nu_0 \sim -0.020$.
  - Oscillation frequency (simulations): $f \sim 163$MHz. Mode number at maximum growth $n = 55 \sim 65$.

Simulation of the e-p Instability for PSR

- Large-scale parallel simulations using the BEST code has been carried out the e-p instability in the PSR.



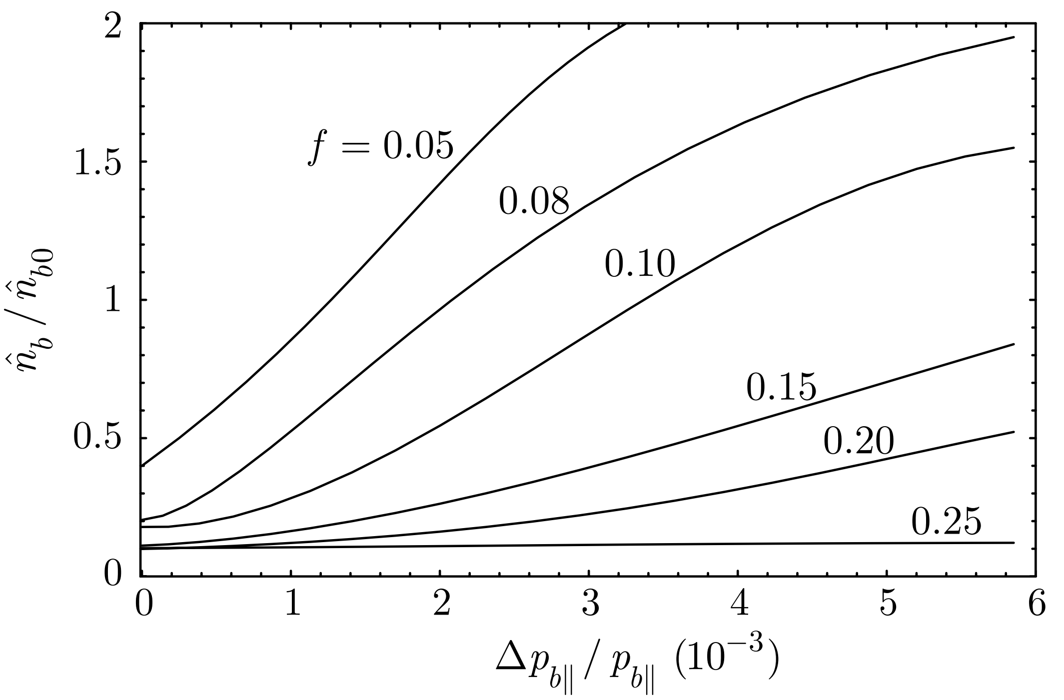
- Simulations results agrees with experimental observations.
  - Characteristic dipole mode structure in unstable linear phase.
  - Mode frequency, growth rate, and wavelength in good agreement.
  - Realistic damping mechanisms in the simulations.

Instability Threshold for Illustrative PSR Parameters

⇨ Important growth rate reduction mechanisms includes

- ❍ Longitudinal Landau damping by the beam ions.
- ❍ Stabilizing effects due to space-charge-induced tune spread.

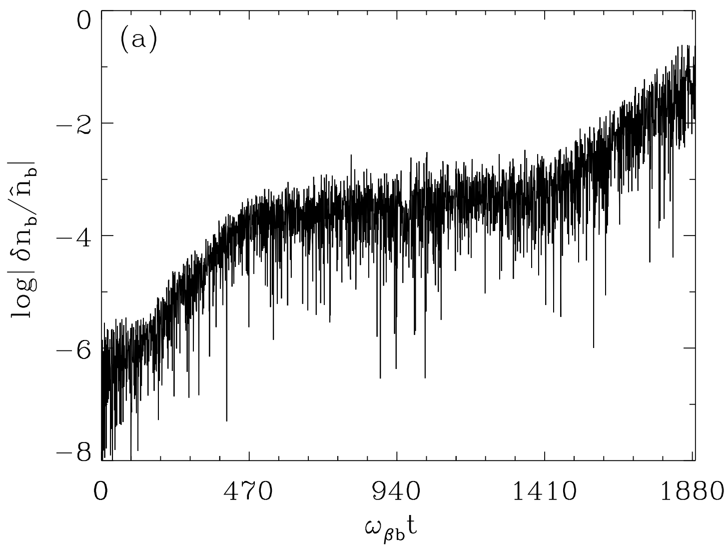
⇨ An instability threshold is observed in the simulations.



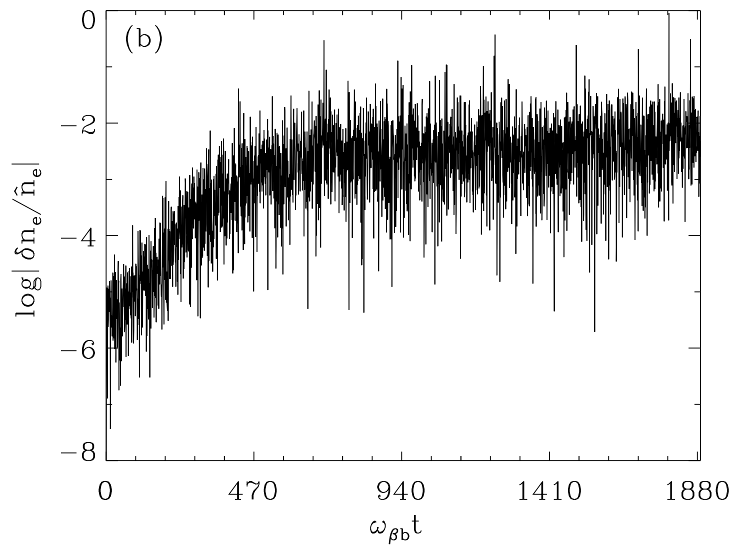
⇨ Larger momentum spread and smaller fractional charge neutralization imply a higher density threshold for the instability to occur.

Nonlinear Phase of the e-p Instability for PSR

- There are two phases — the linear stage and the nonlinear stage.



(a) Proton density perturbation



(b) Electron density perturbation

- The e-p instability in PSR may have entered the second stage of nonlinear growth.
- Possible physical mechanism by Channell: large mass ration induces different nonlinear behaviors.

Conclusions

- ⇨ A 3D multispecies nonlinear perturbative particle simulation method has been developed to study collective processes in intense charged particle beams described self-consistently by the Vlasov-Maxwell equations.
- ⇨ Compared with conventional particle-in-cell simulations, the noise level in nonlinear perturbative particle simulations is significantly reduced.
- ⇨ Implemented in the BEST code, the δf formalism has been tested and applied in different beam parameter regimes.
- ⇨ Linear eigenmodes of high intensity charged particle beams, such as the body modes and the surface modes, have been systematically studied using the BEST code.

Conclusions

- In particular, large-scale parallel simulations have also been carried out to study the ion-electron two-stream instability in the very-high-intensity heavy ion beams envisioned for heavy ion fusion applications, and for the e-p two-stream instability observed in the PSR experiment.

- Important properties of this instability were investigated numerically, and are found to be in qualitative agreement with theoretical predictions and the PSR experiment.

- Numerically, the instability threshold was found to decrease with increasing fractional charge neutralization, and increase with increasing axial momentum spread of the beam particles.

- In the nonlinear phase, the simulation results showed that the instability first saturates at a relatively low level, and subsequently grows to a much larger level.

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