Erratum: Approximate periodically focused solutions to the nonlinear Vlasov-Maxwell equations for intense beam propagation through an alternating-gradient field configuration [Phys. Rev. ST Accel. Beams 2, 074401 (1999)]

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We have detected an algebraic error in our recent paper on Hamiltonian averaging techniques in an alternating-gradient field configuration. As summarized below, the main changes are in the definitions of the coupling coefficients $\delta_x(s)$, $\delta_y(s)$, κ_{fx}, and κ_{fy}. With the corrected definitions, the main results of the original paper remain unchanged, including the coordinate transformation to order ϵ^3, the transformed Hamiltonian and nonlinear Vlasov-Maxwell equations, and the back-transformed properties in the laboratory frame.

The main error occurs in the definition of the third-order Hamiltonian $\mathcal{H}_3$ in Eq. (50), which should include on the right-hand side of Eq. (50) the additional terms

$$x_2 \frac{\partial^2 S_1}{\partial X \partial s} + y_2 \frac{\partial^2 S_1}{\partial Y \partial s} = -\tilde{\kappa}_x(s) x_2 X - \tilde{\kappa}_y(s) y_2 Y \,, \tag{1}$$

where use has been made of Eq. (37), and $\partial S_1/\partial s = -(1/2)[\tilde{\kappa}_x(s)X^2 + \tilde{\kappa}_y(s)Y^2]$. The expression for $\mathcal{H}_3$ in Eq. (51) is then modified to become

$$\mathcal{H}_3 = \beta_x(s)\left[X'^2 - \tilde{\kappa}_x X^2 - X\frac{\partial \psi}{\partial X}\right] + \beta_y(s)\left[Y'^2 - \tilde{\kappa}_y Y^2 - Y\frac{\partial \psi}{\partial Y}\right] + \frac{1}{2}\alpha_x^2(s)X^2 + \frac{1}{2}\alpha_y^2(s)Y^2 + \frac{\partial S_3}{\partial s}. \tag{2}$$

The only changes in the analysis leading to the final expression for the slowly varying Hamiltonian $\mathcal{H}$ defined in Eq. (66) are in the definitions of the coupling coefficients $\delta_x(s)$, $\delta_y(s)$, $\langle\delta_x\rangle$, $\langle\delta_y\rangle$, κ_{fx}, and κ_{fy} in Eqs. (35), (59), and (63). The corrected definitions are given by

$$\begin{aligned} \delta_x(s) &= \alpha_x^2(s), \qquad \delta_y(s) = \alpha_y^2(s)\,, \\ \langle\delta_x\rangle &= \frac{1}{S}\int_0^S ds\, \alpha_x^2(s), \qquad \langle\delta_y\rangle = \frac{1}{S}\int_0^S ds\, \alpha_y^2(s)\,, \\ \kappa_{fx} &= \frac{1}{S}\int_0^S ds[\alpha_x^2(s) - \langle\alpha_x\rangle^2], \qquad \kappa_{fy} = \frac{1}{S}\int_0^S ds[\alpha_y^2(s) - \langle\alpha_y\rangle^2]\,, \end{aligned} \tag{3}$$

where $\alpha_x(s)$ and $\alpha_y(s)$ are defined in Eq. (35). With the corrected definitions given in Eq. (3), the functional form of the transformed Hamiltonian $\mathcal{H}(\tilde{X}, \tilde{Y}, \tilde{X}', \tilde{Y}', s)$ given correct to order ϵ^3 in Eq. (66) remains unchanged, as does the form of the canonical transformation of coordinates in Eqs. (71)–(74), and the definitions of $\alpha_x(s)$, $\alpha_y(s)$, $\beta_x(s)$, and $\beta_y(s)$ in Eq. (35).

Most importantly, the detailed analysis of properties of the nonlinear Vlasov-Maxwell equations in the transformed variables and the back-transformed properties in the laboratory frame presented in Secs. IV and V remain unchanged, provided we make use of the corrected definitions of the coupling coefficients given above in Eq. (3). For example, for the case of a periodic focusing quadrupole field, $\alpha_x(s) = -\alpha_y(s) = \alpha_q(s) \equiv \int_0^s ds\, \kappa_q(s)$, and Eq. (3) reduces to

$$\begin{aligned} \delta_x(s) = \delta_y(s) = \delta_q(s) \equiv \alpha_q^2(s), \qquad \langle\delta_x\rangle = \langle\delta_y\rangle = \langle\delta_q\rangle = \frac{1}{S}\int_0^S ds\, \alpha_q^2(s)\,, \\ \kappa_{fx} = \kappa_{fy} = \kappa_{fq} = \frac{1}{S}\int_0^S ds[\alpha_q^2(s) - \langle\alpha_q\rangle^2]\,, \end{aligned} \tag{4}$$

which are the definitions to be used in Eqs. (67), (68), (79), and (80) and the related analysis of the Vlasov-Maxwell equations in the case of a quadrupole focusing field.

Finally, for the example of a sinusoidal quadrupole lattice $\kappa_q(s) = \hat{\kappa}_q \sin(2\pi s/S)$ considered in Table I, the entries related to the definition of $\delta_q(s)$ should be corrected to read [see Eq. (3)]

$$\delta_q(s) = \frac{\lambda_q^2}{S^2}[1 + \cos^2(k_s s) - 2\cos(k_s s)], \qquad \langle\delta_q\rangle = \frac{3}{2}\frac{\lambda_q^2}{S^2},$$
$$\int_0^s ds[\delta_q(s) - \langle\delta_q\rangle] = \frac{\lambda_q^2}{2\pi S}\left[\frac{1}{4}\sin(2k_s s) - 2\sin(k_s s)\right], \qquad \kappa_{fq}S^2 = \frac{1}{2}\lambda_q^2, \tag{5}$$

where $k_s = 2\pi/S$ and $\lambda_q = \hat{\kappa}_q S^2/2\pi$.

We apologize for the algebraic error, but reiterate that all of the principal results of the original paper remain unchanged, including the coordinate transformation to order ϵ^3, the transformed Hamiltonian and nonlinear Vlasov-Maxwell equations, and the back-transformed properties in the laboratory frame, provided we make use of the corrected coupling coefficients defined above in Eq. (3) [or Eq. (4) for the case of a periodic quadrupole field].