Erratum: Approximate periodically focused solutions to the nonlinear Vlasov-Maxwell equations for intense beam propagation through an alternating-gradient field configuration


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We have detected an algebraic error in our recent paper on Hamiltonian averaging techniques in an alternating-gradient field configuration. As summarized below, the main changes are in the definitions of the coupling coefficients \( \delta_x(s) \), \( \delta_y(s) \), \( \kappa_{fx} \), and \( \kappa_{fy} \). With the corrected definitions, the main results of the original paper remain unchanged, including the coordinate transformation to order \( \epsilon^3 \), the transformed Hamiltonian and nonlinear Vlasov-Maxwell equations, and the back-transformed properties in the laboratory frame.

The main error occurs in the definition of the third-order Hamiltonian \( \mathcal{H}_3 \) in Eq. (50), which should include on the right-hand side of Eq. (50) the additional terms

\[
\begin{align*}
 x_2 \frac{\partial^2 S_1}{\partial x \partial s} + y_2 \frac{\partial^2 S_1}{\partial y \partial s} &= -\tilde{\kappa}_x(s)x_2X - \tilde{\kappa}_y(s)y_2Y, \quad (1)
\end{align*}
\]

where use has been made of Eq. (37), and \( \partial S_1/\partial s = -(1/2) [\tilde{\kappa}_x(s)X^2 + \tilde{\kappa}_y(s)Y^2] \). The expression for \( \mathcal{H}_3 \) in Eq. (51) is then modified to become

\[
\begin{align*}
\mathcal{H}_3 = \beta_x(s) \left[ X'^2 - \tilde{\kappa}_x X^2 - X \frac{\partial \psi}{\partial X} \right] + \beta_y(s) \left[ Y'^2 - \tilde{\kappa}_y Y^2 - Y \frac{\partial \psi}{\partial Y} \right] + \frac{1}{2} \alpha_x^2(s)X^2 + \frac{1}{2} \alpha_y^2(s)Y^2 + \frac{\partial S_3}{\partial s}. \quad (2)
\end{align*}
\]

The only changes in the analysis leading to the final expression for the slowly varying Hamiltonian \( \mathcal{H} \) defined in Eq. (66) are in the definitions of the coupling coefficients \( \delta_x(s) \), \( \delta_y(s) \), \( \langle \delta_x \rangle \), \( \langle \delta_y \rangle \), \( \kappa_{fx} \), and \( \kappa_{fy} \) in Eqs. (35), (59), and (63). The corrected definitions are given by

\[
\begin{align*}
\delta_x(s) &= \alpha_x^2(s), \quad \delta_y(s) = \alpha_y^2(s), \quad \langle \delta_x \rangle = \frac{1}{S} \int_0^S ds \alpha_x^2(s), \quad \langle \delta_y \rangle = \frac{1}{S} \int_0^S ds \alpha_y^2(s), \quad (3)
\end{align*}
\]

\[
\kappa_{fx} = \frac{1}{S} \int_0^S ds[\alpha_x^2(s) - \langle \alpha_x \rangle^2], \quad \kappa_{fy} = \frac{1}{S} \int_0^S ds[\alpha_y^2(s) - \langle \alpha_y \rangle^2],
\]

where \( \alpha_x(s) \) and \( \alpha_y(s) \) are defined in Eq. (35). With the corrected definitions given in Eq. (3), the functional form of the transformed Hamiltonian \( \tilde{\mathcal{H}}(\tilde{X}, \tilde{Y}, \tilde{X}', \tilde{Y}') \) given correct to order \( \epsilon^3 \) in Eq. (66) remains unchanged, as does the form of the canonical transformation of coordinates in Eqs. (71)–(74), and the definitions of \( \alpha_x(s) \), \( \alpha_y(s) \), \( \beta_x(s) \), and \( \beta_y(s) \) in Eq. (35).

Most importantly, the detailed analysis of properties of the nonlinear Vlasov-Maxwell equations in the transformed variables and the back-transformed properties in the laboratory frame presented in Secs. IV and V remain unchanged, provided we make use of the corrected definitions of the coupling coefficients given above in Eq. (3). For example, for the case of a periodic focusing quadrupole field, \( \alpha_x(s) = -\alpha_y(s) = \alpha_q(s) \equiv \int_0^s ds \kappa_{qy} \), and Eq. (3) reduces to

\[
\begin{align*}
\delta_x(s) &= \delta_y(s) = \delta_q(s) = \alpha_q^2(s), \quad \langle \delta_x \rangle = \langle \delta_y \rangle = \langle \delta_q \rangle = \frac{1}{S} \int_0^S ds \alpha_q^2(s), \quad (4)
\end{align*}
\]

which are the definitions to be used in Eqs. (67), (68), (79), and (80) and the related analysis of the Vlasov-Maxwell equations in the case of a quadrupole focusing field.

Finally, for the example of a sinusoidal quadrupole lattice \( \kappa_y(s) = \tilde{\kappa}_y \sin(2\pi s/S) \) considered in Table I, the entries related to the definition of \( \delta_q(s) \) should be corrected to read [see Eq. (3)]
\[ \delta_q(s) = \frac{\lambda_q^2}{S^2} [1 + \cos^2(k_s s) - 2 \cos(k_s s)], \quad \langle \delta_q \rangle = \frac{3}{2} \frac{\lambda_q^2}{S^2}, \]
\[ \int_0^s ds [\delta_q(s) - \langle \delta_q \rangle] = \frac{\lambda_q^2}{2\pi S} \left[ \frac{1}{4} \sin(2k_s s) - 2 \sin(k_s s) \right], \quad \kappa_{f_q} S^2 = \frac{1}{2} \lambda_q^2, \tag{5} \]

where \( k_s = 2\pi / S \) and \( \lambda_q = \kappa_q S^2 / 2\pi \).

We apologize for the algebraic error, but reiterate that all of the principal results of the original paper remain unchanged, including the coordinate transformation to order \( \epsilon^3 \), the transformed Hamiltonian and nonlinear Vlasov-Maxwell equations, and the back-transformed properties in the laboratory frame, provided we make use of the corrected coupling coefficients defined above in Eq. (3) [or Eq. (4) for the case of a periodic quadrupole field].