A Paul trap configuration to simulate intense non-neutral beam propagation over large distances through a periodic focusing quadrupole magnetic field

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This paper considers an intense non-neutral charged particle beam propagating in the z-direction through a periodic focusing quadrupole magnetic field with transverse focusing force, $-\kappa_q(s)\times[x\hat{\mathbf{e}}_x-y\hat{\mathbf{e}}_y]$, on the beam particles. Here, $s=\beta_b ct$ is the axial coordinate, $(\gamma_b-1)m_b c^2$ is the directed axial kinetic energy of the beam particles, q_b and m_b are the charge and rest mass, respectively, of a beam particle, and the oscillatory lattice coefficient satisfies $\kappa_q(s+S)=\kappa_q(s)$, where S is the axial periodicity length of the focusing field. The particle motion in the beam frame is assumed to be nonrelativistic, and the Vlasov-Maxwell equations are employed to describe the nonlinear evolution of the distribution function $f_b(x,y,x',y',s)$ and the (normalized) self-field potential $\psi(x,y,s)=q_b\phi(x,y,s)/\gamma_b^3 m_b\beta_b^2 c^2$ in the transverse laboratory-frame phase space (x,y,x',y'), assuming a thin beam with characteristic radius $r_b\ll S$. It is shown that collective processes and the nonlinear transverse beam dynamics can be simulated in a compact Paul trap configuration in which a long non-neutral plasma column $(L\gg r_p)$ is confined axially by applied dc voltages $\hat{V}=\text{const}$ on end cylinders at $z=\pm L$, and transverse confinement in the $x-y$ plane is provided by segmented cylindrical electrodes (at radius r_w) with applied oscillatory voltages $\pm V_0(t)$ over 90° segments. Here, $V_0(t+T)=V_0(t)$, where $T=\text{const}$ is the oscillation period, and the oscillatory quadrupole focusing force on a particle with charge q and mass m near the cylinder axis is $-m\kappa_q(t)[x\hat{\mathbf{e}}_x-y\hat{\mathbf{e}}_y]$, where $\kappa_q(t)\equiv 8qV_0(t)/\pi m r_w^2$. [S1070-664X(00)01103-4]

I. INTRODUCTION

Periodic focusing accelerators and transport systems[1–6] have a wide range of applications ranging from basic scientific research, to applications such as heavy ion fusion, spallation neutron sources, and nuclear waste treatment, to mention a few examples. Of particular interest, at the high beam currents and charge densities of practical interest, are the combined effects of the applied focusing field and the intense self-fields produced by the beam space charge and current on determining detailed equilibrium, stability, and transport properties.[1] Through analytical studies based on the nonlinear Vlasov-Maxwell equations, and numerical simulations using particle-in-cell models and nonlinear perturbative simulation techniques, considerable progress has been made in developing an improved understanding of the collective processes and nonlinear beam dynamics characteristic of high-intensity beam propagation[7–24] in periodic focusing and uniform focusing transport systems. Nonetheless, it remains important to develop an improved basic understanding of the nonlinear dynamics and collective processes in periodically focused intense charged particle beams, with the goal of identifying operating regimes for stable (quiescent) beam propagation over large distances, including a minimum degradation of beam quality and luminosity.

In this paper, we present in Sec. II a brief summary of the nonlinear Vlasov-Maxwell equations describing the collective processes and nonlinear transverse dynamics of a thin $(r_b\ll S)$, intense charged particle beam propagating through a periodic focusing quadrupole magnetic field with axial periodicity length $S=\text{const}$. In Sec. III, a compact Paul trap[25,26] configuration is described which simulates the equivalent collective processes and nonlinear transverse beam dynamics in a periodic focusing quadrupole transport system. The idea of using a single-species trap to model periodically focused beam propagation has previously been discussed by Okamoto and Tanaka.[27] The emphasis of their work is on solenoidal confinement,[27] whereas the present analysis focuses on periodic quadrupole confinement. In addition, the present analysis treats the case of arbitrary (but periodic) time dependence of the focusing potential.

To briefly summarize, a long non-neutral plasma column $(L\gg r_p)$ is confined axially by applied dc voltages $\hat{V}=\text{const}$ on end cylinders at $z=\pm L$, and transverse confinement in the $x-y$ plane is provided by segmented cylindrical electrodes (at radius r_w) with applied oscillatory voltages $\pm V_0(t)$ over 90° segments (Fig. 1). Here, $V_0(t+T)=V_0(t)$, where $T=\text{const}$ is the oscillation period, and the oscillatory quadrupole focusing force on a particle with charge q and mass m near the cylinder axis is $-m\kappa_q(t)\times[x\hat{\mathbf{e}}_x-y\hat{\mathbf{e}}_y]$, where $\kappa_q(t)\equiv 8qV_0(t)/\pi m r_w^2$. This configuration offers the possibility of simulating intense beam propagation over large distances in a compact configuration which is *stationary* in the laboratory frame. The Paul trap analogy described in the present paper is intended to simu-

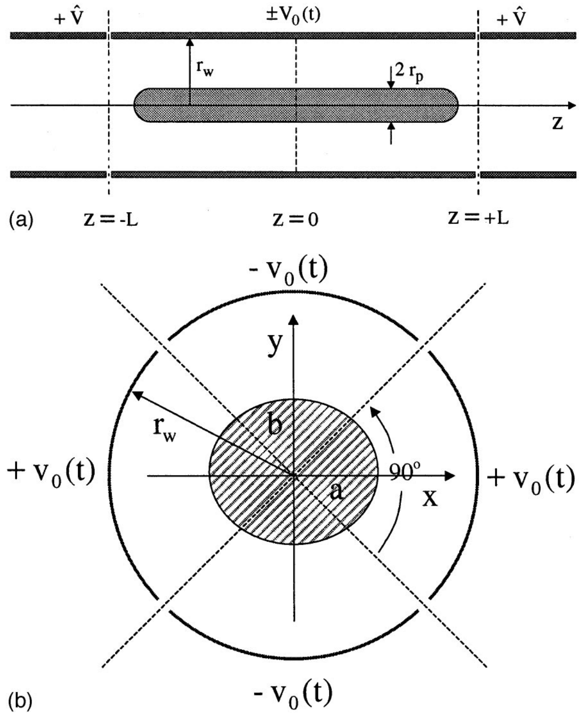

FIG. 1. (a) Axial confinement of a long ($L \gg r_p$) non-neutral plasma column is provided by applied dc voltages $\hat{V}=$const on end cylinders at $z=\pm L$; (b) Transverse confinement of the non-neutral plasma column is provided by cylindrical electrodes at radius $r=r_w$ with applied oscillatory voltages $\pm V_0(t)$ over 90° segments with $V_0(t+T)=V_0(t)$ and $\int_0^T dt V_0(t)=0$.

late continuous beam propagation in a periodic focusing transport line (not a storage ring). In this regard, as indicated in Sec. III, it is important that the trapped plasma be sufficiently long ($L \gg r_p$) that the characteristic bounce frequency for axial motion in Fig. 1 be much smaller than the characteristic transverse oscillation frequency ($\hat{\omega}_z \ll \hat{\omega}_q$) in the applied oscillatory voltage $V_0(t)$.

II. THEORETICAL MODEL FOR INTENSE BEAM PROPAGATION THROUGH A PERIODIC FOCUSING QUADRUPOLE MAGNETIC FIELD

We consider a thin, intense charged particle beam with characteristic radius r_b and average axial momentum $\gamma_b m_b \beta_b c$ propagating in the z-direction through a periodic focusing quadrupole magnetic field with axial periodicity length S. Here, $r_b \ll S$ is assumed, $(\gamma_b-1)m_b c^2$ is the directed axial kinetic energy of the beam particles, $\gamma_b=(1-\beta_b^2)^{-1/2}$ is the relativistic mass factor, $V_b=\beta_b c$ is the average axial velocity, q_b and m_b are the particle charge and rest mass, respectively, and c is the speed of light *in vacuo*. In addition, the particle motion in the beam frame is assumed to be nonrelativistic. We introduce the scaled time variable $s=\beta_b ct$, and the (dimensionless) transverse velocities $x'=dx/ds$ and $y'=dy/ds$. Then, within the context of the assumptions summarized above, the nonlinear beam dynamics in the transverse, laboratory-frame phase space (x,y,x',y') is described self-consistently by the nonlinear Vlasov-Maxwell equations for the distribution function $f_b(x,y,x',y',s)$ and the normalized self-field potential $\psi(x,y,s)=q_b\phi(x,y,s)/\gamma_b^3 m_b \beta_b^2 c^2$, where $\phi(x,y,s)$ is the electrostatic potential. For a thin beam ($r_b \ll S$), the transverse focusing force on a beam particle produced by the periodic quadrupole field can be approximated over the cross section of the beam by

$$\mathbf{F}_{\rm foc}=-\kappa_q(s)[x\hat{\mathbf{e}}_x-y\hat{\mathbf{e}}_y], \tag{1}$$

where (x,y) is the transverse displacement of a particle from the beam axis, and the s-dependent focusing coefficient $\kappa_q(s+S)=\kappa_q(s)$ is defined by

$$\kappa_q(s)=\frac{q_b B_q'(s)}{\gamma_b m_b \beta_b c^2}. \tag{2}$$

Here, the field gradient $B_q'(s)$ is defined by $B_q'(s)=(\partial B_x^q/\partial y)_{(0,0)}=(\partial B_y^q/\partial x)_{(0,0)}$. Note from Eq. (2) that $\kappa_q(s)$ has the dimensions of $(\text{length})^{-2}$. In terms of the normalized self-field potential $\psi(x,y,s)=q_b\phi(x,y,s)/\gamma_b^3 m_b \beta_b^2 c^2$ and the distribution function $f_b(x,y,x',y',s)$, the nonlinear beam dynamics and collective processes in the laboratory-frame transverse phase space (x,y,x',y') are described self-consistently by the Vlasov-Maxwell equations[1,23]

$$\left\{\frac{\partial}{\partial s}+x'\frac{\partial}{\partial x}+y'\frac{\partial}{\partial y}-\left(\kappa_q(s)x+\frac{\partial\psi}{\partial x}\right)\frac{\partial}{\partial x'}\right.$$
$$\left.-\left(-\kappa_q(s)y+\frac{\partial\psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\}f_b=0, \tag{3}$$

and

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi=-\frac{4\pi q_b^2}{\gamma_b^3 m_b \beta_b^2 c^2}\int dx'dy' f_b. \tag{4}$$

Here, $n_b(x,y,s)=\int dx'dy' f_b$ is the number density of the beam particles. Moreover, the laboratory-frame Hamiltonian $\hat{H}_\perp(x,y,x',y',s)$ for transverse single-particle motion consistent with Eqs. (3) and (4) is given (in dimensionless units) by

$$\hat{H}_\perp(x,y,x',y',s)=\tfrac{1}{2}(x'^2+y'^2)+\tfrac{1}{2}\kappa_q(s)(x^2-y^2)$$
$$+\psi(x,y,s). \tag{5}$$

The nonlinear Vlasov-Maxwell equations (3) and (4) are rich in physics content and are widely used to describe the stability and transport properties of an intense non-neutral beam propagating through a periodic focusing quadrupole field $\kappa_q(s+S)=\kappa_q(s)$. While considerable progress has been made in analytical and numerical studies of Eqs. (3) and (4),[7–24] detailed calculations of the equilibrium and stability behavior are generally complex because the quadrupole focusing coefficient $\kappa_q(s)$ is both s-dependent and oscillatory, with $\int_0^S ds\,\kappa_q(s)=0$ for a periodic focusing lattice. Indeed, only recently has a canonical transformation been developed[23,24] that utilizes an expanded generating function that transforms away the rapidly oscillating terms in Eq. (5), leading to a Hamiltonian in the transformed variables,

$\mathcal{H}(\tilde{X},\tilde{Y},\tilde{X}',\tilde{Y}',s)=(1/2)(\tilde{X}'^2+\tilde{Y}'^2)+(1/2)\kappa_{fq}(\tilde{X}^2+\tilde{Y}^2)+\psi(\tilde{X},\tilde{Y},s)$, where $\kappa_{fq}=\text{const}$ (independent of s).

III. COMPACT PAUL TRAP CONFIGURATION TO MODEL PERIODICALLY FOCUSED INTENSE BEAM PROPAGATION OVER LARGE DISTANCES

In practical accelerator applications, if the spacing between quadrupole magnets corresponds (for example) to $S=2m$, and the transverse nonlinear beam dynamics described by Eqs. (3)–(5) is to be followed in detail for 500 lattice periods, then the length of the transport system that is required is 1 km. The obvious question arises as to whether or not it is possible to *model* the nonlinear transverse beam dynamics described by Eqs. (3)–(5) in a *compact* laboratory configuration. The answer is *yes*, and the key is to recognize that the particle motion in the frame of the beam is nonrelativistic, and that the oscillatory quadrupole focusing terms in Eqs. (5) can be simulated in the laboratory frame by applying oscillatory voltages to cylindrical electrodes in a modified Paul trap,[25,26] as illustrated in Fig. 1. A Paul trap[28,29] utilizes oscillatory voltages applied to external electrodes to provide transverse confinement of the non-neutral plasma in the $x-y$ plane, whereas transverse confinement in a Malmberg-Penning trap[30–35] is provided by an applied axial magnetic field $B_0\hat{\mathbf{e}}_z$.

A. Trap configuration

To model an axially continuous charged particle beam (or a very long charge bunch), we consider a long non-neutral plasma column [Fig. 1(a)] with length $2L$ and characteristic radius $r_p \ll L$, confined axially by applied dc voltages $\hat{V}=\text{const}$ on end cylinders at $z=\pm L$. The particles making up the (nonrelativistic) non-neutral plasma in Fig. 1(a) have charge q and mass m. With regard to transverse confinement of the particles in the $x-y$ plane, there is no applied axial magnetic field ($\mathbf{B}_0=B_0\hat{\mathbf{e}}_z=0$). Rather, segmented cylindrical electrodes (at radius r_w) have applied oscillatory voltages $\pm V_0(t)$ over 90° segments with the polarity illustrated in Fig. 1(b). Here, the applied voltage $V_0(t)$ is oscillatory with

$$V_0(t+T)=V_0(t), \qquad \int_0^T dt V_0(t)=0, \tag{6}$$

where $T=\text{const}$ is the period, and $f_0=1/T$ is the oscillation frequency. While different electrode shapes will result in an oscillatory quadrupole potential near the cylinder axis, the configuration shown in Fig. 1(b) is particularly simple and amenable to direct calculation. Neglecting end effects ($\partial/\partial z=0$), and representing the applied electric field by $\mathbf{E}_a=-\nabla_\perp\phi_a(x,y,t)$, where $\nabla_\perp\cdot\mathbf{E}_q=0$ and $\nabla_\perp\times\mathbf{E}_q\simeq 0$, it is readily shown that the solution to $\nabla_\perp^2\phi_a(x,y,t)=0$ that satisfies the appropriate boundary conditions at $r=r_w$ in Fig. 1(b) is given by

$$\phi_a(x,y,t)=\frac{4V_0(t)}{\pi}\sum_{l=1}^{\infty}\frac{\sin(l\pi/2)}{l}\left(\frac{r}{r_w}\right)^{2l}\cos(2l\theta) \tag{7}$$

for $0\leqslant r\leqslant r_w$ and $0\leqslant\theta\leqslant 2\pi$. Near the cylinder axis ($r\ll r_w$), Eq. (7) readily gives to lowest order,

$$q\phi_a(x,y,t)=\tfrac{1}{2}m\kappa_q(t)(x^2-y^2), \tag{8}$$

where the oscillatory quadrupole focusing coefficient $\kappa_q(t)$ is defined by

$$\kappa_q(t)\equiv\frac{8qV_0(t)}{m\pi r_w^2}. \tag{9}$$

From Eqs. (6) and (9), note that $\kappa_q(t+T)=\kappa_q(t)$ and $\int_0^T dt\kappa_q(t)=0$. Moreover, $\kappa_q(t)$ has dimensions of $(\text{time})^{-2}$. Most importantly, from Eq. (7), the leading-order correction to Eq. (8) is of order $(1/3)(r/r_w)^4$. Therefore, for example, if the characteristic radial dimension r_p of the plasma column in Fig. 1 satisfies $r_p/r_w\lesssim 0.1$, then the corrections to the simple quadrupole potential in Eq. (7) are smaller than one part in 10^4 over the transverse region occupied by the plasma particles. That is, for sufficiently small r_p/r_w, Eq. (8) is a highly accurate representation of the applied quadrupole focusing potential $\phi_a(x,y,t)$. Use of 60° electrodes instead of 90° electrodes could in principle be used to minimize the potential contribution proportional to $(r/r_w)^6$.

We now construct the Hamiltonian for the transverse particle motion, neglecting axial variations ($\partial/\partial z=0$). Denoting the (dimensional) transverse particle velocities by $\dot{x}=dx/dt$ and $\dot{y}=dy/dt$, and the self-field electrostatic potential due to the plasma space charge by $\phi_s(x,y,t)$, it readily follows that the (dimensional) Hamiltonian $H_\perp(x,y,\dot{x},\dot{y},t)$ describing the transverse particle motion is given by

$$H_\perp(x,y,\dot{x},\dot{y},t)=\tfrac{1}{2}m(\dot{x}^2+\dot{y}^2)+\tfrac{1}{2}m\kappa_q(t)(x^2-y^2)+q\phi_s(x,y,t), \tag{10}$$

where use has been made of Eq. (8). The striking feature of the transverse Hamiltonian in Eq. (10) is that it is identical in functional form to the transverse Hamiltonian defined in Eq. (5) provided we make the replacements

$$\begin{aligned} &t\rightarrow s,\\ &(\dot{x},\dot{y})\rightarrow(x',y'),\\ &\frac{q}{m}\phi_s(x,y,t)\rightarrow\psi(x,y,s),\\ &\kappa_q(t)[\text{Eq. (9)}]\rightarrow\kappa_q(s)[\text{Eq. (2)}],\\ &\frac{1}{m}H_\perp(x,y,\dot{x},\dot{y},t)\rightarrow\hat{H}_\perp(x,y,x',y',s), \end{aligned} \tag{11}$$

in Eq. (10). Therefore, the collective processes and nonlinear transverse dynamics described by Eq. (10) and the configuration in Fig. 1 are fully equivalent to the collective processes and nonlinear transverse dynamics described by Eq. (5) for an intense non-neutral beam propagating through a periodic focusing quadrupole magnetic field, provided we make the replacements in Eq. (11). For example, intense beam propagation through 500 quadrupole magnet lattice periods S is equivalent to studying the transverse dynamics of the compact non-neutral plasma configuration in Fig. 1 (which is axially *stationary* in the laboratory frame) for 500 oscillation periods T of the voltage $V_0(t)$.

For completeness, consistent with Eq. (10) and Fig. 1, we summarize here the nonlinear Vlasov-Poisson equations describing the self-consistent evolution of the distribution function $f(x,y,\dot{x},\dot{y},t)$ and self-field electrostatic potential $\phi_s(x,y,t)$ in the transverse phase space $(x,y,\dot{x},\dot{y})$. Of course, the characteristics of the nonlinear Vlasov equation correspond to the single-particle orbit equations calculated from Eq. (10), with $d\mathbf{x}_\perp/dt=m^{-1}\partial H_\perp/\partial\dot{\mathbf{x}}_\perp$ and $d\dot{\mathbf{x}}_\perp/dt=-m^{-1}\partial H_\perp/\partial\mathbf{x}_\perp$. It readily follows that the nonlinear Vlasov-Poisson equations for $f(x,y,\dot{x},\dot{y},t)$ and $\phi_s(x,y,t)$ consistent with the Hamiltonian in Eq. (10) can be expressed as

$$\left\{\frac{\partial}{\partial t}+\dot{x}\frac{\partial}{\partial x}+\dot{y}\frac{\partial}{\partial y}-\left(\kappa_q(t)x+\frac{q}{m}\frac{\partial}{\partial x}\phi_s\right)\frac{\partial}{\partial\dot{x}}-\left(-\kappa_q(t)y+\frac{q}{m}\frac{\partial}{\partial y}\phi_s\right)\frac{\partial}{\partial\dot{y}}\right\}f_b=0, \tag{12}$$

and

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_s=-4\pi q\int d\dot{x}d\dot{y}f, \tag{13}$$

where $n(x,y,t)=\int d\dot{x}d\dot{y}f$ is the particle number density. As expected, the collective processes and transverse plasma dynamics described by the nonlinear Vlasov-Poisson equations (12) and (13) for the non-neutral plasma configuration in Fig. 1 are *identical* to those described by Eqs. (3) and (4) for an intense beam propagating through a periodic focusing quadrupole magnetic field, provided we make the replacements in Eq. (11).

As noted earlier, the Paul trap analogy described in this section is intended to simulate the transverse dynamics of a *continuous* beam propagating in a periodic focusing transport line. Furthermore, the Hamiltonian in Eq. (10) and the nonlinear Vlasov-Poisson equations (12) and (13) describe only the transverse dynamics of the long non-neutral plasma column $(L\gg r_p)$ in Fig. 1, and z-variations and axial particle motions are not included in the description. While such a model is expected to provide a good description of the transverse dynamics of the plasma column for $L\gg r_p$, there are important limitations on the range of applicability of the Paul trap analogy for simulating the propagation of a continuous beam through a periodic focusing lattice. Most importantly, the non-neutral plasma column illustrated in Fig. 1 is confined axially, and the particles execute axial bounce motion between the ends of the plasma column (at $z=\pm L$). If we denote the characteristic thermal speed of a particle by v_{Th} (assumed for present purposes to be similar in the axial and transverse directions), then the characteristic bounce frequency for the axial motion of a particle is $\hat{\omega}_z=2\pi/T_z$, where $T_z\sim 4L/v_{Th}$ is the period. We denote the characteristic oscillation frequency for transverse particle motion in the oscillatory quadrupole potential by $\hat{\omega}_q=2\pi/T_q$ (see Sec. III B), where T_q is the period for transverse motion. At low-to-moderate density, the period T_q and characteristic plasma radius r_p are related approximately by $T_q\sim 2r_p/v_{Th}$. Therefore, in an approximate sense, the transverse and axial oscillation frequencies and periods stand in the ratio $\hat{\omega}_z/\hat{\omega}_q=T_q/T_z\sim r_p/2L\ll 1$ (by assumption). On a time scale $t\sim T_z$, the (finite-length) effects of the axial bounce motion of particles in the Paul trap configuration illustrated in Fig. 1 can become important, and limit the validity of the Paul trap analogy with the propagation of a continuous beam through a periodic quadrupole lattice. For sufficiently large $L\gg r_p$, however, the axial bounce period T_z can be very long. As illustrative parameters, consider the case where $r_p=1$ cm, $2L=200$ cm, and the frequency $f_0=1/T$ of the applied oscillatory voltage $V_0(t)$ is $2\pi f_0\sim 4\hat{\omega}_q$. In this case, $T_z\sim 200T_q\sim 800T$, where T is the oscillation period of $V_0(t)$. In this case, a typical particle in Fig. 1 experiences the effects of 800 oscillation periods of the quadrupole focusing potential (800 equivalent lattice periods) before it executes one axial bounce in the trap.

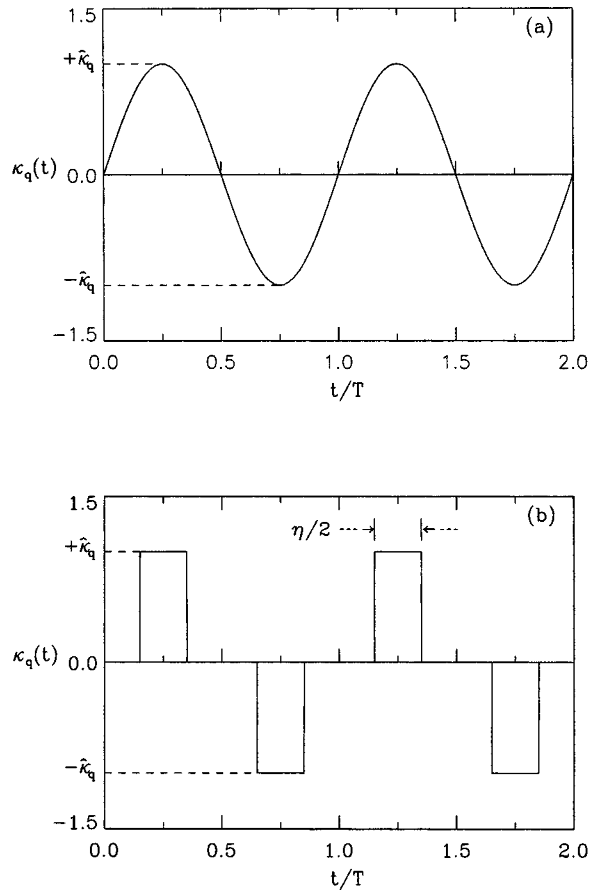

FIG. 2. Illustrative oscillatory waveforms for the quadrupole focusing coefficient $\kappa_q(t)=(8q/\pi m)V_0(t)$ corresponding to (a) the sinusoidal waveform $\kappa_q(t)=\hat{\kappa}_q\sin(2\pi t/T)$, where $\hat{\kappa}_q=$const, and (b) a periodic step-function with waveform with maximum amplitude $\hat{\kappa}_q$ and filling factor η.

B. Operating range

Typical oscillatory waveforms for the quadrupole focusing coefficient $\kappa_q(t)=(8q/\pi m)V_0(t)$ defined in Eq. (9) are illustrated in Fig. 2. Here, Fig. 2(a) corresponds to a sinusoidal waveform with $\kappa_q(t)=\hat{\kappa}_q \sin(2\pi t/T)$, where $\hat{\kappa}_q$ $=\text{const}$ and $T=1/f_0$ is the oscillation period, and Fig. 2(b) corresponds to a periodic step-function lattice with maximum amplitude $\hat{\kappa}_q$ and filling factor η.

The oscillatory applied potential, $(m/2)\kappa_q(t)(x^2-y^2)$, in Eq. (10) [or, equivalently, $(1/2)\kappa_q(s)(x^2-y^2)$ in Eq. (5)] typically results in a non-neutral plasma column (or intense charged particle beam) that has a pulsating elliptical cross section in the $x-y$ plane.[23,26] In this regard, it is convenient to denote the on-axis $(r=0)$ plasma density by $\hat{n}$ and the corresponding plasma frequency by $\hat{\omega}_p \equiv (4\pi\hat{n}q^2/m)^{1/2}$. From Eq. (10), we further denote the characteristic (angular) oscillation frequency $\hat{\omega}_q$ for the transverse motion of a single particle in the (maximum) focusing field by $\hat{\omega}_q \equiv |\hat{\kappa}_q|^{1/2}$ $=|8q\hat{V}_0/\pi m r_w^2|^{1/2}$, where $\hat{V}_0=|V_0(t)|_{\max}$ is the maximum applied voltage. Transverse confinement[23] of the non-neutral plasma by the field requires $\hat{\omega}_p/\sqrt{2}<\hat{\omega}_q$. On the other hand, applicability of Hamiltonian averaging techniques[23,24] typically requires that the oscillation frequency f_0 of the applied voltage $V_0(t)$ be sufficiently large and that the maximum voltage $\hat{V}_0$ be sufficiently small that $2\pi f_0$ exceeds $\hat{\omega}_q$ by a sufficiently large amount. Combining these inequalities gives

$$\frac{1}{\sqrt{2}}\,\hat{\omega}_p<\hat{\omega}_q \ll 2\pi f_0, \tag{14}$$

or equivalently,

$$\frac{1}{\sqrt{2}\,2\pi}\left(\frac{4\pi\hat{n}q^2}{m}\right)^{1/2}<\frac{1}{2\pi}\left|\frac{8q\hat{V}_0}{\pi m r_w^2}\right|^{1/2} \ll f_0. \tag{15}$$

The inequalities in Eq. (15) are expected to assure robust confinement of the plasma particles by the oscillatory voltage in Fig. 1. With regard to the right-most inequalities in Eqs. (14) and (15), to assure applicability of Hamiltonian averaging techniques,[23,24] and also to avoid the important envelope instability[4] associated with an overly strong focusing field, the oscillation frequency f_0 of the applied voltage $V_0(t)$ should be several times larger than $\hat{\omega}_q/2\pi$.

Equation (15) applies to either a single-species pure ion plasma or to a pure electron plasma. For a non-neutral electron plasma ($q=-e$ and $m=m_e$), which is relatively simple to create and confine in a practical sense,[31–35] Eq. (15) becomes

$$6.35\times 10^3(\hat{n})^{1/2}<1.07\times 10^7\,\frac{(\hat{V}_0)^{1/2}}{r_w} \ll f_0, \tag{16}$$

where $\hat{n}$, $\hat{V}_0$, r_w, and f_0 are expressed in units of cm^{-3}, volts, cm, and s^{-1}, respectively. As illustrative design parameters for a pure electron plasma, we take $\hat{V}_0=100\,\text{V}$ and $r_w=10\,\text{cm}$. Equation (16) then gives the requirements that $\hat{n}<2.8\times 10^6\,\text{cm}^{-3}$ and that f_0 exceed several tens of MHz, which are both tractable requirements from a practical standpoint.[31–35] For a pure ion plasma, the requirements on the oscillation frequency f_0 are less stringent. For example, for protons ($m=m_p$, $q=+e$, and $m_e/m_p=1/1836$), assuming $V_0=100\,\text{V}$ and $r_w=10\,\text{cm}$, Eq. (15) gives $\hat{n}<2.8$ $\times 10^6\,\text{cm}^{-3}$, and f_0 should be in the MHz range in order to satisfy the right-most inequality in Eq. (15).

In concluding this section, we reiterate that the main purpose of this paper is to show the analogy between the transverse particle dynamics in a long non-neutral plasma column ($L \gg r_p$) confined in a Paul trap [Eqs. (10), (12), and (13)] and the transverse particle dynamics in a continuous charged particle beam propagating through a periodic focusing quadrupole field [Eqs. (3)–(5)]. While it is not the purpose of this paper to present an experimental design, it should be noted that many of the experimental techniques [see, for example, Refs. 31–35] for plasma formation, density diagnostics, transverse temperature diagnostics, etc., developed by the non-neutral plasma trap community over the years are expected to be applicable. For example, a pure electron plasma column many thermal Debye lengths in diameter can be formed using a spiral tungsten filament.[31–35] The electrons would flow into the trap region from the left (say) in Fig. 1, and then be trapped axially by applying dc end voltages.

IV. CONCLUSIONS

In summary, in this paper we presented in Sec. II a brief description of the nonlinear Vlasov-Maxwell equations describing the collective processes and nonlinear transverse dynamics of a thin ($r_b \ll S$), intense charged particle beam propagating through a periodic focusing quadrupole magnetic field with axial periodicity length $S=\text{const}$. In Sec. III, a compact Paul trap configuration was described, which fully simulates the equivalent collective processes and nonlinear transverse beam dynamics in a periodic focusing transport system. This configuration (Fig. 1) offers the possibility of simulating intense beam propagation over large distances in a compact configuration which is *stationary* in the laboratory frame.

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