

Nonlinear δF Simulation Studies of Intense Charged Particle Beams with Large Pressure Anisotropy*

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Abstract

In this paper, a 3D nonlinear perturbative particle simulation code (BEST) is used to systematically study the stability properties of intense nonneutral charged particle beams with pressure anisotropy ($P_{\perp} > P_{\parallel}$). The most unstable modes are identified and their eigenfrequencies and radial mode structure are determined for axisymmetric perturbations with $\partial/\partial\theta = 0$.

1 INTRODUCTION

It's well known that in plasmas with strongly anisotropic distributions ($T_{\parallel b}/T_{\perp b} \ll 1$) a collective instability may develop if there is sufficient coupling between the transverse and longitudinal degrees of freedom [1, 2, 3]. Such anisotropies develop naturally in accelerators, where the longitudinal temperature of the accelerated beam of charged particles with charge q accelerated by a voltage V is reduced according to $T_{\parallel bf} = T_{\parallel bi}^2/2qV$ (for a nonrelativistic beam). At the same time, the transverse temperature may increase due to nonlinearities in the applied and self-field forces, nonstationary beam profiles, and beam mismatch. These processes provide the free energy to drive collective instabilities and may lead to a deterioration of beam quality.

2 LINEAR THEORY

Wang and Smith [2] investigated the kinetic stability properties of an intense particle beam assuming a KV distribution in the limit of large energy anisotropy ($T_{\parallel b}/T_{\perp b} \rightarrow 0$) by expanding the solution of the linearized Vlasov equation in a series of Gluckstern eigenfunctions $\delta\varphi_n(r) = (1/2)[P_{n-1}(1 - 2r^2/r_b^2) + P_n(1 - 2r^2/r_b^2)]$, where $P_n(x)$ is the n th order Legendre Polynomial. The expansion yields a dispersion relation, expressible in terms of an infinite matrix determinant. For long-wavelength perturbations with $k_z^2 r_b^2 \ll 1$, one-half of the modes are identified as transverse (T_n) Gluckstern modes with $\delta\varphi \propto \delta\varphi_n$. The other half consists of modes corresponding in the limit $\nu \rightarrow 0$ to an ordinary cold-beam longitudinal mode (L_1) with $\delta\varphi \propto I_0(k_z r)$ inside the beam and $\omega^2 = (\hat{\omega}_{pb}^2/2)(k_z r_b)^2 \ln(r_w/r_b)$, plus a less-known class of "coupling" modes (L_n) with $\delta\varphi \propto \delta\varphi_n$ and $\omega^2 = [\hat{\omega}_{pb}^2/8n(n+1)](k_z r_b)^2 \int_0^{2\pi} (dx/2\pi) P_n(\cos x)$. The latter modes are the result of the interaction between transversely oscillating particles and the longitudinal perturbed

potential. Here, $\nu = \nu_0(1 - s_b)^{1/2}$ is the depressed tune, where $s_b = \hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta b}^2$ is the normalized beam intensity, $\hat{\omega}_{pb}^2 = 4\pi\hat{n}_b e_b^2/\gamma_b m_b$ is the plasma frequency-squared, $\nu_0 = \omega_{\beta b}$ is the transverse betatron frequency associated with the applied focusing field, r_b is the beam radius, and r_w is the radius of the perfectly conducting wall.

As a general rule, for a KV distribution, instability arises in the regions of parameter space where two or more modes interact resonantly. The transverse modes (T_n) are not significantly affected by longitudinal perturbations, and therefore the instability due to their interaction is a consequence of the fact that the KV distribution has a highly inverted population in phase space [2, 3, 4]. The most dangerous $T_n - L_k$ instabilities are due to $T_2 - L_1$ interactions [2] in the region where $\nu/\nu_0 \simeq 0.44$ with maximum growth rate $Im\omega/\nu_0 \simeq 0.03$, and to $T_2 - L_2$ interactions in the region $0.2 \leq \nu/\nu_0 \leq 0.32$ with maximum growth rate $Im\omega/\nu_0 \simeq 0.15$. The latter mode has a much higher growth rate due to the similar transverse structure of the L_2 and T_2 modes. The growth rate is a maximum for $k_z^2 r_b^2 \geq 1$ in both cases.

To remedy the problem arising from an unphysical KV distribution, Davidson and Strasburg [1] employed a warm-fluid model to investigate the stability properties of intense charged beams with pressure anisotropy[5]. The kinetic and fluid descriptions predict instability for different ranges of beam intensity. In the following section we compare these results with the simulation results using the nonlinear δf simulation technique [6, 7].

3 DESCRIPTION OF THE BEST NONLINEAR δF SIMULATION CODE

The theoretical models described in Sec. 2 use simplified assumptions for the background distribution. In practice, the transverse distribution function is close to thermal equilibrium with temperature $T_{\perp b}$, and the longitudinal distribution can be described by a drifting Maxwellian distribution with temperature $T_{\parallel b} \ll T_{\perp b}$. This distribution is stable with respect to transverse perturbations [4]. For an arbitrary equilibrium distribution one cannot solve the stability problem analytically and must employ numerical simulation techniques. To investigate stability properties numerically, we use the nonlinear δf method [6] described below, as implemented in the Beam, Equilibrium, Stability and Transport (BEST) code [7].

In the smooth-focusing approximation, the transverse focusing force is modeled by $\mathbf{F}_{foc} = -\gamma_b m_b \omega_{\beta b}^2 \mathbf{x}_{\perp}$, where

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$\gamma_b = (1 - \beta_b^2)^{1/2}$ is the relativistic mass factor, and $\beta_b c$ is the axial beam velocity. The solutions to the nonlinear Vlasov-Maxwell equations are expressed as $f_b = f_b^0 + \delta f_b$, $\varphi = \varphi^0 + \delta\varphi$ and $A_z = A_z^0 + \delta A_z$, where $(f_b^0, \varphi^0, A_z^0)$ are known equilibrium solutions. The perturbed potentials satisfy the equations [7]

$$\nabla^2 \delta\varphi = -4\pi e_b \int d^3p \delta f_b, \quad (1)$$

$$\nabla^2 \delta A_z = -\frac{4\pi}{c} e_b \int d^3p v_z \delta f_b, \quad (2)$$

where $\delta f_b(\mathbf{x}, \mathbf{p}, t)$ is given by the weighted Klimontovich representation,

$$\delta f_b = \frac{N_b}{N_{sb}} \sum_{i=1}^{N_{sb}} w_{bi} \delta(\mathbf{x} - \mathbf{x}_{bi}) \delta(\mathbf{p} - \mathbf{p}_{bi}). \quad (3)$$

Here, N_{sb} is total number of beam simulation particles, N_b is total number of actual beam particles, and the weight function is defined by $w_b \equiv \delta f_b / f_b$.

The nonlinear particle simulations are carried out by iteratively advancing the particle motion, including the weights they carry, according to

$$\frac{d\mathbf{x}_{bi}}{dt} = (\gamma_b m_b)^{-1} \mathbf{p}_{bi}, \quad (4)$$

$$\frac{d\mathbf{p}_{bi}}{dt} = -\gamma_b m_b \omega_{\beta b}^2 \mathbf{x}_{\perp bi} - e_b (\nabla\varphi - \frac{v_{zbi}}{c} \nabla_{\perp} A_z), \quad (5)$$

$$\frac{dw_{bi}}{dt} = -(1 - w_{bi}) \frac{1}{f_{b0}} \frac{\partial f_{b0}}{\partial \mathbf{p}} \cdot \delta \left(\frac{d\mathbf{p}_{bi}}{dt} \right), \quad (6)$$

$$\delta \left(\frac{d\mathbf{p}_{bi}}{dt} \right) = -e_b (\nabla\delta\varphi - \frac{v_{zbi}}{c} \nabla_{\perp} \delta A_z), \quad (7)$$

and updating the fields by solving the perturbed Maxwell's equations with appropriate boundary conditions at the cylindrical, perfectly conducting wall at radius r_w . The δf approach is fully equivalent to the original nonlinear Vlasov-Maxwell equations, but the noise associated with representation of the background distribution f_{b0} in conventional particle-in-cell (PIC) simulations is removed.

4 SIMULATION RESULTS

Here we present the simulation results for a continuous, anisotropic beam in a constant focusing field. The equilibrium distribution function is assumed to be

$$f_0(r, \mathbf{p}_{\perp}) = \frac{\hat{n}_b}{(2\pi\gamma_b m_b)^{3/2} \gamma_b T_{\perp b} T_{\parallel b}^{1/2}} \times \exp \left\{ -\frac{(p_{\parallel} - \gamma_b m_b \beta_b c)^2}{2\gamma_b^3 m_b T_{\parallel b}} \right\} \times \exp \left\{ -\frac{p_{\perp}^2 / 2\gamma_b m_b + \gamma_b m_b \omega_{\beta b}^2 r^2 / 2 + e_b (\varphi_0 - \beta_b A_{z0})}{T_{\perp b}} \right\}, \quad (8)$$

where \hat{n}_b is the beam density at $r = 0$, and $T_{\perp b}$ and $T_{\parallel b}$ are the transverse and longitudinal temperatures of the beam particles. The equilibrium self-field potentials (φ_0, A_{z0}) are determined numerically from Maxwell's

equations [7]. It is also assumed that the beam is located inside a grounded, cylindrical, perfectly conducting wall at radius $r_w = 3r_b$, where $r_b = [\langle r^2 \rangle]^{1/2}$ is the rms beam radius. Random initial perturbations are introduced to the particle weights, and the beam is propagated from $t = 0$ to $t = 800\omega_{\beta b}^{-1}$. The initial temperature ratio is taken to be $T_{\parallel b}/T_{\perp b} = 0.04$. The simulations are performed for a wide range of normalized beam intensities ranging from $s_b = 0.1$ to $s_b = 0.95$, and detailed stability properties have been determined for the range of intensity parameters satisfying $s_b \geq 0.5$ assuming axisymmetric perturbations with $\partial/\partial\theta = 0$. Shown in Fig.1 is the time history of the density perturbation $\delta n_b = \int d^3p \delta f_b$ for normalized beam intensity $s_b = 0.7$. After the initial linear growth, the instability saturates at the moderately large level $|\delta n_b^{max}/\hat{n}_b| \simeq 0.05$.

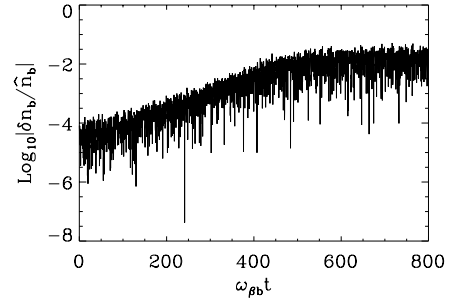


Figure 1: Time history of the normalized density perturbation $\delta n_{max}/\hat{n}_b$ for beam intensity $s_b = 0.7$ at fixed z and $r = 0.2r_b$.

The net change in the longitudinal momentum distribution $\delta F_b(p_z)/\hat{F}_{0b}$ at the conclusion of the simulation is shown in Fig.2. Here $\delta F_b(p_z) = \int d^2p_{\perp} d^3x \delta f_b$ and $\hat{F}_{0b} = \hat{n}_b / (2\pi\gamma_b^3 m_b T_{\parallel b})^{1/2}$. The formation of tails in axial momentum space in Fig.2 and the consequent saturation of the instability are attributed to quasilinear stabilization. Figures 3 and 4 show plots of the real and imaginary parts

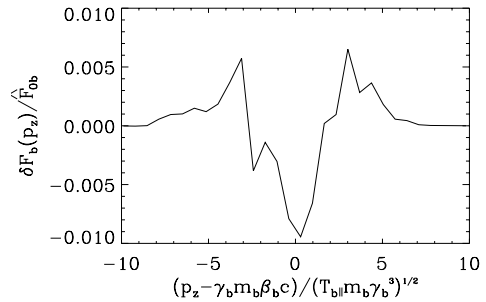


Figure 2: Perturbed longitudinal momentum distribution $\delta F_b(p_z)/\hat{F}_{0b}$ at time $t = 800\omega_{\beta b}^{-1}$, for normalized beam intensity $s_b = 0.7$.

of the complex oscillation frequency ω versus normalized

axial wavenumber $k_z r_b$. The instability has a finite bandwidth with maximum growth rate $Im\omega/\omega_{\beta b} \simeq 0.02$ at $k_z r_b = 2.5$. The unstable mode structure is shown in Fig.5

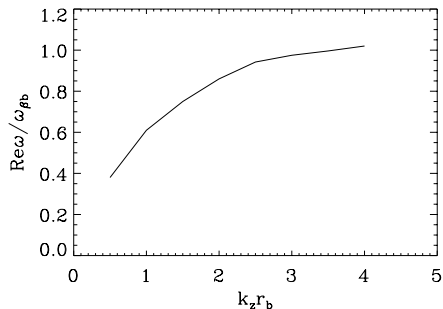


Figure 3: Normalized eigenfrequency $Re\omega/\omega_{\beta b}$ plotted versus $k_z r_b$ for $s_b = 0.7$.

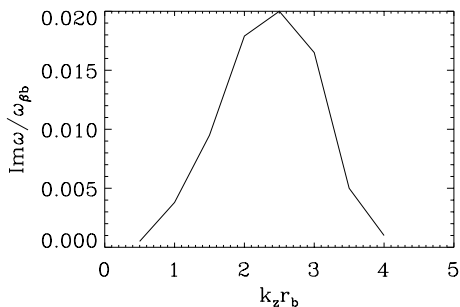


Figure 4: Normalized growth rate $Im\omega/\omega_{\beta b}$ plotted versus $k_z r_b$ for $s_b = 0.7$.

for $k_z r_b = 2.5$. The dispersion relation and mode structure (Fig.5) are similar to the well-known longitudinal (L_1) mode for a cold beam [2]. For long wavelengths $k_z^2 r_b^2 \ll 1$, the dispersion relation is linear with $Re\omega$ proportional to $k_z r_b$. For short wavelengths $k_z^2 r_b^2 \gg 1$, the transverse beam size is unimportant and $Re\omega \simeq 1.03\omega_{\beta b}$. The de-

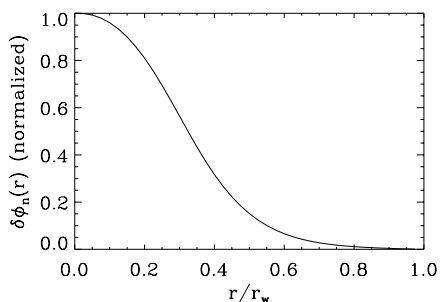


Figure 5: Radial mode structure of the unstable eigenfunction for $k_z r_b = 2.5$ and $s_b = 0.7$.

pendence of the maximum growth rate $(Im\omega)_{max}/\omega_{\beta b}$ on

beam intensity s_b is shown in Fig.6. The maximum growth rate $(Im\omega)_{max}/\omega_{\beta b} \simeq 0.038$ occurs for $s_b \simeq 0.8$, with no instability in the region $s_b \leq 0.5$.

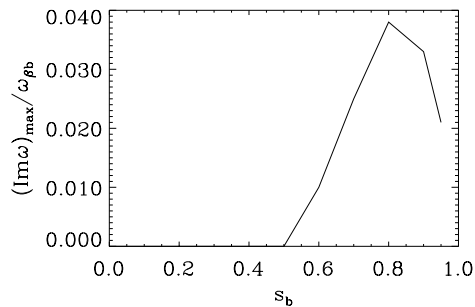


Figure 6: Plot of $(Im\omega)_{max}/\omega_{\beta b}$ versus normalized beam intensity s_b .

Finally, in the present simulations, the instability is found to be absent if the ratio of initial longitudinal and transverse temperatures is greater than the threshold value $T_{||b}/T_{\perp b} = 0.07$.

5 CONCLUSIONS

The BEST code [7], which implements the nonlinear δf scheme, has been used to investigate the stability properties of intense charged particle beams with large temperature anisotropy ($T_{||b}/T_{\perp b} \ll 1$). The simulation results clearly show that moderately intense beams $s_b \geq 0.5$ are linearly unstable to short wavelength perturbations with $k_z^2 r_b^2 \geq 1$, provided the ratio of longitudinal and transverse temperatures is smaller than some threshold value. The mode structure, growth rate and the onset of the instability are qualitatively similar to what is predicted for a KV beam for the unstable $T_2 - L_1$ mode interaction. In the nonlinear saturation stage, the total distribution function is still far from equipartitioned, and free energy is available to drive an instability of the hydrodynamic type [1].

6 REFERENCES

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