Analytical and Numerical Studies of the Complex Interaction of a Fast Ion Beam Pulse with a Background Plasma

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Abstract
Plasma neutralization of an intense ion beam pulse is of interest for many applications, including plasma lenses, heavy ion fusion, high energy physics, etc. Comprehensive analytical, numerical and experimental studies are underway to investigate the complex interaction of a fast ion beam with a background plasma. The positively charged ion beam attracts plasma electrons, and as a result the plasma electrons have a tendency to neutralize the beam charge and current. A suite of particle-in-cell codes has been developed to study the propagation of an ion beam pulse through the background plasma. For quasi-steady-state propagation of the ion beam pulse, an analytical theory has been developed using the assumption of long charge bunches and conservation of generalized vorticity. The analytical results agree well with the results of the numerical simulations. The visualization of the data obtained in the numerical simulations shows complex collective phenomena during beam entry into and exit from the plasma.

1. Introduction
Neutralization of the ion beam charge and current by a background plasma is an important issue for many applications involving the transport of positive charges in plasma, including heavy ion inertial fusion [1], positron for electron-positron colliders [2], high-density laser-produced proton beams for the fast ignition of inertial confinement fusion targets [3], etc. In these applications, plasma is used to reduce the high self-electric and self-magnetic fields of the beam.

The case where the beam propagates through a cold plasma, with the plasma density large compared with the beam density, can be studied by the use of linear perturbation theory [4]. In this paper, we focus on the nonlinear case where the plasma density has an arbitrary value compared with the beam density. The transport of stripped, pinched ion beams has also been discussed in [1], where the assumptions of current and charge neutrality were made to determine self-consistent solutions for the electric and magnetic fields. Rosenbluth et al., have considered the equilibrium of an isolated, charge-neutralized, self-pinched ion beam pulse in the absence of background plasma [5]. In contrast, we consider here the case where “fresh” plasma is always available in front of the beam, and there are no electrons co-moving with the beam.

The electron response frequency is of order the electron plasma frequency, \( \omega_p = (4\pi n_e e^2/m_e)^{1/2} \), where \( n_e \) is the background plasma density. For heavy ion fusion applications, the ion pulse propagation time through the chamber is much longer than the inverse electron plasma frequency \( \omega_p^{-1} \). Therefore, a beam-plasma quasi-steady state forms during beam propagation. The steady-state propagation (in the beam frame) of an ion beam pulse through a background plasma has been investigated in Refs. [6–9].

In recent calculations [6,7], we studied the nonlinear quasi-equilibrium properties of an intense ion beam pulse propagating through a cold background plasma, assuming that the beam pulse duration \( \tau_b \) is much longer than the inverse electron plasma frequency, i.e., \( \omega_p \tau_b \gg 1 \). In a related study [8], we extended the previous results to general values of the parameter \( \omega_p \tau_b \). Here, complex collective phenomena are found during beam entry into and exit from the plasma.

2. Basic equations for ion beam pulse propagation in background plasma

Treating the beam ions and plasma ions as infinitely massive, the electron momentum equation together with Maxwell’s equations comprise a complete system of equations describing the electron response to a propagating ion beam pulse. The electron momentum equation is given by,

\[
\frac{dp_e}{dt} = -e \left( E + \frac{1}{c} V_e \times B \right),
\]

where \(-e\) is the electron charge, \(V_e\) is the electron velocity, \(p_e = \gamma_e m_e V_e\) is the electron momentum, \(m_e\) is the electron rest mass, and \(\gamma_e\) is the relativistic mass factor. Maxwell’s equations for the self-generated electric and magnetic fields, \(\mathbf{E}\) and \(\mathbf{B}\), are given by

\[
\nabla \times \mathbf{B} = \frac{4\pi e}{c} (Z_b n_b V_b - n_e V_e) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},
\]

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},
\]

where \(V_b\) is the ion beam flow velocity, \(n_e\) and \(n_b\) are the number densities of the plasma electrons and beam ions, respectively (the background plasma is uniform with density \(n_p\), and far away from the beam \(n_e \rightarrow n_p\)), and \(Z_b\) is the ion beam charge state. The plasma ions are assumed to remain stationary with \(V_i = 0\) and \(n_i = n_p\). In reality the plasma ions tend to be expelled by the ion beam self-electric field, but for short ion pulses, the plasma ions do not have time to move. The assumption of immobile plasma ions is valid for sufficiently short ion pulses with \(2\lambda_b < r_b \sqrt{M/m_e} \) [6]. Here, \(r_b\) and \(2\lambda_b\) are the ion beam radius and length, respectively, and \(M\) is the plasma ion mass.

Note that considerable simplification can be achieved by applying the conservation of generalized vorticity \(\Omega\) [6]. If \(\Omega\) is initially equal to zero ahead of the beam, and all
streamlines inside of the beam originate from the region ahead of the beam, then $\Omega$ remains equal to zero everywhere, i.e.,

$$\Omega = \nabla \times \mathbf{p} - \frac{e}{c} \mathbf{B} = 0.$$  \hfill (4)

If most electrons are dragged along with the beam and originate from the region of large magnetic field, the situation may be different [5,9].

3. Collective phenomena during beam entry into and exit from the plasma

In plasma focusing schemes, the ion beam pulse after acceleration enters the plasma for focusing or transport purposes. It is important to access the transition to steady-state propagation in the background plasma [1,2]. As the intense ion beam enters the plasma, the plasma electrons readily move in the strong electric and magnetic fields. Because the plasma electron density is comparable with the ion beam density, the plasma electrons can generate current and space charge as high as the ion beam current and charge. As a result, the ion-beam self fields change significantly in the plasma. Consequently, the electron motion during the entry of the intense beam into the plasma exhibits highly complex collective behavior as shown in Figs. 1–4. The simulations presented in Figs. 1–4 are obtained using the edPIC particle-in-cell code [10]. For fast intense ion beams, the self-magnetic field $B$ of the beam ions affects considerably the electron motion. For the parameters in Figs. 1 and 2, the electron cyclotron frequency $\omega_c = eB/m_ec$ is comparable with the electron plasma frequency $\omega_p$. As a result, the electrons penetrate into the beam initially only in the beam center, where the magnetic field is small. Therefore, when the beam density is increased relative to the background plasma density, $\omega_c$ increases compared with $\omega_p$, thereby increasing the effect of the magnetic field on the electron dynamics (compare Figs.

![Fig. 1. Beam entry into the background plasma. The beam propagates in the x-direction. Shown in the figure are normalized electron density $(n_e/n_p)$ obtained in particle-in-cell simulations. The beam velocity is $V_b = 0.5c$, and the beam density is $n_b = 0.5n_p$. The beam pulse dimensions correspond to radius $r_b = 1.5c/\omega_p$ and half length $l_b = 7.5c/\omega_p$ (the beam pulse duration $\tau_b$ is long compared with the inverse plasma frequency $\omega_p$); the beam density profile is flattop with smooth edges at 20% of the beam size; see Ref. [4] for more details. Shown in the figure are contours at four different times after the beam enters the plasma at successive time intervals of $3\pi/\omega_p$. White contours show the position of the edges of the ion beam.](image-url)
Fig. 2. Beam entry into the background plasma similar to Fig. 1, but for a more intense ion beam. The beam density is \( n_b = 5n_p \); the beam pulse dimensions correspond to radius \( r_b = 1.5c/\omega_p \), and half length \( l_b = 30c/\omega_p \) (the beam pulse duration \( \tau_b \) is long compared with the inverse plasma frequency \( \omega_p, \tau_b = 120 \)). Shown in the figure are contours at four different times after the beam enters the plasma at successive time intervals of \( 20\tau_p/\omega_p \).

Fig. 3. Beam entry into the background plasma for the same conditions as in Fig. 1 but at later times. Figure 3(a) is taken at a time interval \( 3\pi/\omega_p \) later than Fig. 1(d), and Fig. 3(b) is taken at a time interval \( 30\pi/\omega_p \) later than Fig. 3(a). Note that the hole velocity in the beam frame is approximately \( 6c/30\pi \approx c/16 \) or 1/8th of the beam velocity.
1 and 2). Figure 2 shows the development of hose-like structures and electron holes during the beam entry, which are absent in Fig. 1.

Because the ion beam collects electrons predominantly in the transverse direction, large electron holes (absence of electrons) appear near the plasma boundaries. In Fig. 2, the electron holes “break” after some time, and an electron stream moves from the “bottom” of the hole upward with a speed even larger than the beam speed. In contrast, the holes in Fig. 1 do not break, but trail along beside the beam, slowly lagging behind the beam with velocity approximately \( V_b/8 \), as can be seen in Fig. 3.

Figure 4 shows the excitation of plasma waves during beam exit from the plasma. In contrast to steady-state propagation, where the plasma waves establish a stationary stripe-like pattern [4,6,8], after the beam exits the plasma the plasma waves form a nonstationary periodic pattern resembling butterfly-wing motion.

In summary, steady-state ion beam propagation through a background plasma has been described in detail in previous publications [4–8]. The analytical results agree well with the results of PIC numerical simulations for ion beam charge and current neutralization for the case of steady-state ion beam propagation in a background plasma. In contrast to steady-state ion beam propagation, the visualization of the time-dependent data obtained in the numerical simulations presented here shows largely unexplored, complex collective phenomena during beam entry into and exit from the plasma. Further visualization material is also available on the website [11].

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References