Design considerations of wobblers for HIF

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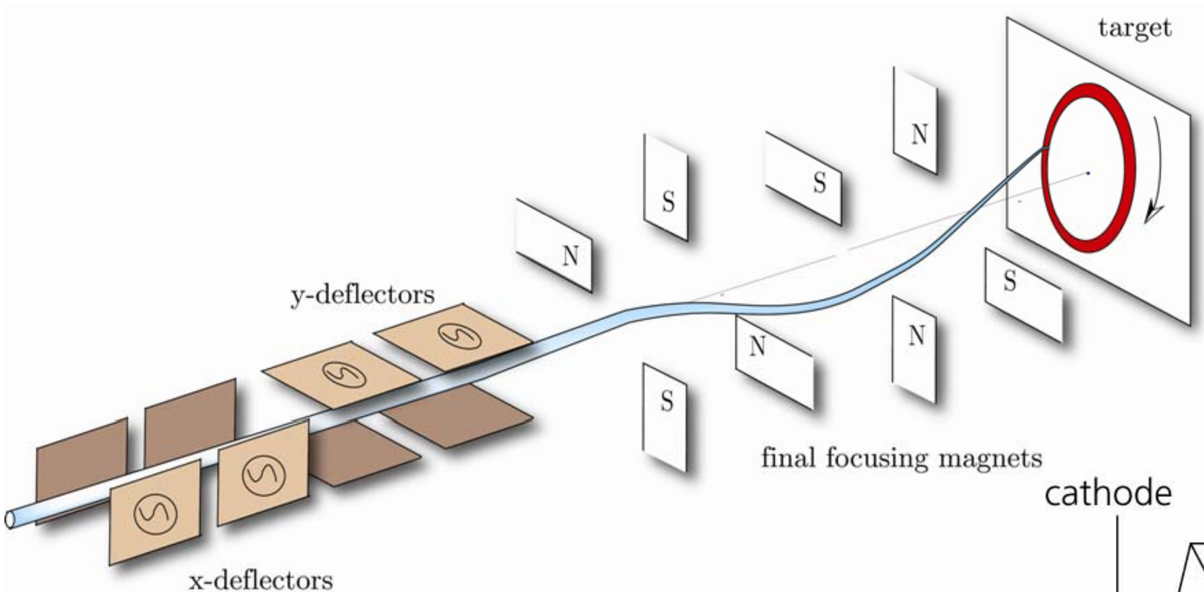
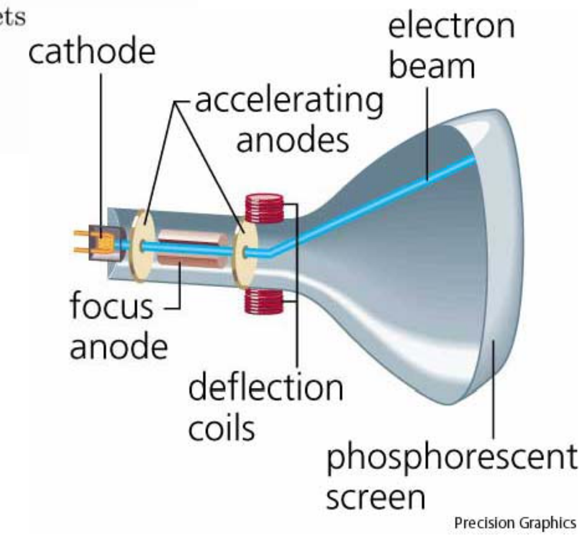
Outline

- **Wobbler fields control the centroid.**
  - Focusing lattice controls the envelope.
  - Envelope and centroid dynamics are decoupled.
- **Time modulation stabilizes RTI.**
  - Courant-Snyder theory for dynamics stabilization.
  - Slower modulation is necessary.
  - What is the beam modulation form.
- **A wobbler design for NDCX-II.**

Basic idea: scan the beam for better target performance



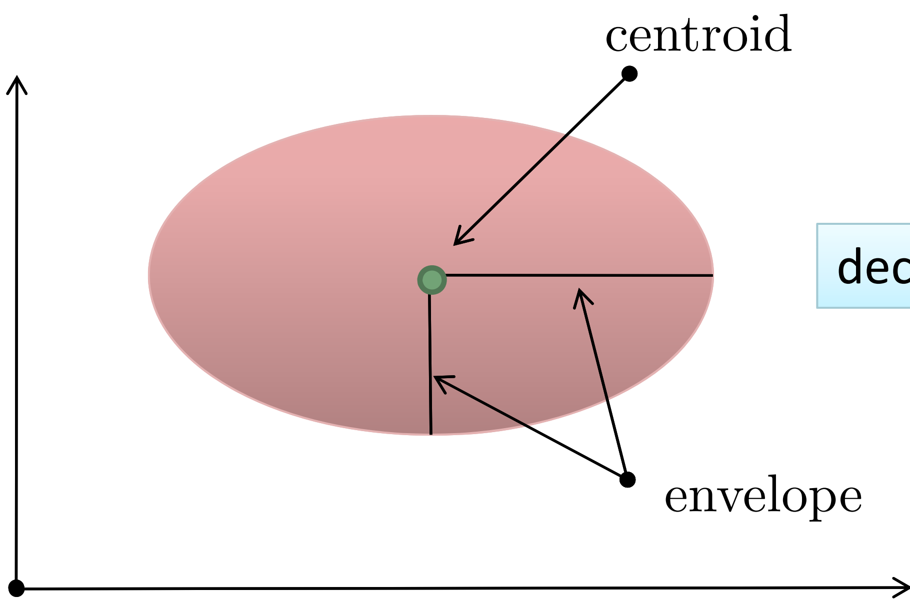
- Kawata 1993, 2009, 2012, reduce instability amplitude.
- Piriz et al, 2009, 2011, NO dynamic stabilization of ablative RTI.
- Friedman, 2012, mimi-wobbler.



Cathode ray tube

Basic beam physics: envelope dynamics and centroid dynamics

- Wobbler fields control the centroid
- Focusing lattice controls the envelope



decoupled

- What are the second order effects?
  - Are wobbler and envelope coupled?
  - *Does the space charge force affect the centorid?*
  - *Do the wobbler fields change the envelope?*
  - *Does the focusing lattice affect the centroid? (trivial)*

- Is it practical in terms of engineering?
  - *Field strength*
  - *Field frequency*

Centriod and enevelope dynamics: starting from Vlasov-Maxwell Eqs.

Space charge

Focusing

Wobbler

$$\frac{\partial f}{\partial s} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} - \left(\kappa_x x + \frac{\partial \psi}{\partial x} - F_x \right) \frac{\partial f}{\partial v_x} - \left(\kappa_y y + \frac{\partial \psi}{\partial y} - F_y \right) \frac{\partial f}{\partial v_y} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi = -\frac{2\pi K_b}{N_b} \int f \, dv_x dv_y$$

for each slice

Lee 87, Sharp 92, Barnard 96, Lund 04, Lund & Barnard 09

Moment $\langle \chi \rangle \equiv \left(\int \chi f \, dx dy dv_x dv_y \right) / N_b.$

Vlasov $\longrightarrow$ Rate eq.

$$\frac{d\langle\chi\rangle}{ds} = \left\langle \frac{\partial\chi}{\partial s} + v_x \frac{\partial\chi}{\partial x} + v_y \frac{\partial\chi}{\partial y} - \left(\kappa_x x + \frac{\partial\psi}{\partial x} - F_x\right)\frac{\partial\chi}{\partial v_x} - \left(\kappa_y y + \frac{\partial\psi}{\partial y} - F_y\right)\frac{\partial\chi}{\partial v_y} \right\rangle$$

Centroid:

$$\mu \equiv \langle x \rangle, \;\; \nu \equiv \langle y \rangle.$$

$$\mu'' = \langle v_x \rangle' = -\kappa_x \mu + F_x - \left\langle \frac{\partial\psi}{\partial x} \right\rangle,$$

$$\nu'' = \langle v_y \rangle' = -\kappa_y \nu + F_y - \left\langle \frac{\partial\psi}{\partial y} \right\rangle.$$

Envelope and emittance defined relative to the centroid

$$a \equiv \sqrt{\left\langle x-\mu \right\rangle^2}\,,\; \varepsilon_x \equiv 2\sqrt{a^2\left\langle \left(v_x-\mu'\right)^2 \right\rangle - \left\langle \left(v_x-\mu'\right)\left(x-\mu\right) \right\rangle^2}\,,$$

$$b \equiv \sqrt{\left\langle y-\nu \right\rangle^2}\,,\; \varepsilon_y \equiv 2\sqrt{b^2\left\langle \left(v_y-\nu'\right)^2 \right\rangle - \left\langle \left(v_y-\nu'\right)\left(y-\nu\right) \right\rangle^2}\,.$$

$$a'' + \kappa_x a = \frac{\varepsilon_x^2}{4a^3} - \frac{1}{a}\left\langle \frac{\partial \psi}{\partial x}\left(x-\mu\right) \right\rangle$$

$$b'' + \kappa_x b = \frac{\varepsilon_y^2}{4b^3} - \frac{1}{b}\left\langle \frac{\partial \psi}{\partial y}\left(y-\nu\right) \right\rangle$$

No wobbler field

Centroid enters through space charge

$$\frac{d}{ds}\left(\frac{\varepsilon_y^2}{8}\right) = \frac{d}{ds}\left(\frac{b^2}{2}\right)\left\langle \frac{\partial \psi}{\partial y}\left(y-\nu\right) \right\rangle - b^2\left\langle \frac{\partial \psi}{\partial y}\left(v_y-\nu'\right) \right\rangle\,.$$

$$\frac{d}{ds}\left(\frac{\varepsilon_x^2}{8}\right) = \frac{d}{ds}\left(\frac{a^2}{2}\right)\left\langle \frac{\partial \psi}{\partial x}\left(x-\mu\right) \right\rangle - a^2\left\langle \frac{\partial \psi}{\partial x}\left(v_x-\mu'\right) \right\rangle\,.$$

Does the space-charge affect centroid dynamics?

$$\mu'' = \left\langle v_x \right\rangle' = -\kappa_x \mu + F_x - \left\langle \frac{\partial \psi}{\partial x} \right\rangle$$

$$\nu'' = \left\langle v_y \right\rangle' = -\kappa_y \nu + F_y - \left\langle \frac{\partial \psi}{\partial y} \right\rangle$$

Focusing lattice

$$-\left(\left\langle \frac{\partial \psi}{\partial x} \right\rangle, \left\langle \frac{\partial \psi}{\partial y} \right\rangle \right) = -\left\langle \nabla \psi \right\rangle = \frac{N_b}{2\pi K_b} \int_{wall} \left(\nabla\psi\nabla\psi - \left|\nabla\psi\right|^2 \mathbf{I} \right) \cdot d\mathbf{s},$$

If the wall is faraway, then space-charge does not affect the centorid.

Lee 87, Sharp 92

Are the centroid and enevelope decoupled?

$$a'' + \kappa_x a = \frac{\varepsilon_x^2}{4a^3} - \frac{1}{a}\left\langle \frac{\partial \psi}{\partial x}\left(x - \mu\right)\right\rangle$$
$$b'' + \kappa_x b = \frac{\varepsilon_y^2}{4b^3} - \frac{1}{b}\left\langle \frac{\partial \psi}{\partial y}\left(y - \nu\right)\right\rangle$$

- Centroid enters through space charge

If the wall is far away,

then $\left\langle \frac{\partial \psi}{\partial x}\left(x - \nu\right)\right\rangle \approx \left\langle \frac{\partial \psi}{\partial X} X\right\rangle$,

centroid and envelope are decoupled.

Assume $\quad n\left(X, Y, s\right) = \frac{N_b}{2\pi ab} S\left(\frac{X^2}{2a^2} + \frac{Y^2}{2b^2}\right)$

$$\left\langle \frac{\partial \psi}{\partial X} X\right\rangle = \frac{K_b}{2(a+b)}$$

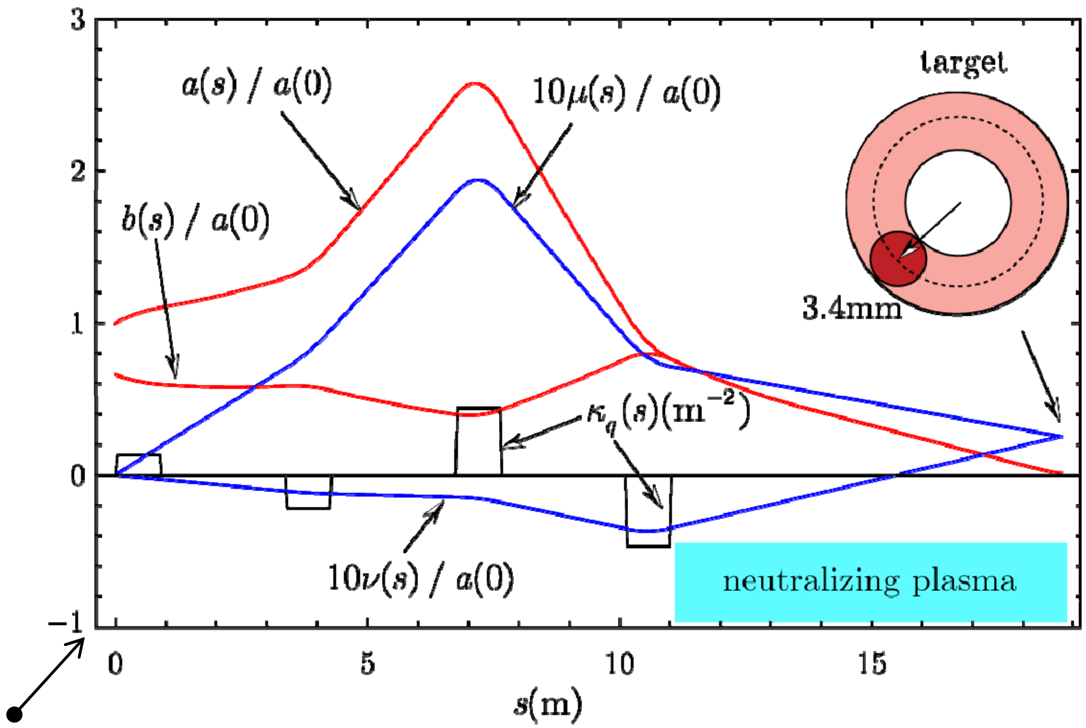
$$\varepsilon_x = const.$$

Davidson & Qin 90

Driver example : drift compression and final focus with wobbler

Qin, Davidson, Barnard and Lee 2004

$Cs^+, \quad m = 132au, \quad (\gamma - 1)mc^2 = 2.43GeV, \quad I = 2895A$



$(a_0, b_0) = (40, 22.8)mm$

$\rightarrow (a_f, b_f) = (1.2, 1, 2)mm$

$(\mu, \nu) = 2.4mm$

Assuming no coupling

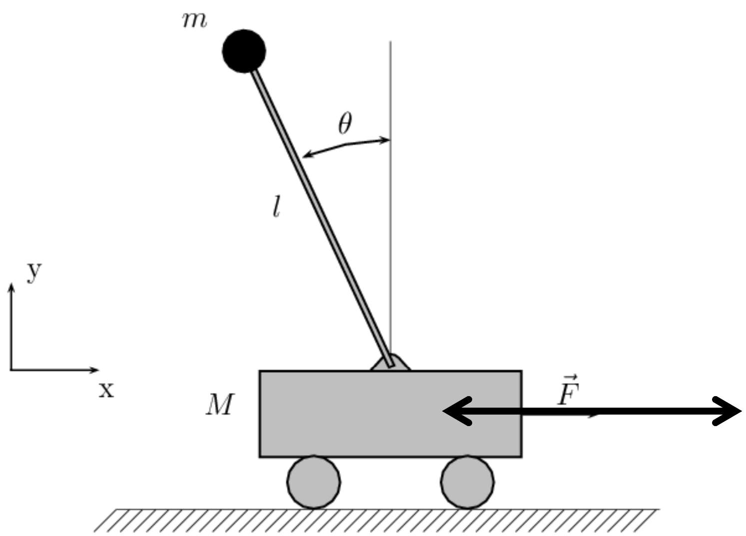
Wobbler fields (not shown) at 0.4MV/m, 67MHz

Dynamic Stabilization of RTI by wobbler

- Wolf, 1970, dynamic stabilization of classical RTI.
- Troyon & Gruber, 1971, need both viscosity & surface tension.
- Betti, McCrory, & Verdon, 1993, dynamic stabilization of ablative RTI.
- Piriz et al, 2009, 2011, NO dynamic stabilization of ablative RTI.
- Kawata 1993, 2009, 2012, reduce instability without change growth rate.

$$F = 1 + \alpha \, \mathrm{Sin}(\omega t)$$

Dynamic Stabilization of an inverted pendulum



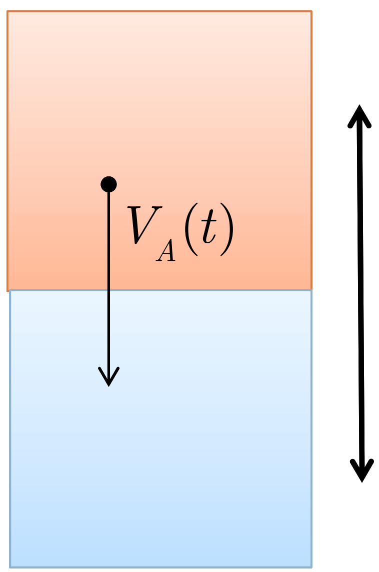
Easiest to implement

Most effective

- F is random
- F is feedback controlled
- **F is pre-determined & time-dependent**

- <u>Kawata 1993, 2009, 2012</u>, reduce instability without change growth rate by an analogy of feedback control.

Sharp boundary model for ablative RTI



Oscillatory acceleration

e.g., $g(t) = 1 + q\sin(\omega t)$

Surface displacement

$$\frac{d^2\eta}{dt^2} + \beta k V_A(t)\frac{d\eta}{dt} + [\alpha(t) - kAg(t)]\eta = 0$$

Atwood number

Ablative velocity

Betti 1993, Piriz 2011

Dynamic stabilization and ablative stabilization

Ablative stabilization

$$\eta = \exp\left[-\frac{\beta}{2}\int_0^t kV_A(t')dt'\right]\xi$$

Dynamic stabilization

$$\frac{d^2\xi}{dt^2} + \Big[\underbrace{\alpha(t) - kAg(t) - \frac{\beta^2 k^2 V_A^2}{4} - \frac{\beta k}{2}\frac{dV_A}{dt}}_{-\gamma^2\left[1+\delta g(t)\right]}\Big]\xi = 0$$

measure t using $1/\gamma$.

$\gamma \sim 3\ \mathrm{ns}^{-1}$

$$\frac{d^2\xi}{dt^2} + \Big[\underbrace{1+\delta g(t)}_{g(t)}\Big]\xi = 0$$

Second simplest physics problem

Betti 93, Takabe 85: $\delta g(t) \sim 4.5\sin(\omega t)$

Growth-rate depends on the modulation in a complex way

- Piriz 2011, NO dynamic stabilization of ablative RTI for a sequence of delta-function modulations, therefore NO dynamic stabilization for all modulations.

- Kawata 1993, 2009, 2012, reduce instability without change growth rate by an analogy of feedback control.

- The modulation can be designed to minimize the growth rate, with correct theoretical (mathematical) treatment of the problem.

Floquet theory and its (incorrect) application

For $\dot{x} = B(t)x$, where x is a vector,
and $B(t+T) = B(t)$ is a matrix,
the fundamental matrix solution $\phi(t)$ satisfies

$$\phi(t+T) = \phi(t)\underbrace{\phi^{-1}(0)\phi(T)}_{\text{determines the growth rate}}.$$

For the 2D dynamic stabilization problem, need to solve
$\dot{x} = B(t)x$ for two independent solutions to obtain the growth rate.

Betti 93 solves $\dot{x} = B(t)x$ once obtain the growth rate,
which is not accurate.

Courant-Snyder theory for dynamic stabilization

$$w''(s) + g(s)w(s) = w^{-3}(s) \qquad \varphi = \int_{t_0}^{t_0+T} \frac{ds}{w^2(s)}$$

$$M = \begin{pmatrix} w(T) & 0 \\ \dot{w}(T) & \dfrac{1}{w(T)} \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} w_0^{-1} & 0 \\ -\dot{w}_0 & w_0 \end{pmatrix}$$

Growth rate for ξ is $\ln|\mu|$

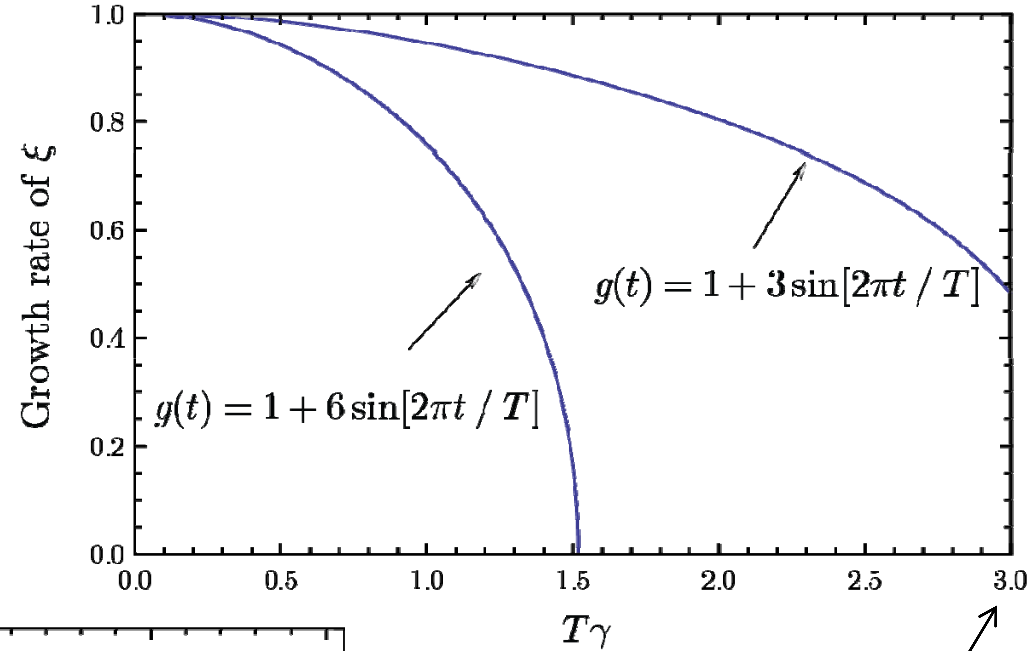
μ : the eigenvalue of $M(t)$ with largest amplitude.

μ : independent of choices of t_0 and initial conditions for w.

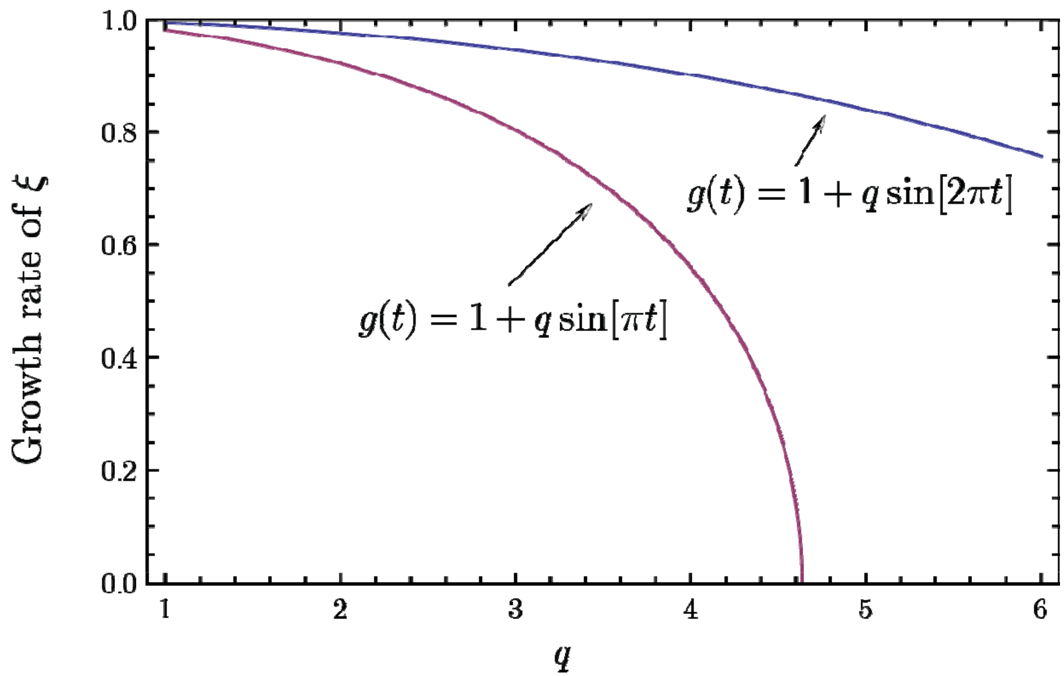
Slower modulation is better

$\gamma \sim 3\ \mathrm{ns}^{-1}$

$T \sim 2/\gamma \simeq 0.7\ \mathrm{ns}$

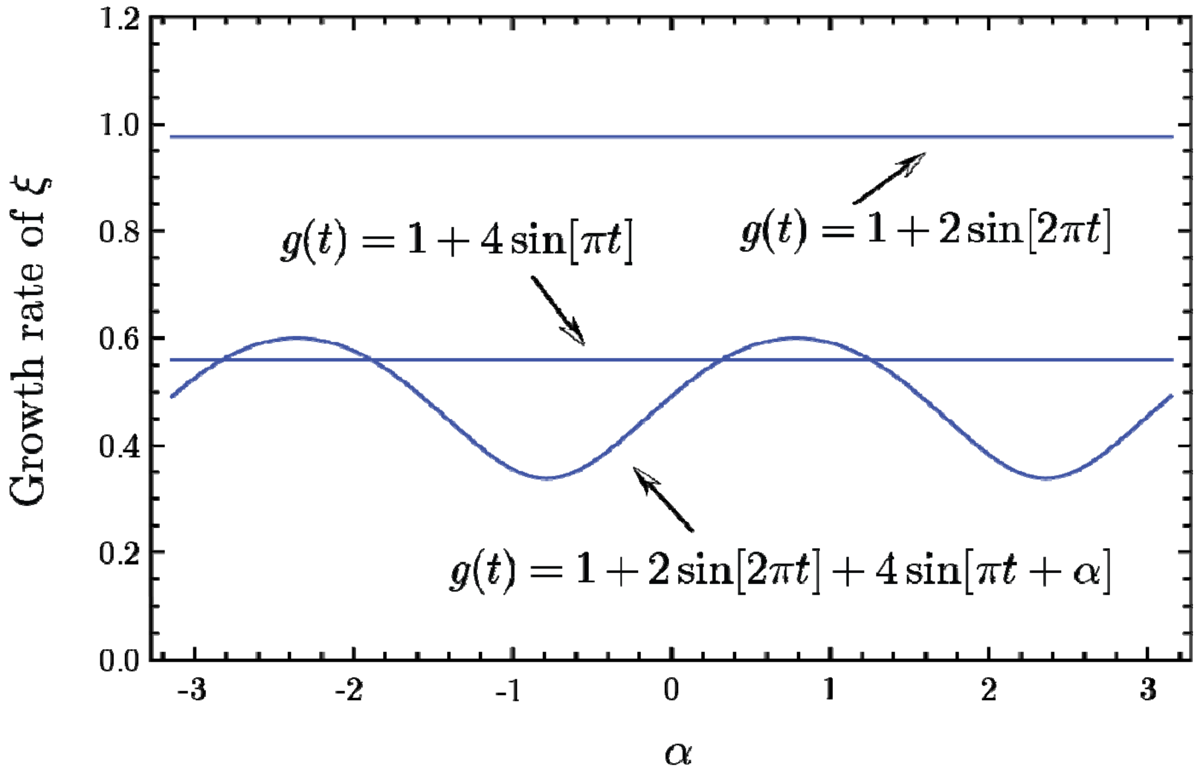


$T \sim 1\ \mathrm{ns}$



What is the best modulation?

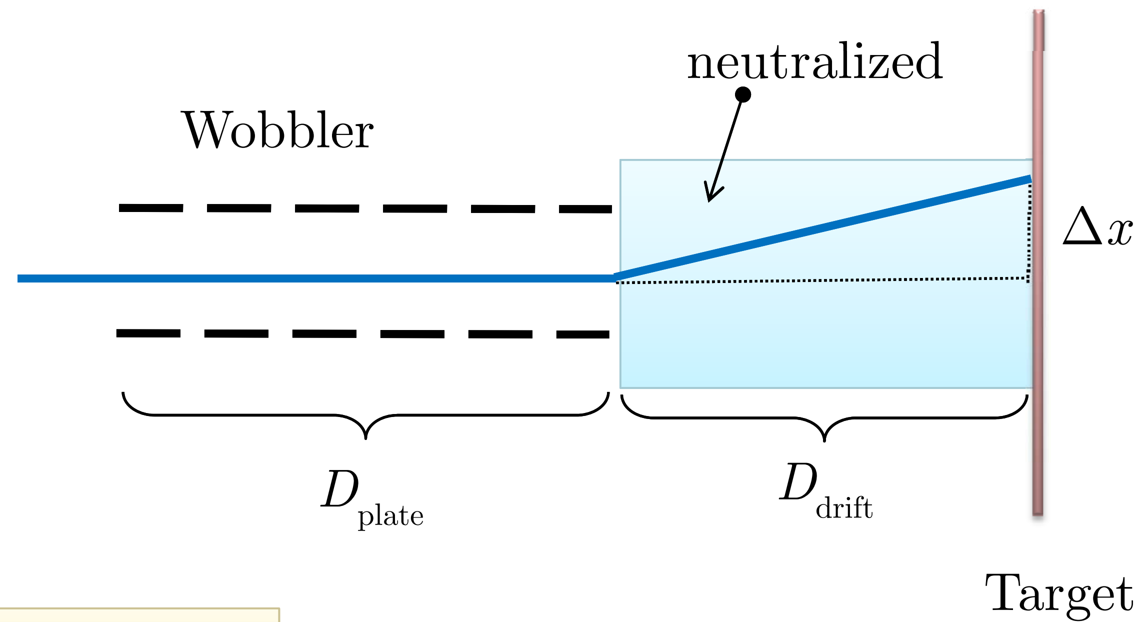
Complicated dependence on modulation forms



Demonstration of wobbler concept on NDCX-II

$$A = 7,\;\; q = 1,\; (\gamma\text{-}1)mc^2 = 3\text{MeV},\; l_{\text{pulse}} = 140\text{ns}$$

$$\Delta x = 5\text{mm},\; D_{\text{drift}} = 1\text{m},\; D_{\text{plate}} = 0.5\text{m}$$



E=6000Volts/meter @ 7MHz

Conclusions

- **Wobbler fields control the centroid.**
  - Focusing lattice controls the envelope.
  - Envelope and centroid dynamics are decoupled.
- **Time modulation stabilizes RTI.**
  - Courant-Snyder theory for dynamics stabilization.
  - Slower modulation is necessary.
  - Best modulation form is yet to be found.
- **A wobbler design for NDCX-II.**