Collective Interaction Processes in Intense Heavy Ion Beam-Plasma Systems

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Outline of Presentation

- Introduction

- Nonlinear stability theorem for collective interactions.

- Collective interactions and instabilities:

  - One-component intense ion beam (Harris, Weibel).
  - Partially neutralized intense ion beam (electron cloud instability).
  - Intense ion beam propagating through background plasma (multispecies Weibel, two-stream).
  - Effects of velocity tilt on two-stream instability.

- Use of neutral plasmas for focusing/compressing high-intensity charge bunches - Examples of dynamic beam equilibria.

Normalized Beam Intensity

- Consider a long coasting charge bunch with volume number density n_b propagating through a transverse focusing field with average focusing frequency $\omega_{\beta\perp}$.

- A convenient dimensionless measure of the self-field intensity(self-electric plus self-magnetic) is the dimensionless parameter

$$s_b \equiv \omega_{pb}^2 / 2\gamma_b^2 \omega_{\beta\perp}^2,$$

where $\omega_{pb}^2 = 4\pi n_b e_b^2 / \gamma_b m_b$ is the relativistic plasma frequency-squared.

- The condition for transverse confinement of the beam particles (applied focusing force exceeds the defocusing self-field force) is

$$0 < s_b = \omega_{pb}^2 / 2\gamma_b^2 \omega_{\beta\perp}^2 < 1.$$

- In terms of the normalized tune depression, ν / ν_0, for the special case of uniform beam density, this condition can be expressed as

$$1 > \nu / \nu_0 \equiv \left(1 - s_b\right)^{1/2} > 0.$$

Nonlinear Stability Theorem*

- Determine the class of beam distribution functions $f_b(\mathbf{x}, \mathbf{p}, t)$ that can propagate quiescently over large distances at high space-charge intensity.

- Analysis makes use of global (spatially-averaged) conservation constraints satisfied by the nonlinear Vlasov-Maxwell equations to determine a sufficient condition for stability of an intense charged particle (or charge bunch) propagating in the z - direction with average axial velocity $V_b = const.$ along the axis of a perfectly-conducting cylindrical pipe with wall radius $r = (x^2 + y^2)^{1/2} = r_w$.

* *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, 2001), R. C. Davidson and H. Qin, Chapter 4.

" Kinetic Stability Theorem for High-Intensity Charged Particle Beams Based on the Nonlinear Vlasov-Maxwell Equations," R. C. Davidson, Physical Review Letters **81**, 991 (1998).

" Three-Dimensional Kinetic Stability Theorem for High-Intensity Charged Particle Beams," R. C. Davidson, Physics of Plasmas **5**, 3459 (1998).

Theoretical Model and Assumptions

- Model makes use of fully nonlinear Vlasov-Maxwell equations for the self-consistent evolution of the distribution function $f_b(\mathbf{x},\mathbf{p},t)$ and self-generated electric and magnetic fields

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A} \quad \mathbf{B} = \nabla \times A$$

- Vlasov-Maxwell equations are Lorentz transformed to the beam frame (‘primed coordinates) where the (time-independent) confining potential of the applied focusing force is assumed to be of the form (smooth-focusing approximation)

$$\Psi'_{sf}(\mathbf{x}') = \frac{1}{2}\gamma_b m_b \omega_{\beta\perp}^2 (x'^2 + y'^2) + \frac{1}{2}\gamma_b m_b \omega_{\beta z}^2 z'^2,$$

where $\omega_{\beta z}$ and $\omega_{\beta\perp}$ are constant focusing frequencies.

- Particle motions in the beam frame are assumed to be nonrelativistic.

Nonlinear Stability Theorem

- Consider general perturbations about a quasi-steady equilibrium distribution function $f_{eq}(\mathbf{x}',\mathbf{p}')$. For $f_{eq} = f_{eq}(H')$, using the global conservation constraints, it can be shown that

$$\frac{\partial}{\partial H'} f_{eq}(H') \leq 0$$

is a sufficient condition for linear and nonlinear stability.

- Here, H' is the single-particle Hamiltonian defined by

$$H' = \frac{1}{2m_b}\mathbf{p}'^2 + \Psi'_{sf}(\mathbf{x}') + q_b\phi'(\mathbf{x}'),$$

where $\phi'(\mathbf{x}')$ is the equilibrium space-charge potential.

Nonlinear Kinetic Stability Theorem

Therefore a necessary condition for instability is that the beam distribution function have some nonthermal feature such as:

- An inverted population in phase space.
- Or a strong energy anisotropy.
- Or that the beam have directed kinetic energy relative to background charge components.
- Or dissipation mechanisms (e.g., finite wall resistivity).

Collective Instabilities in Intense Charged Particle Beams

One-Component Beams

- Electrostatic Harris instability

$$T_{\perp b} >> T_{\parallel b}$$

- Electromagnetic Weibel instability

$$T_{\perp b} >> T_{\parallel b}$$

- Resistive wall instability

Propagation Through Background Electrons

- Electron-ion two-stream (Electron Cloud) instability

Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

Harris Instability in Intense One-Component Beams

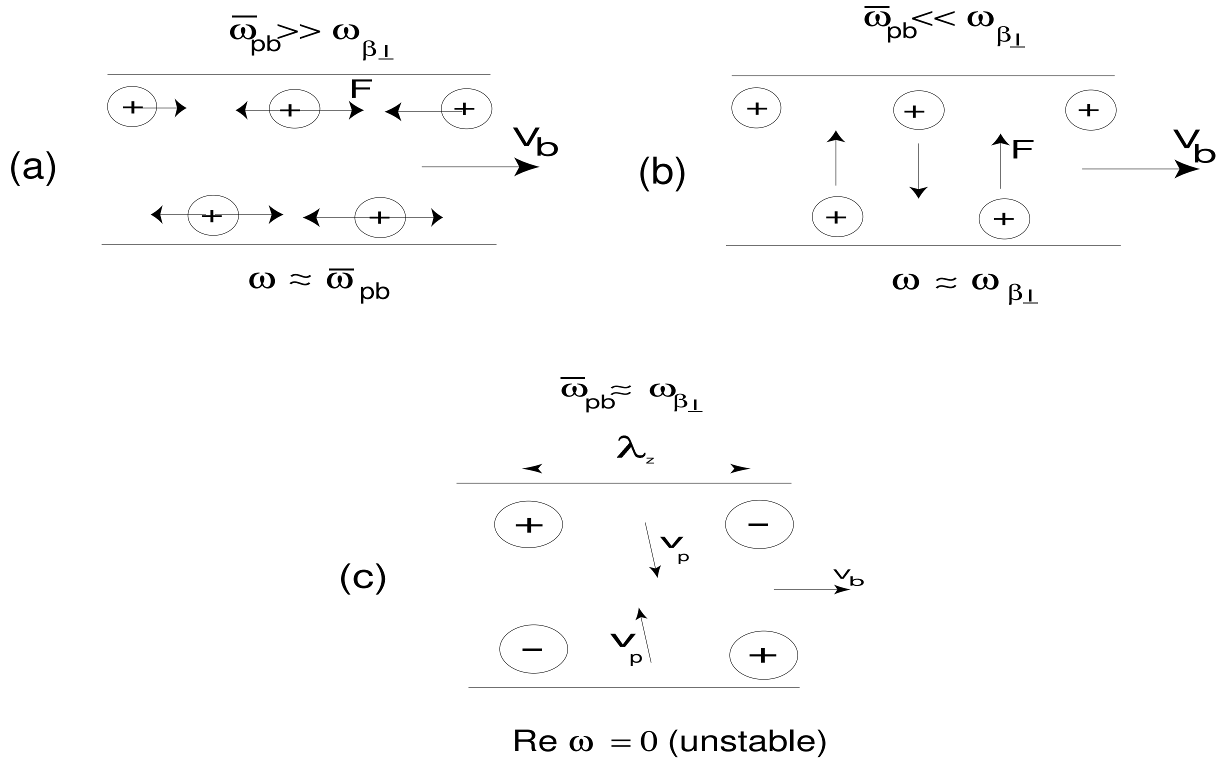
- Electrostatic Harris instability* can play an important role in multispecies plasmas with temperature anisotropy $T_{\parallel j} < T_{\perp j}$.
- Harris instability is inherently three-dimensional and involves a coupling of the longitudinal and transverse particle dynamics.
- Harris-like instability*** also exists in intense one-component beams provided the anisotropy is sufficiently large and the beam intensity is sufficiently large.

* E. G. Harris, Phys. Rev. Lett. **2**, 34 (1959).

* I. Haber et al., Phys Plasmas **6,** 2254 (1999).

*** E. A. Startsev, R. C. Davidson and H. Qin, Physical Review Special Topics on Accelerators and Beams **8**, 124201(2005); Physics of Plasmas **9**, 3138 (2002); Laser and Particle Beams **20**, 585 (2002); Physical Review Special Topics on Accelerators and Beams **6**, 084401 (2003).

Harris Anisotropy Instability Mechanism*



- Instability requirements

$$\lambda_z > v_{||}^{th}\frac{2\pi}{\omega_{\beta\perp}} \qquad \Rightarrow \qquad \frac{T_{||b}}{T_{\perp b}} < \frac{1}{k_z^2 r_b^2}$$

*E. A. Startsev, R. C. Davidson and H. Qin, *Phys. Rev. ST Accel. Beams* **8**, 124201 (2005).

Description of Beam Equilibrium Stability and Transport (BEST) Code

- The solutions to the nonlinear Vlasov-Maxwell equations are expressed as

$$f_b = f_b^0 + \delta f_b, \qquad \phi = \phi^0 + \delta\phi,$$

where (f_b^0, ϕ^0, A_z^0) are known equilibrium solutions $(\partial/\partial t = 0)$.

- Use particles (markers) to represent only $\delta f_b(\mathbf{x}, \mathbf{p}, t)$ part of the distribution

$$\delta f_b = \frac{N_b}{N_{sb}} \sum_{i=1}^{N_{sb}} w_{bi} \delta(\mathbf{x} - \mathbf{x}_{bi}) \delta(\mathbf{p} - \mathbf{p}_{bi}),$$

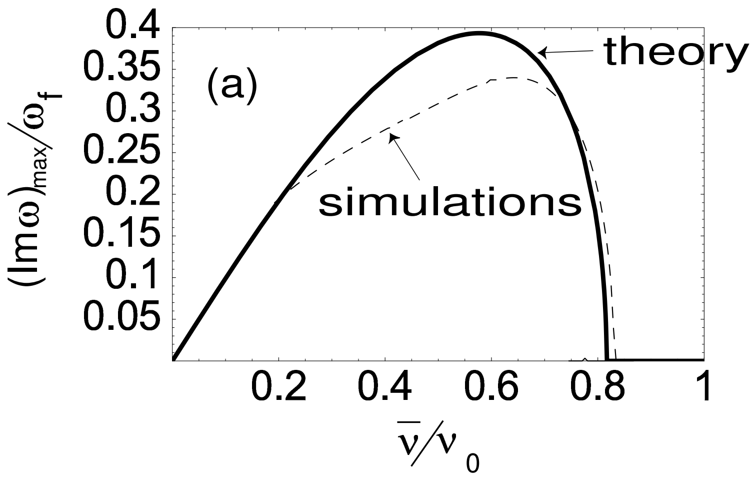
$$w_b \equiv \delta f_b / f_b$$

- Noise associated with f_b^0 is removed.

- Statistical noise is significantly reduced by a factor

$$\epsilon_{\delta f} / \epsilon_f = \bar{w}_b.$$

The $m = 1$ Dipole Mode has the Highest Growth Rate



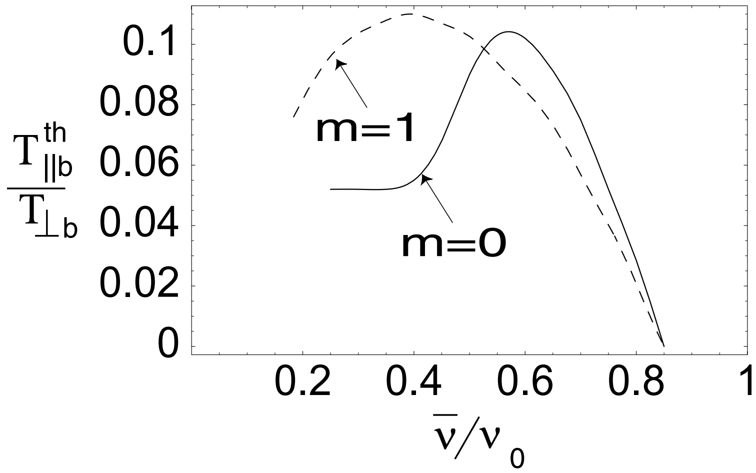
- Wall radius $r_w = 3r_b$, anisotropy $T_{||b}/T_{\perp b} = 0$.

- The m=1 dipole mode is purely growing with $Re\omega = 0$ and $(Im\omega)_{max}/\omega_f \simeq 0.34$ for $\bar{\nu}/\nu_0 \simeq 0.62$.

- The instability is absent for $\bar{\nu}/\nu_0 > 0.82$.

- This is good news for intense beams with sufficiently small tune shift $\delta\nu/\nu_0 = (\bar{\nu} - \nu_0)/\nu_0$.

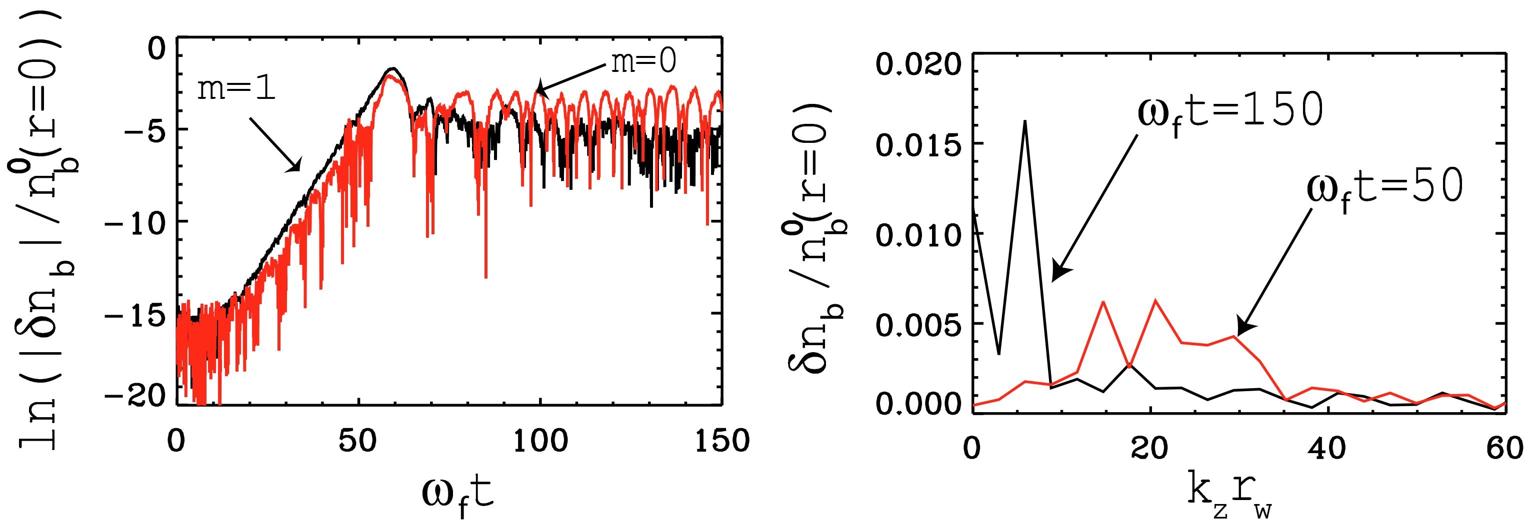
Longitudinal Threshold Temperature $T_{||b}^{th}$ Versus Normalized Tune Depression $\bar{\nu}/\nu_0$



- System is stable for

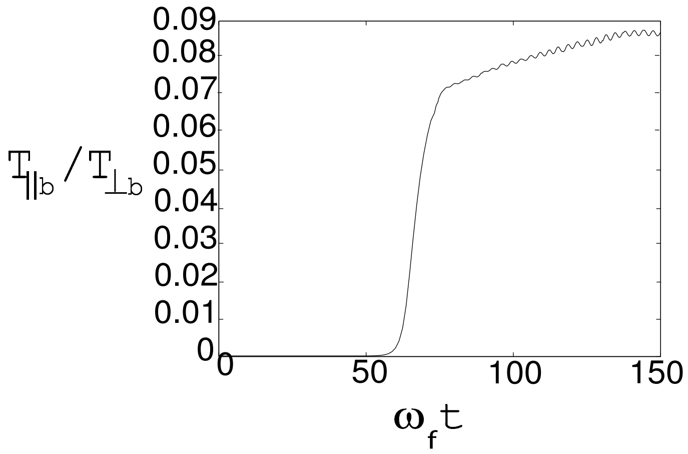
$$\frac{T_{||b}}{T_{\perp b}} > \left(\frac{T_{||b}^{th}}{T_{\perp b}}\right)_{max} = 0.11$$

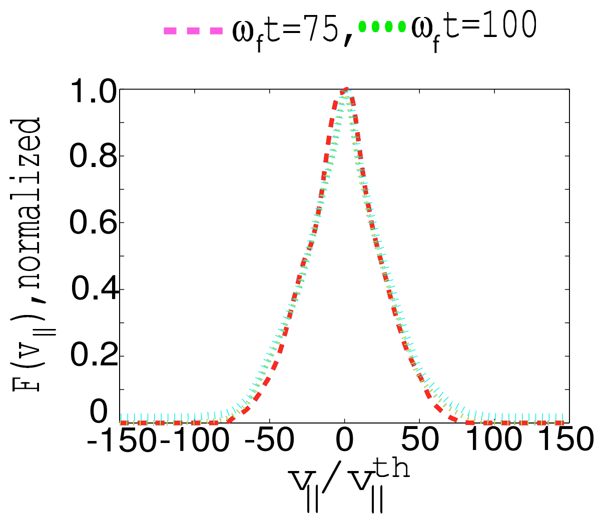
Nonlinear Stage of the Instability is Dominated by the Long-Wavelength m=0 Mode ($\partial/\partial\theta = 0$)

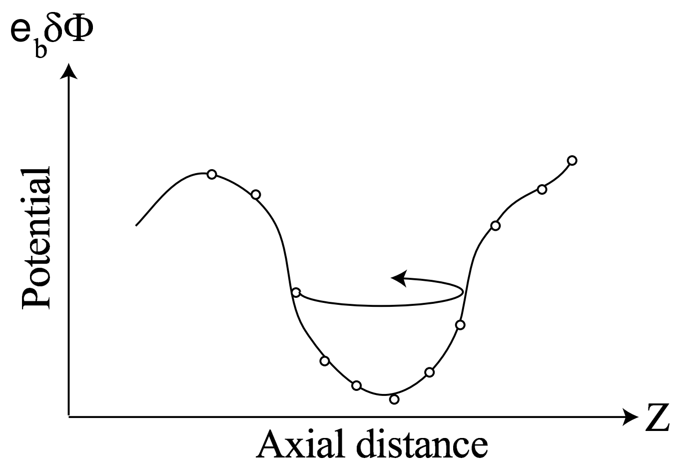


Results obtained using the BEST nonlinear δf simulation code, with $\bar{\nu}/\nu_0 = 0.6$ and initial temperature ratio $T_{||b}/T_{\perp b} = 10^{-4}$.

Instability Saturates Nonlinearly by Particle Trapping and Quasilinear Relaxation



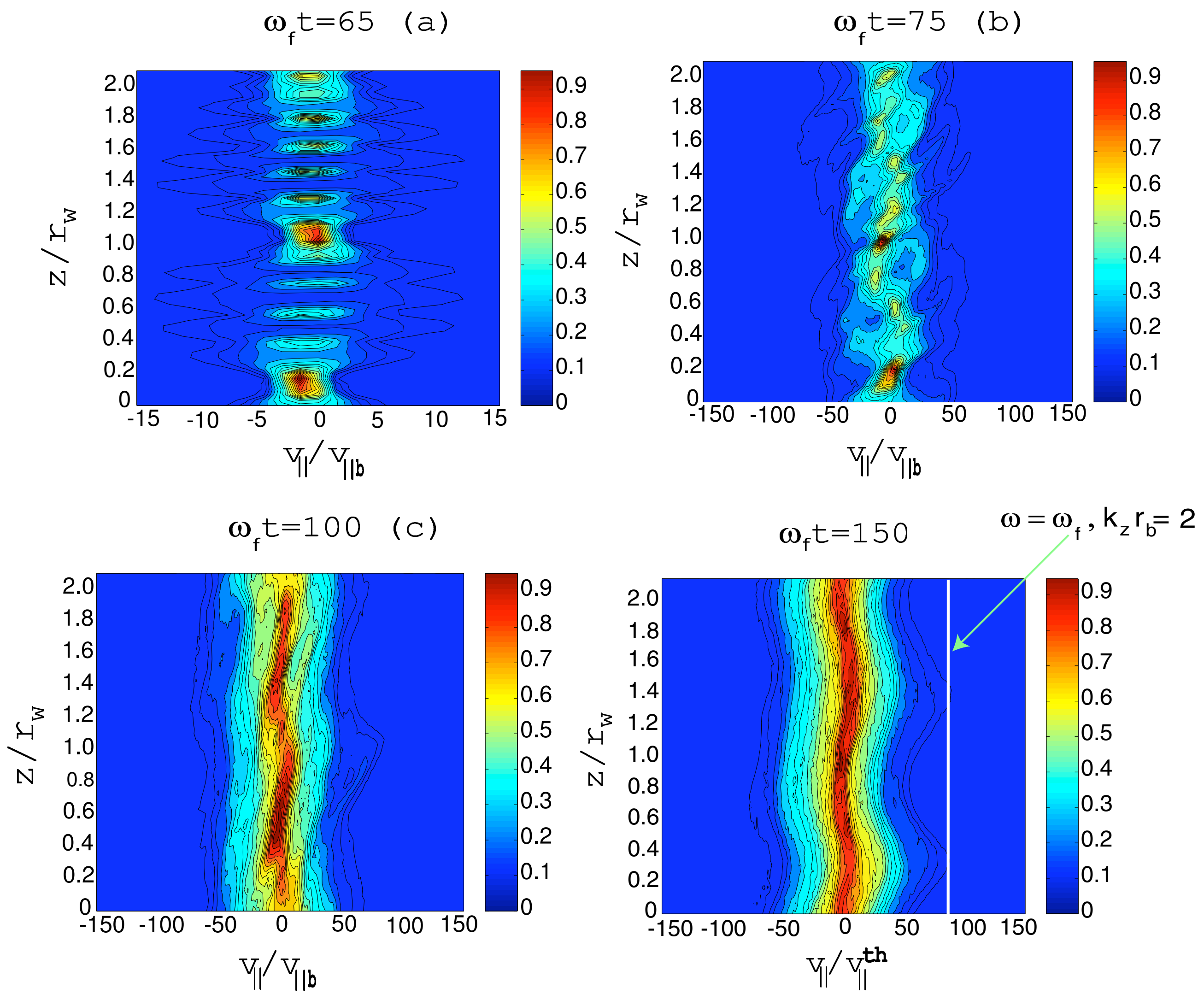




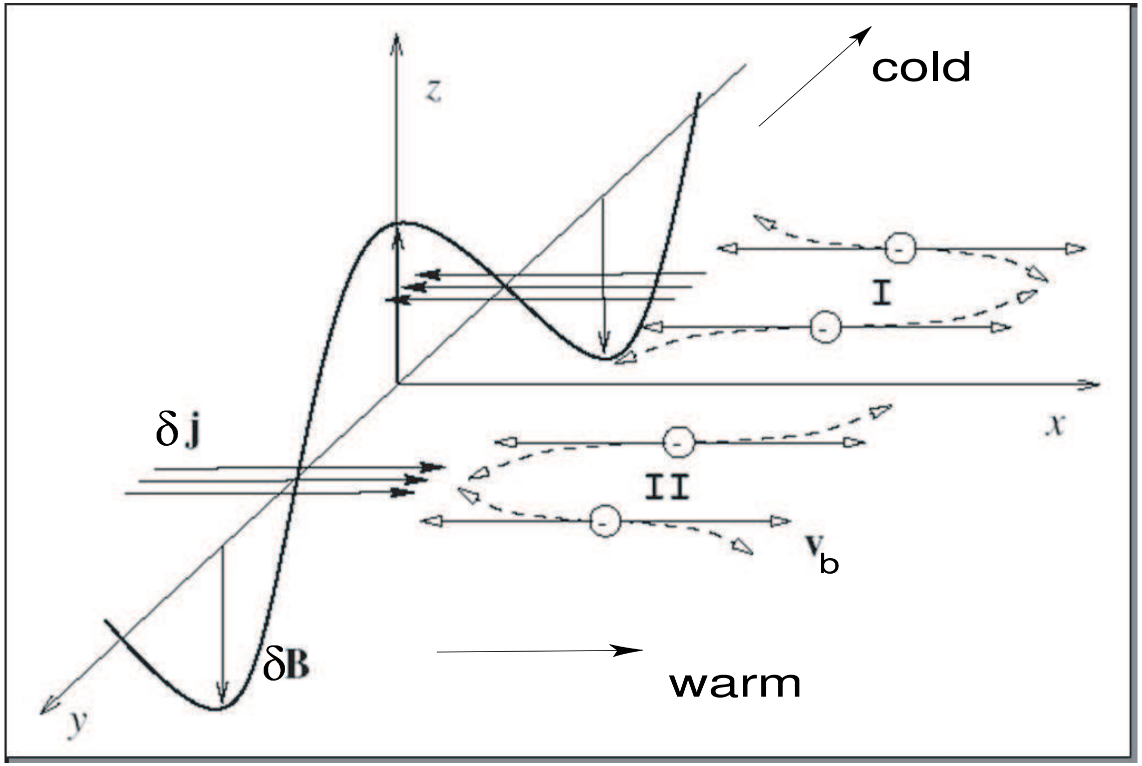
- Instability saturates nonlinearly for

$$(T_{\|b})_f \sim |e_b \delta\Phi_{max}| \sim m_b \left(\frac{\gamma^2}{k_z^2}\right)_{max}$$

Instability Saturates Nonlinearly by Particle Trapping and Quasilinear Relaxation (Longitudinal Phase Space Projection)



Basic Mechanism for Electromagnetic Weibel Instability ($T_{||b} \ll T_{\perp b}$) with Magnetic Perturbation $\delta B \neq 0$*



* E.A. Startsev and R.C. Davidson, *Physics of Plasmas* **10**, 4829 (2003).

Electromagnetic Weibel Instability in Intense Charged Particle Beams with Large Temperature Anisotropy

- Finite transverse geometry introduces the Weibel instability threshold*:

$$T^{th}_{\parallel b}/T_{\perp b} \approx 10^{-0.7} r_b^2 \widehat{\omega}_{pb}^2/c^2 \sim (v^{th}_{\perp b}/c)^2$$

- Electrostatic Harris instability saturates at much larger longitudinal temperature, but is active only for very intense beams with normalized tune $\bar{\nu}/\nu_0 < 0.82$

$$(T^{th}_{\parallel b}/T_{\perp b})^{Weibel} \ll (T^{th}_{\parallel b}/T_{\perp b})^{Harris} \approx 0.1$$

- Electromagnetic Weibel Instability is likely to be an important instability mechanism in relativistic one-component charged particle beams with $\bar{\nu}/\nu_0 > 0.82$, but not in intense ion beams with $\bar{\nu}/\nu_0 < 0.82$.

- Nonlinear δf PIC code BEST is being modified to allow detailed simulations of electromagnetic Weibel instability.

* E.A. Startsev and R.C. Davidson, *Physics of Plasmas* **10**, 4829 (2003).

Conclusions

- We have generalized the analysis of the classical Harris and Weibel instabilities to the case of a one-component intense charged particle beam with anisotropic temperature.

- For a long, coasting beam, the delta-f particle-in-cell code BEST and the eigenmode code bEASt have been used to determine the detailed 3D stability properties over a wide range of temperature anisotropy and beam intensity.

- Intense beams with $\bar{\nu}/\nu_0 < 0.82$ and $T_{||b}/T_{\perp b} < 0.11$ are linearly unstable to electrostatic perturbations (Harris-type instability).

- The instability is kinetic in nature and is due to the coupling of the particles' transverse betatron motion with the longitudinal plasma oscillations excited by the perturbation.

- The nonlinear saturation is governed by longitudinal particle trapping.

- The final longitudinal velocity distribution is not Maxwellian and can be characterized by a remnant temperature anisotropy ($T_{||b}/T_{\perp b} \simeq 0.11$), where $T_{||b} \equiv m_b < v_{||}^2 >$.

Electron-Ion Two-Stream (Electron Cloud) Instability

- A background population of electrons can result when energetic beam ions strike the chamber wall or ionize background gas atoms.

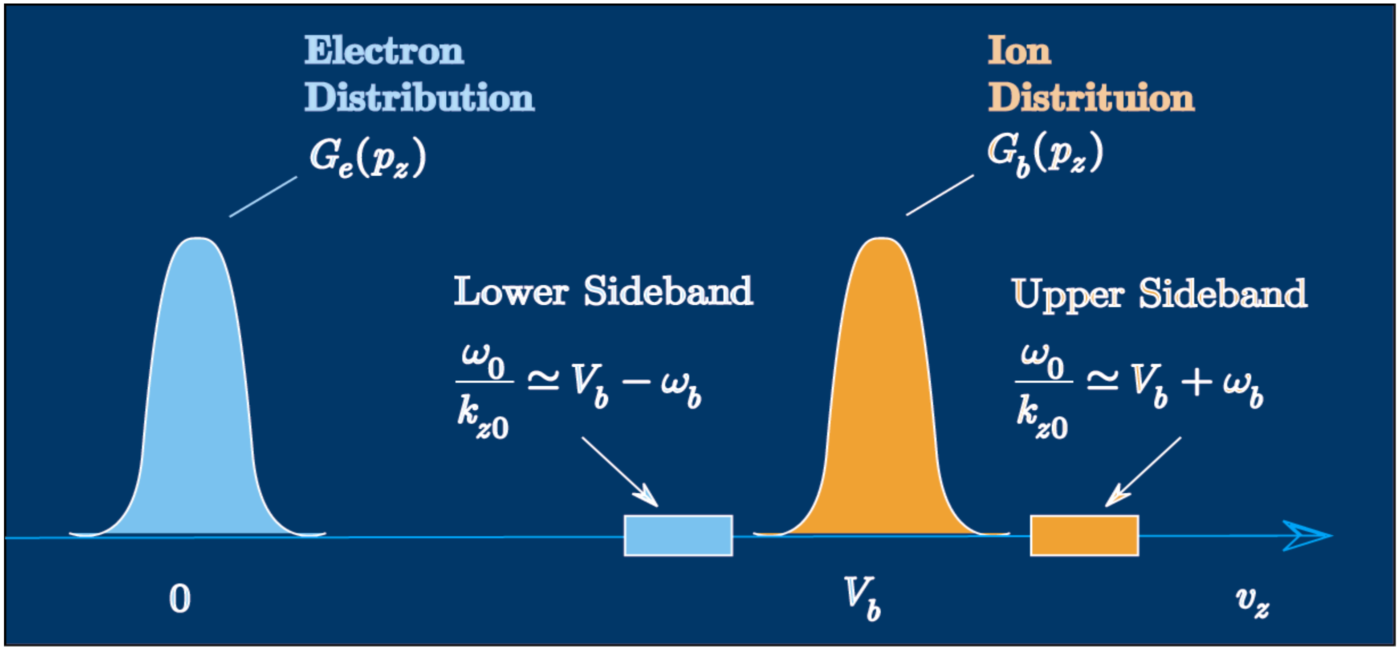
- Relative streaming motion of beam ions through the background electrons provides the free energy to drive the classical two-stream instability, appropriately modified to include the effects of dc space charge, relativistic kinematics, transverse beam geometry, etc.

- Experimental evidence for two-stream instability in proton machines such as the Proton Storage Ring (PSR) experiment at Los Alamos National Laboratory.

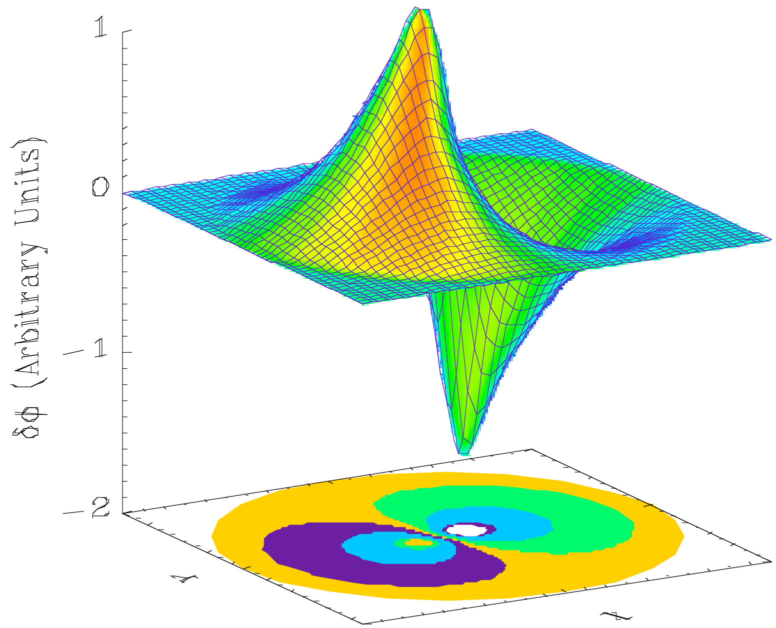
Electron-Ion Two-Stream (Electron Cloud) Instability

- An intense ion beam supports collective oscillations (sideband oscillations) with phase velocity ω/k_z upshifted and downshifted relative to the average beam velocity $\beta_b c$.

- An (unwanted) electron component provides the free energy to drive the classical two-stream instability.

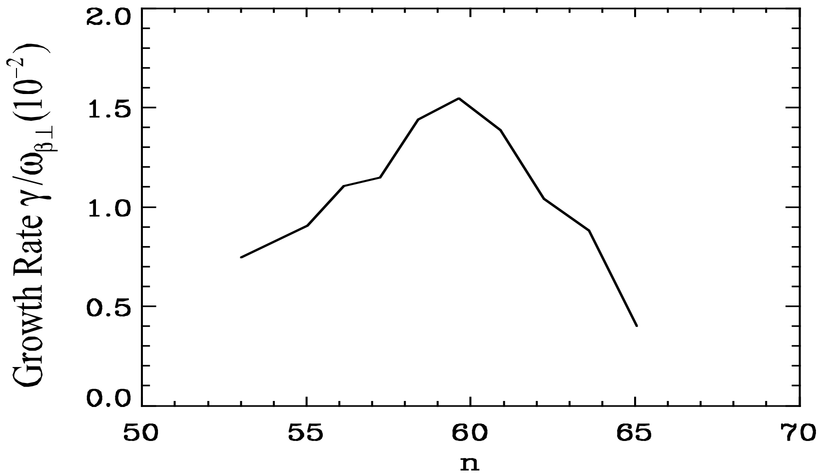


Electron-Ion Two-Stream (Electron Cloud) Instability



The $x - y$ projection (at fixed value of z) of the perturbed electrostatic potential $\delta\phi(x, y, t)$ for the ion-electron two-stream instability growing from a small initial perturbation, shown at $\omega_{\beta\perp} t = 200$.

Electron-ion Two-stream Instability in the Presence of Background Electrons



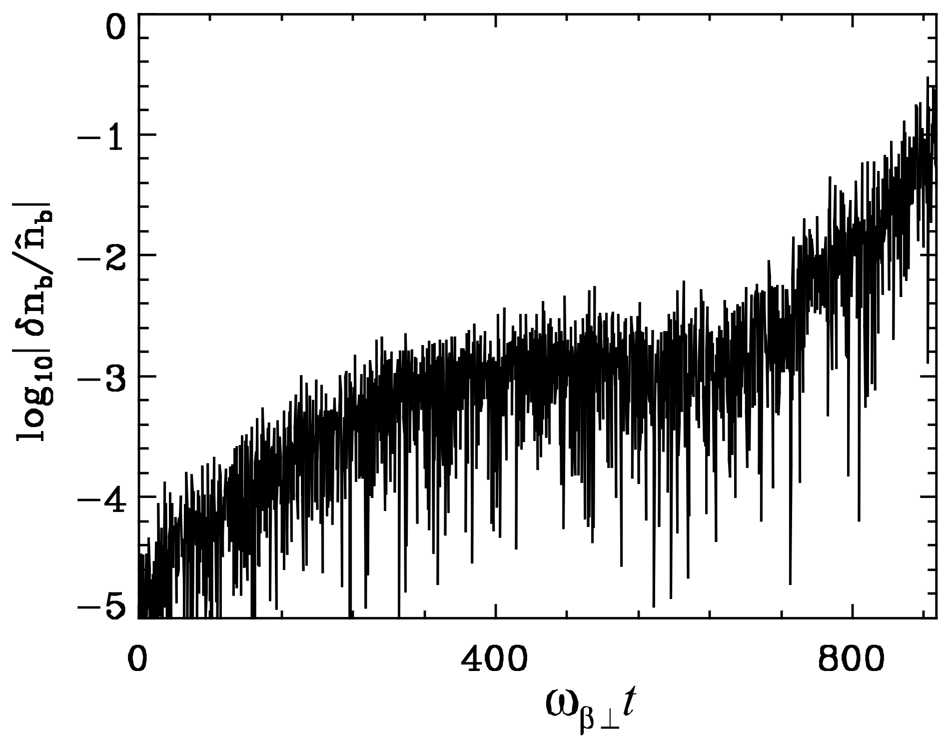
Illustrative parameters in PSR (coasting beam)

- Space-charge-induced tune shift: $\delta\nu / \nu_0 = -0.02;\ \hat{\omega}^2_{pb} / 2\gamma_b^2\omega^2_{\beta\perp} = 0.079.$

- Mode oscillation frequency (simulations): f ~163 MHz. Mode number at maximum growth n = 55-65.

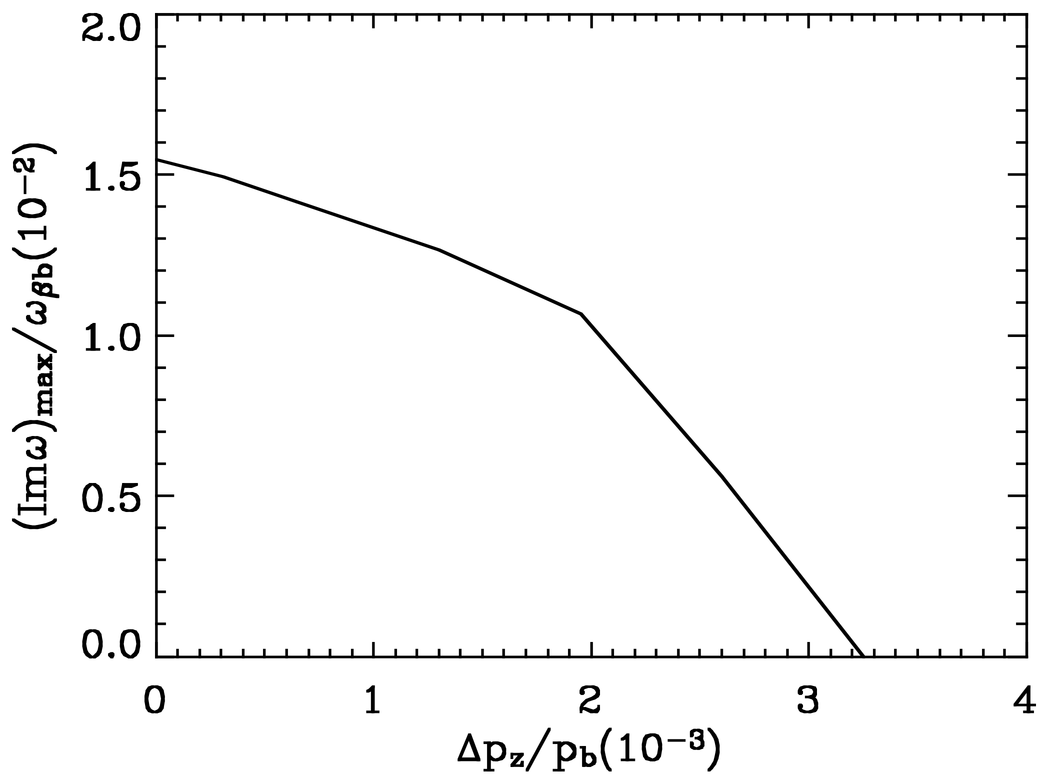
- $\lambda_b = 9.13\times 10^8 cm^{-1},\ \lambda_e = 9.25\times 10^7 cm^{-1},\ T_{b\perp} = 4.41 keV,$
  $T_{e\perp} = 0.73 keV,\ \phi_0(r_w) - \phi_0(0) = -3.08\times 10^3 Volts.$

Electron-Ion Two-Stream (Electron Cloud) Instability



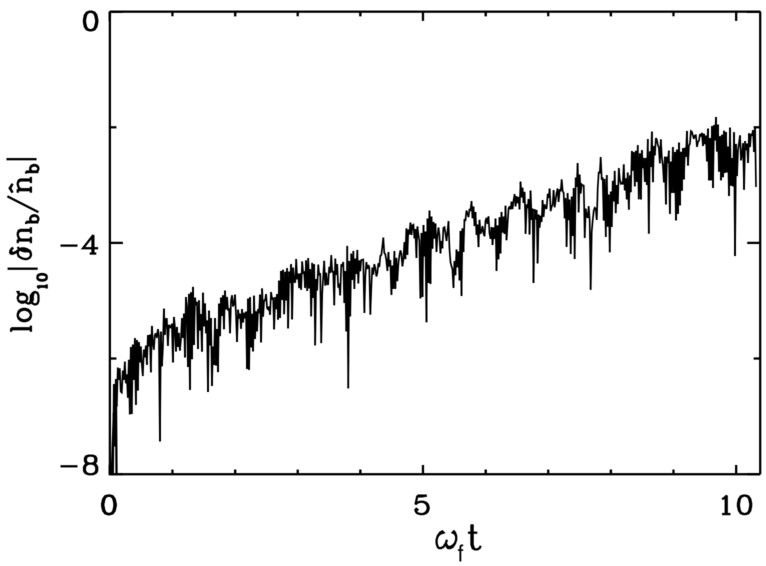
Time history of perturbed density $\delta n_b/\hat{n}_b$ at a fixed spatial location and proton line density $1.83\times10^9 cm^{-1}$. After an i nitial transition period, the $m=1$ dipole-mode perturbation grows exponentially, saturates, and enters a subsequent nonlinear phase.

Electron-Ion Two-Stream (Electron Cloud) Instability



The maximum linear growth rate $(Im\omega)_{\max}$ of the ion-electron two-stream instability decreases as the longitudinal momentum spread of the beam ions increases.

Electron-Ion Two-Stream (Electron Cloud) Instability



Time history of perturbed density $\delta n_b/\tilde{n}_b$ at a fixed spatial location. After an initial transition period, the $m = 1$ dipole-mode perturbation grows exponentially.

Growth Rate Reduction Mechanisms

- Axial momentum spread in the beam ions.
- Proximity of a conducting wall.
- Reduction in fractional charge neutralization.
- Rounded beam density profiles
  - Spread in transverse oscillation frequency.

Background plasma can be used to focus and compress intense ion beam pulses

The electrons in a large-volume background plasma can be used to charge neutralize and current neutralize an intense ion charge bunch, thereby greatly facilitating the transverse focusing and longitudinal compression of the charge bunch to a small spot size.

Space-charge-dominated heavy ion beams:

- R. C. Davidson and H. Qin, Physical Review Special Topics on Accelerators and Beams **8,** 064201 (2005).

- P. K. Roy, S. S. Yu et al. ,Physical Review Letters **95,** 234801 (2005).

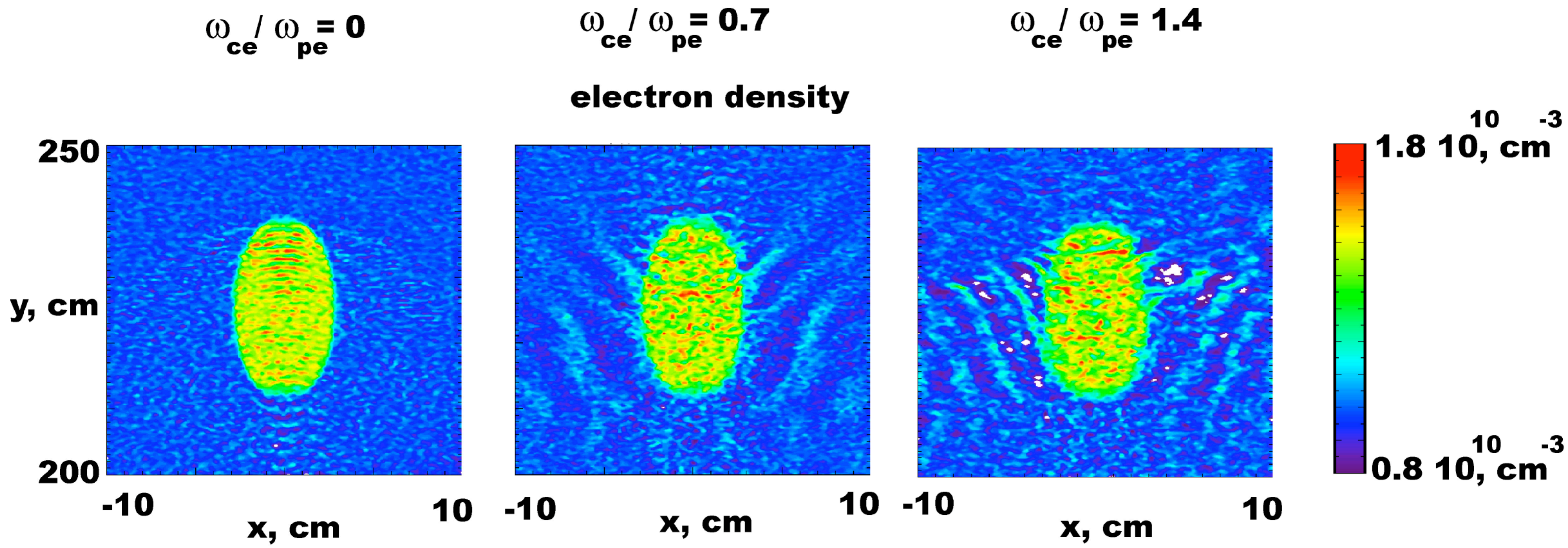
- I.D. Kaganovich, E. A. Startsev, R. C. Davidson and D. R. Welch, Nuclear Instruments and Methods in Physics Research **A544,** 383 (2005).

- I. D. Kaganovich, E. A. Startsev, R. C. Davidson and J. Stimatze, *Proceedings of the 2005 Particle Accelerator Conference,* 2062 (2005).

Analytical studies show that the solenoidal magnetic field influences the neutralization by plasma if $\omega_{ce} > \beta\omega_{pe}$

Plots of electron charge density contours in (x,y) space, calculated in 2D slab geometry using the LSP code with parameters:

Plasma: $n_p = 10^{11} cm^{-3}$; Beam: $V_b = 0.2c$, 48.0A, r_b=2.85cm and pulse duration τ_b=4.75 ns. A solenoidal magnetic field of 1014 G corresponds to $\omega_{ce} = \omega_{pe}$.

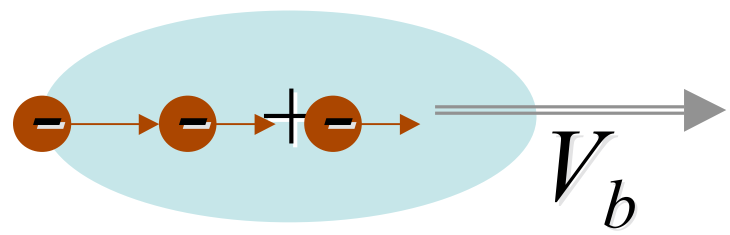


- In the presence of a solenoidal magnetic field, whistler waves are excited, which propagate at an angle with the beam velocity and can perturb the plasma ahead of the beam pulse.

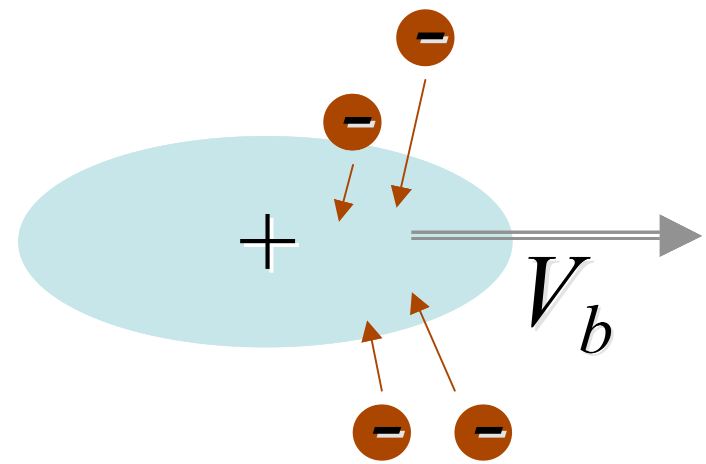
Two ways for neutralization of ion beam pulse

Electrons can move into the beam pulse:

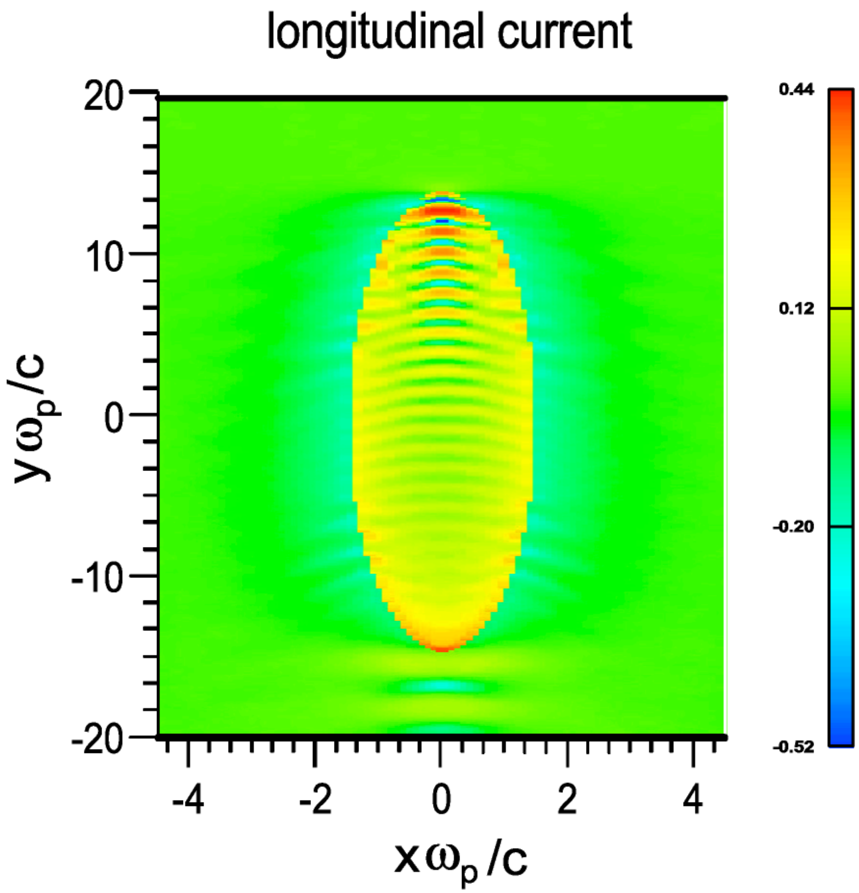
Longitudinally => beam charge and current are neutralized.



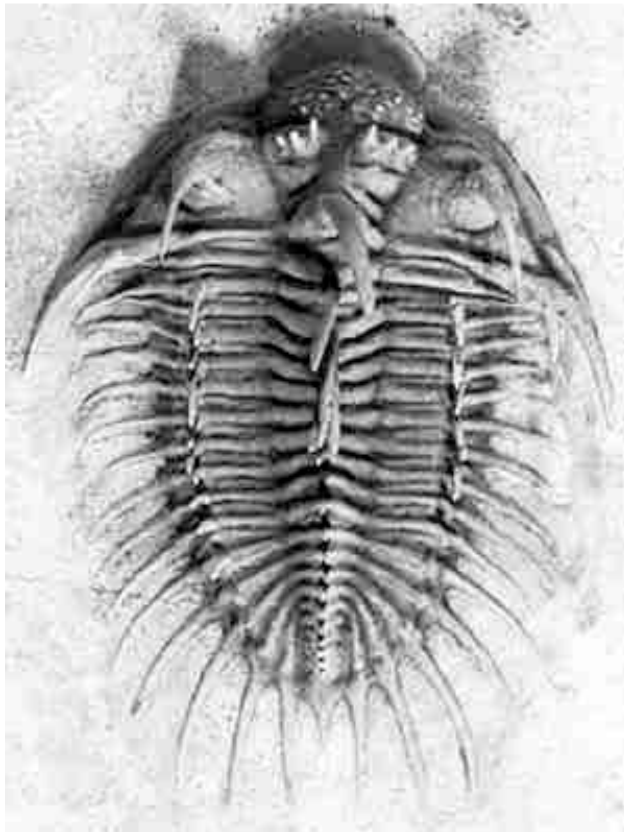
Transversely => beam charge is neutralized, but not current neutralized; a large self-magnetic field can be generated.



Steady- State Results*



Normalized electron current $j_y/(ecn_p)$



http://www.trilobites.com

*I. Kaganovich, *et. al*, IEEE Trans. Plasma Science **33**, 556 (2005).

Nonlinear Theory*

- Important issues:
  - Finite length of the beam pulse
  - General value of n_b/n_p ($n_b >> n_p$)
  - 2D treatment

- Approximations:
  - Fluid approach
  - Conservation of generalized vorticity
  - Long dense beams $l_b >> r_b$.

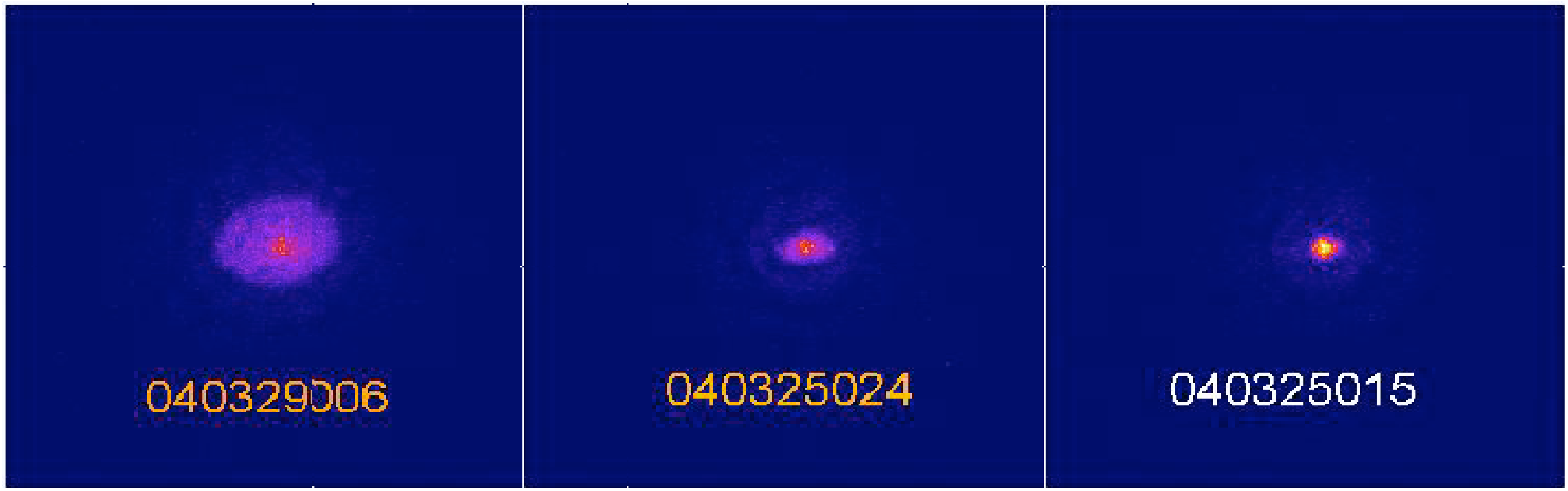
- Exact analytical solution

*I. Kaganovich, *et. al*, Nuclear Instruments and Methods in Physics Research **A 544**, 383 (2005); Physics of Plasmas **8**, 4180 (2001).

Results and Conclusions

- A nonlinear fluid theory has been developed that describes the quasi-steady-state propagation of an intense ion beam pulse in a background plasma.
  - Provides benchmark for numerical codes and experiments.
  - Provides robust analysis of beam propagation through background plasma.
- Simulations of current and charge neutralization performed for conditions relevant to intense ion beams shows:
  - Very good charge neutralization: key parameter is $\omega_p l_b/V_b$,
  - Very good current neutralization: key parameter is $\omega_p r_b/c$.
- Plasma wave breaking heats the electrons whenever $n_p < n_b$.
- Applied solenoidal magnetic field inhibits the self-magnetic field whenever $\omega_{ce} >> \beta_b\, \omega_{pe}$.
- For a solenoidal magnetic field, collective excitations are generated at an angle relative to the solenoidal magnetic field for $\omega_{ce} > \beta_b\, \omega_{pe}$.

Measurements on the Neutralized Transport Experiment (NTX) demonstrate achievement of smaller spot size using volumetric plasma

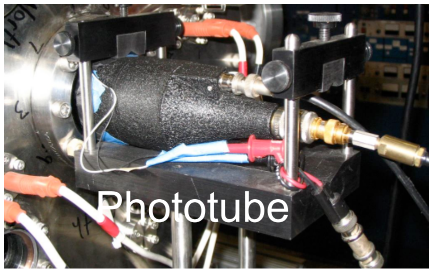


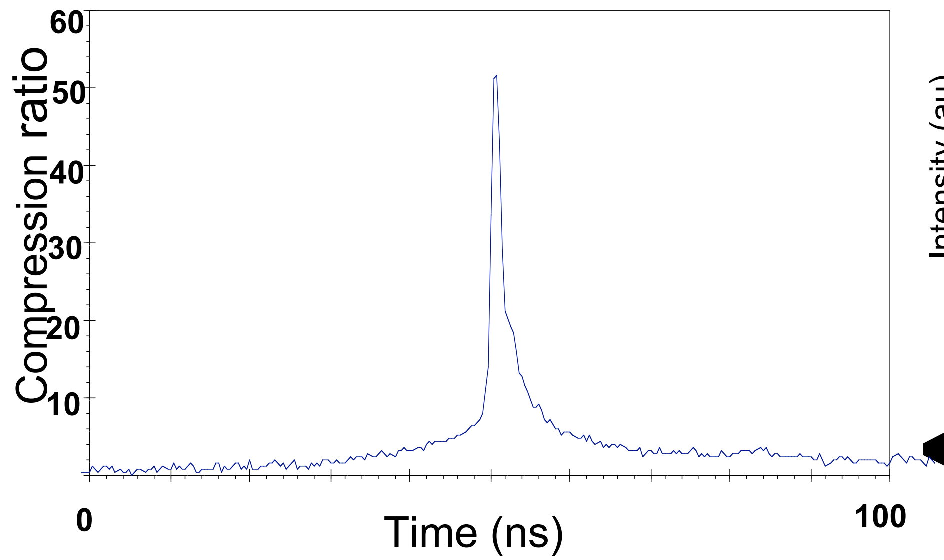
Neither plasma plug nor volumetric plasma.

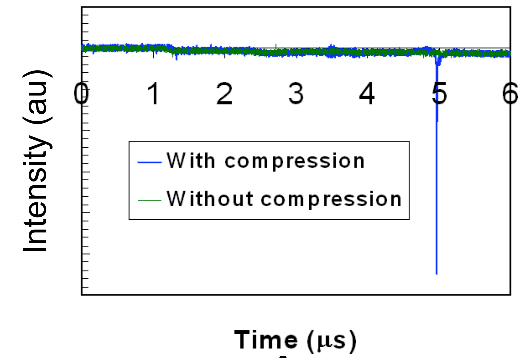
Plasma plug.

Plasma plug and volumetric plasma.

50 Fold Beam Compression Achieved in Neutralized Drift Compression Experiment (NDCX)

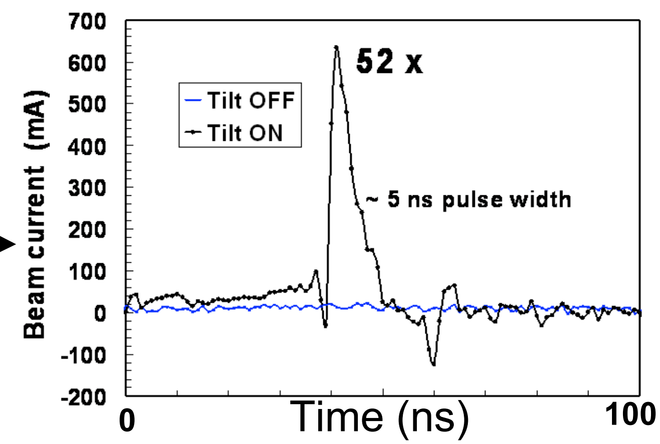




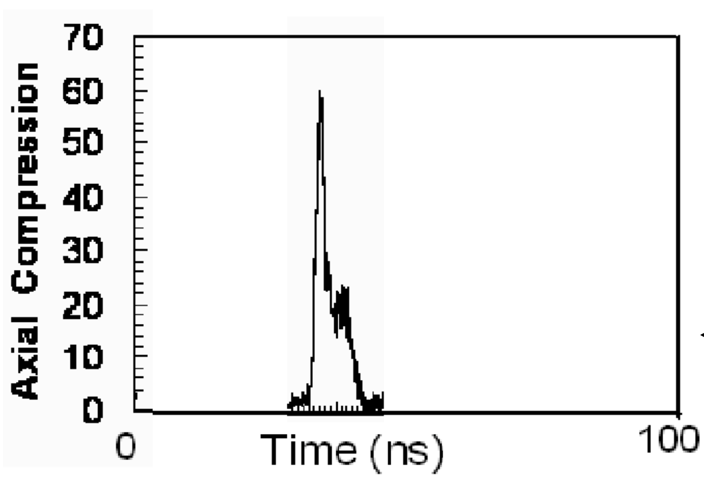


Optical data

Corroborating data from Faraday cup



LSP simulation



Class of Exact Dynamically-Compressing Neutralized Beam Equilibria with Linear Velocity Tilt in a Solenoidal Magnetic Field

Assuming complete charge and current neutralization ($\underline{E}^s = 0 = \underline{B}^s$) for an intense ion beam pulse propagating through background plasma, Vlasov equation supports class of exact, dynamically-compressing solutions

$$f_b(\mathbf{x}, \mathbf{p}, t) = f_b(W_\perp, W_z)$$

where W_z and $W_\perp$ are defined by

$$W_z = \frac{z^2}{z_b^2(t)} + \frac{z_b^2(t)}{z_{b0}^2 v_{T0}^2} \left(v_z - \frac{z}{z_b(t)} \frac{dz_b(t)}{dt} \right)^2$$

$$W_\perp = \frac{x^2+y^2}{r_b^2(t)} + \frac{r_b^2(t)}{r_{b0}^2 v_{\perp 0}^2} [(v_x + \hat{\Omega}_L y - \frac{x}{r_b(t)} \frac{dr_b(t)}{dt})^2 + (v_y - \hat{\Omega}_L y - \frac{y}{r_b(t)} \frac{dr_b(t)}{dt})^2]$$

and Ω_L is the Larmor frequency

$$\Omega_L = -\frac{e_b}{2m_b c} B_z[\gamma_b(z + v_b t)]$$

Class of Exact Dynamically-Compressing Neutralized Beam Equilibria with Linear Velocity Tilt in a Solenoidal Magnetic Field

In the definitions of $W_\perp$ and W_z , the quantities $z_b(t)$ and $r_b(t)$ solve

$$\frac{d^2 z_b(t)}{dt^2} = \frac{z_{b0}^2 v_{T0}^2}{z_b^3(t)}$$

$$\frac{d^2 r_b(t)}{dt^2} + \hat{\Omega}_L^2 r_b(t) = \frac{r_{b0}^2 v_{\perp 0}^2}{r_b^3(t)}$$

and are related to the rms axial and transverse dimensions of the charge bunch.

The constants $z_{b0}^2 v_{z0}^2$ and $r_{b0}^2 v_{\perp 0}^2$ play the roles of scaled transverse and longitudinal emittances.

Dynamically-Compressing Neutralized Beam Equilibria

Many choices of beam equilibria are possible. As one example, consider

$$f_b(W_\perp, W_z) = const.\sqrt{(1 - W_z)}e^{-W_\perp}, \text{ where } 0 \leq W_z < 1$$

Then, the density profile $n_b(r, z, t)$ is given dynamically by

$$n_b(r, z, t) = \hat{n_{b0}} \left[\frac{r_{b0}^2}{r_b^2(t)} \frac{z_{b0}}{z_b(t)}\right] \left(1 - \frac{z^2}{z_b^2(t)}\right) e^{-r^2/r_b^2(t)}, \text{ where } 0 \leq z^2 < z_b^2(t)$$

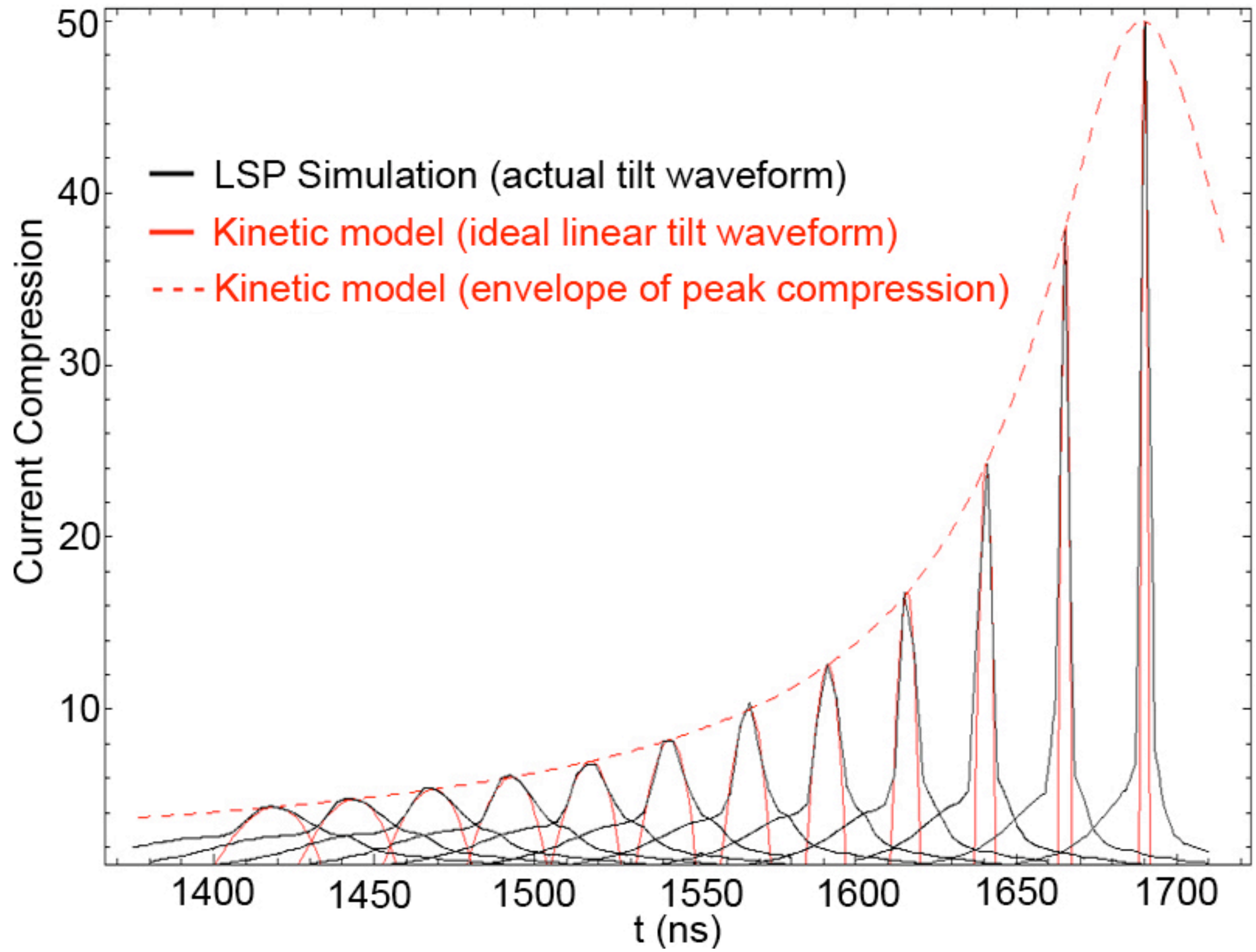
where $r_b(t)$ and $z_b(t)$ solve the envelope equations introduced earlier. For this choice of $f_b(W_\perp, W_z)$ note that the line density is parabolic with

$$\lambda_b(z, t) = \lambda_{b0} \left[\frac{z_{b0}}{z_b(t)}\right] \left(1 - \frac{z^2}{z_b^2(t)}\right), \text{ where } 0 \leq z^2 < z_b^2(t)$$

Comparison with longitudinal current compression in NDCX-1A

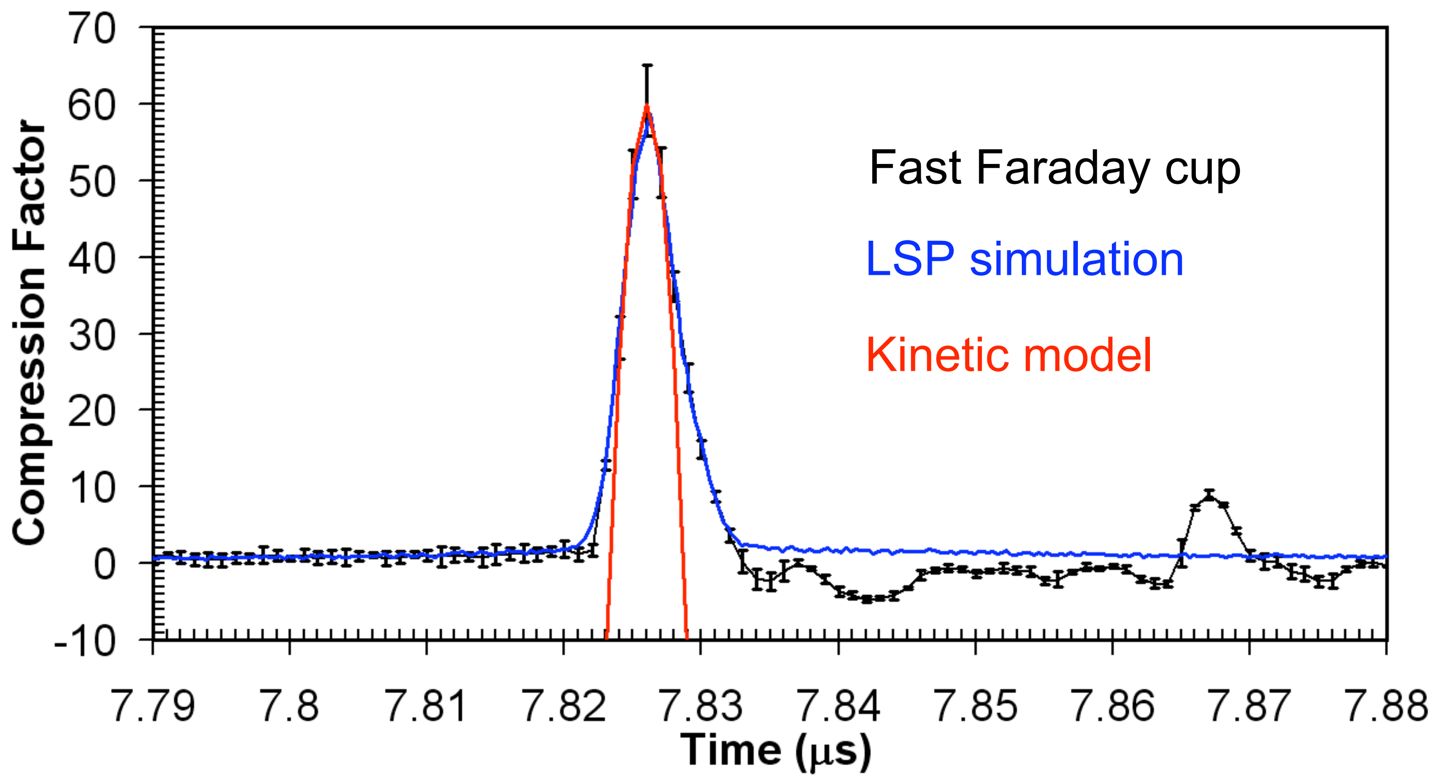
Parabolic waveform of main compression peak is described well by kinetic model

Deviation from ideal (linear) tilt waveform produce pedestals around main parabolic peaks



Comparison with longitudinal current compression in NDCX-1A

Comparison to NDCX-1A experiments at peak compression



Multispecies Weibel Instability

In the collisionless regime, the large directed kinetic energy of the beam ions propagating through a background plasma provides the free energy to drive the electromagnetic Weibel instability.

Assumptions

- Charge and current neutralization

$$\sum_{j=b,e,i} n_j^0(r)e_j = 0 \quad \text{and} \quad \sum_{j=b,e,i} n_j^0(r)e_j\beta_j c = 0$$

- Electromagnetic perturbations with $\partial/\partial\theta = 0 = \partial/\partial z$ and polarization

$$\delta\mathbf{E} = \delta E_r \hat{\mathbf{e}}_r + \delta E_z \hat{\mathbf{e}}_z \ , \quad \delta\mathbf{B} = \delta B_\theta \hat{\mathbf{e}}_\theta$$

Multispecies Weibel Instability

Express

$$\delta E_z(r,t) = \delta\hat{E}_z(r)\exp(-i\omega t)$$

Obtain the eigenvalue equation

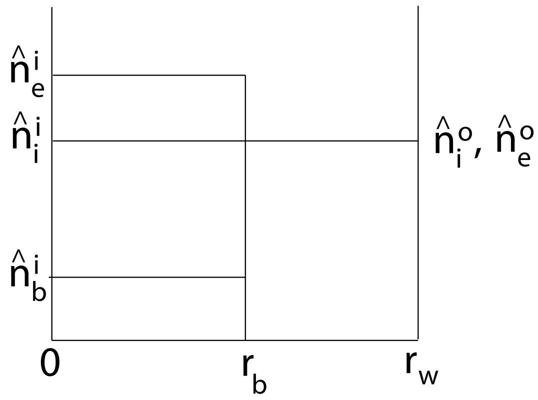
$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1+\sum_{j=b,e,i}\frac{\beta_j^2\omega_{pj}^2(r)}{\omega^2}+\frac{(\sum_{j=b,e,i}\beta_j\omega_{pj}^2(r))^2}{\omega^2[\omega^2-\sum_{j=b,e,i}\omega_{pj}^2(r)]}\right)\frac{\partial}{\partial r}\delta\hat{E}_z\right]+\left(\frac{\omega^2}{c^2}-\sum_{j=b,e,i}\frac{\omega_{pj}^2(r)}{\gamma_j^2c^2}\right)\delta\hat{E}_z=0$$

where $\omega_{pj}(r) = [4\pi n_j^0(r)e_j^2/\gamma_j m_j]^{1/2}$ and $\gamma_j = (1-\beta_j^2)^{-1/2}$.

Slow-wave Weibel instability driven by the terms proportional to

$$\sum_{j=b,e,i}\beta_j^2\omega_{pj}^2(r) \quad \text{and} \quad \sum_{j=b,e,i}\beta_j\omega_{pj}^2(r)$$

Multispecies Weibel Instability



An intense ion beam with step-function density profile propagates through background plasma with uniform density.

Treating the plasma ions as stationary ($\beta_i = 0$) and assuming local charge and current neutralization gives

$$\hat{n}_e^i = Z_b \hat{n}_b^i + Z_i \hat{n}_i^i$$

$$\beta_e = \frac{Z_b \hat{n}_b^i}{Z_b \hat{n}_b^i + Z_i \hat{n}_i^i} \beta_b$$

Multispecies Weibel Instability

For the case of uniform density profiles the eigenvalue equation can be solved exactly to obtain a closed dispersion relation for the complex oscillation frequency ω.

For perturbations with short transverse wavelength the characteristic growth rate of the Weibel instability scales as $Im\omega \sim \Gamma_w$ where

$$\Gamma_w^2 \equiv \beta_e^2 \widehat{\omega}_{pi}^{i^2} + (\beta_b - \beta_e)^2 \widehat{\omega}_{pb}^{i^2}$$

and $\widehat{\omega}_{pj}^i = (4\pi \widehat{n}_j^i e_j^2 / \gamma_j m_j)^{1/2}$.

Multispecies Weibel Instability

The full dispersion relation has been solved numerically over a wide range of beam-plasma parameters.

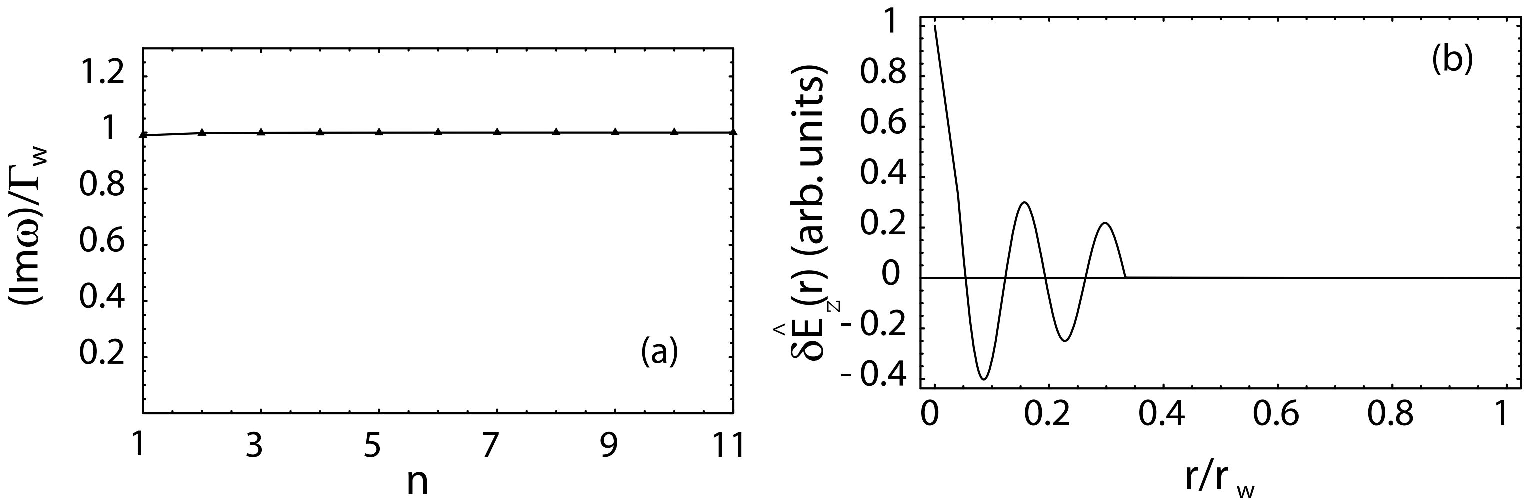
As an illustrative example consider an intense cesium ion beam ($Z_b = 1$) propagating through background argon plasma ($Z_i = 1$) with

$$\beta_b = 0.2 \ , \quad \beta_i = 0 \ , \quad \beta_e = 0.1$$

$$\hat{n}_b^i = \hat{n}_e^i/2 = \hat{n}_i^i$$

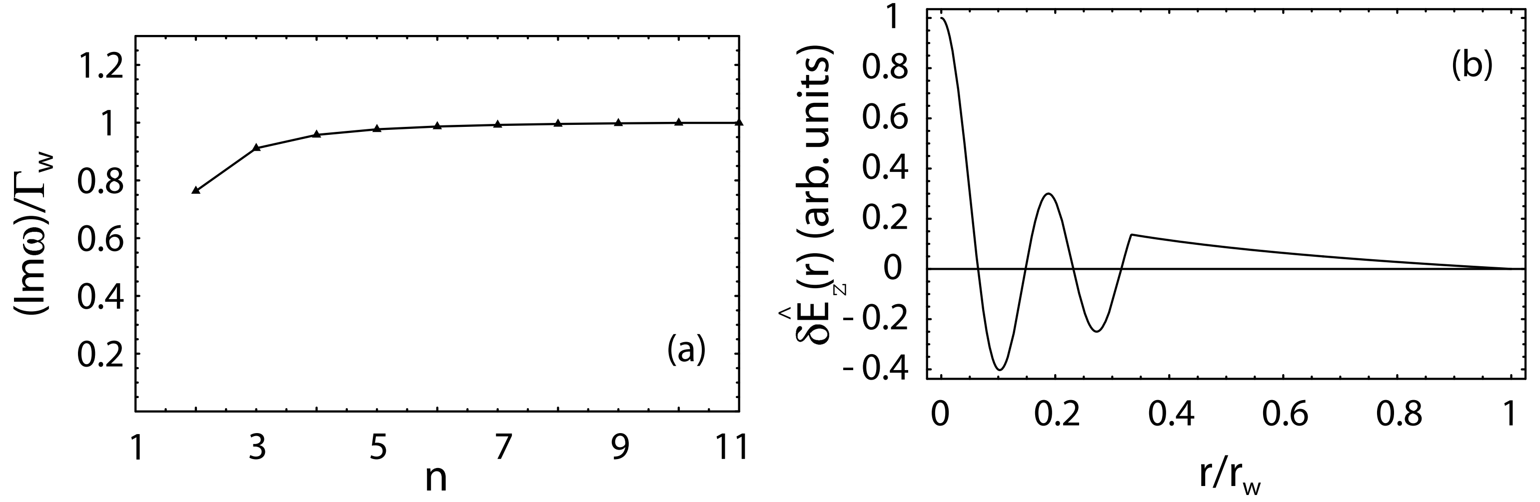
The background plasma provides complete charge and current neutralization.

Multispecies Weibel Instability



Plots of (a) Weibel instability growth rate $(Im\omega)/\Gamma_w$ versus mode number n, and (b) eigenfunction $\delta\hat{E}_z(r)$ versus r/r_w for $n = 5$. System parameters are $r_b = r_w/3$, $\hat{\omega}^i_{pe} r_b/c = 1/3$ and $\hat{n}^0_j = 0$ (vacuum region outside beam).

Multispecies Weibel Instability



Plots of (a) Weibel instability growth rate $(Im\omega)/\Gamma_w$ versus mode number n, and (b) eigenfunction $\delta\hat{E}_z(r)$ versus r/r_w for $n = 5$. System parameters are $r_b = r_w/3$, $\hat{\omega}^i_{pe} r_b/c = 3$ and $\hat{n}^0_j \equiv 0$ (vacuum region outside beam).

Multispecies Weibel Instability

- An effective means to reduce the growth rate of the Weibel instability is to decrease the value of fractional current neutralization f_m.

- The B_θ self-magnetic field constrains the transverse particle motion and likelihood of filamentation in the transverse plane.

Minimizing Effects of the Multispecies Weibel Instability

- Multispecies Weibel instability is unlikely to have a deleterious effect on the beam quality provided

$$\Gamma_W \tau_p < 1,$$

where $\tau_p = L_p/V_b$ is the interaction time of the beam ions with the background plasma. Equivalently, $\Gamma_W \tau_p < 1$ gives

$$L_p < \alpha \frac{c}{\omega_{pb}} = 2.3 \times 10^7 \alpha \frac{A_b^{1/2}}{[n_b(cm^{-3})]^{1/2}} cm.,$$

where the constant α is defined by

$$\alpha = \left[\left(1 - \frac{Z_b n_b}{n_e} \right)^2 + \frac{Z_i}{Z_b} \frac{m_b}{m_i} \frac{Z_b n_b}{n_e} \left(1 - \frac{Z_b n_b}{n_e} \right) \right]^{-1/2} .$$

- For Aluminium beam ions ($A_b = 13$) in background Argon plasma ($A_i = 18$) and $n_b/n_i = 1/2$, we obtain

$$L_p < 1.27m, \quad 12.7m, \quad 127m$$

for

$$n_b = 10^{12} cm^{-3}, \quad 10^{10} cm^{-3}, \quad 10^8 cm^{-3}.$$

Conclusions

- A wide variety of collective instabilities in intense beams and beam-plasma systems have been investigated.
- Growth rate reduction (or elimination) mechanisms have been identified.
- Numerical simulations are playing a critical role in determining threshold conditions and nonlinear dynamics.
- See related publications at http://nonneutral.pppl.gov.
- Related papers on collective excitations at HIF06 Symposium by Kaganovich, Qin, Startsev, Sefkow and Welch.

Back-up Vugraphs