Stability properties of the electron return current for intense ion beam propagation through background plasma

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Abstract

When an intense ion beam propagates through a dense background plasma, its current is partially neutralized by the electron plasma return current. Due to the non-uniformity of the background plasma electrons longitudinal velocity profile $\bar{v}(r)$, the flow can be unstable. The instability is similar to the Kelvin-Helmholz instability for the non-uniform flow of an incompressible neutral fluid, with the electrostatic potential playing the role of pressure. For the case of electron return current flow, the significant new feature is the presence of the partially self-neutralized magnetic field of the ion beam, which significantly affects the evolution of small-amplitude excitations. In this paper the stability properties of the flow of electrons making up the plasma return current is investigated using the macroscopic cold-fluid-Maxwell equations. It is shown that this flow may become unstable, but the instability growth rates are exponentially small. This unstable body mode is qualitatively different from previously studied surface-mode excitations of the electron plasma return current for an intense ion beam with a sharp radial boundary, which is found to be stable due to the stabilizing influence of the partially neutralized magnetic field of the ion beam (Roy, P. K. 2005).

I. INTRODUCTION

Ion beam propagation in neutralizing background plasma is of interest for many applications, including ion-beam-driven high energy density physics and heavy ion fusion. The background plasma is needed to neutralize the ion beam space charge and beam current so that it can be transported and efficiently focused either ballistically or by the remnant unneutralized self-magnetic field or applied magnetic field (Roy, P. K. 2005; Logan et. al., 2007; Sefkow et. al., 2007; Welch et. al., 2007). The ion beam current is neutralized by the opposing background plasma electron return current (Kaganovich et. al., 2007; Kaganovich et. al., 2005; Kaganovich et. al., 2001), which implies that the plasma electrons inside the beam flow with average velocity $v_{e0} = Z_b(n_b/n_0)v_b$ relative to the electrons outside of the beam. Here, v_b is the beam velocity, Z_b is the charge state of the beam ions, and n_b and n_0 are the beam density and background electron densities, respectively. One of the main disadvantages of using plasma to transport and focus intense ion beams is that the ion beam propagation in background plasma may be subject to collective instabilities. There is a growing body of literature dedicated to studying collective beam-plasma interactions. For a recent review of collective beam-plasma instabilities see Ref. (Davidson et. al., 2004; Davidson et. al., 2009). In a recent paper (Startsev et. al., 2009) we have reconsidered the stability of the background electron return current, found to be strongly unstable by many previous authors (see for example Refs. (Rose et. al., 2003; Rose et. al., 2005)). We have shown that the unneutralized magnetic field in the return current layer is responsible for complete stabilization of this particular instability. In that paper (Startsev et. al., 2009) we analyzed only the surface modes by making an explicit assumption that the beam radius a is much greater than the characteristic width of the layer with unneutralized magnetic field. In the present paper we show that the return current can still support unstable body modes that are localized at the beam center. In particular, it is found that the instability develops only if the background electron flow velocity profile $\bar{v}(r)$ satisfies the two resonance conditions $\omega - k_z \bar{V}(r_1) = -\omega_{pe}$ and $\omega - k_z \bar{v}(r_2) = \omega_{pe}$ simultaneously at different radial locations r_1 and r_2 from the beam center. Here ω is the mode oscillation frequency, k_z is the axial wave-number in the flow direction, and ω_{pe} is the background electron plasma frequency. The first resonance condition guarantees the existence of the body-mode excitation, whereas the second resonance condition assures that the mode is unstable. The body mode has a maximum growth rate for beams with radius of the order of the collisionless electron skin

depth, $a \sim c/\omega_{pe}$.

The analysis in the present paper makes use of the formalism developed by one of the authors (Khudik et. al., 1995). The organization of this paper is the following. In Sec. II, the Maxwell equations and cold fluid equations describing the motion of the electron fluid responding to injection of the heavy ion beam into background plasma are analyzed. We use the conservation of generalized vorticity to derive a single nonlinear vector equation for the electron fluid velocity. The steady-state solution of this equation describes the equilibrium electron flow around the beam. This nonlinear equation is linearized around the steadystate solution, and is used to show that the unstable perturbations are short-wavelength, electrostatic perturbations with a negligible perturbed magnetic field component. In Sec. III, the eigenvalue equation for the electrostatic potential perturbations is derived. In Sec. IV we present a detailed analysis of the eigenvalue equation in planar geometry for perturbations with zero perturbed azimuthal flow velocity. The approximate expressions for the mode frequencies are found, and an estimate of the maximum growth rate is presented. Finally, the conclusions are summarized in Sec. V.

II. THEORETICAL MODEL

The analysis presented in this paper is carried out for nonrelativistic ion beams with $v_b^2/c^2 \ll 1$. We assume that the beam propagates along the z-axis (longitudinal direction) with average velocity $\mathbf{v_b} = \mathbf{e}_z v_b$. The equation describing the dynamics of the cold, neutralizing election background are the momentum equation for the flow velocity ${\bf v}$ of the cold electron fluid,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{e}{m} \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right), \tag{1}$$

and Maxwell's equations for the electric E field and magnetic field B,

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} e(Z_b n_b \mathbf{v}_b - n \mathbf{v}),$$
(2)

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} e(Z_b n_b \mathbf{v}_b - n \mathbf{v}), \tag{3}$$

$$\nabla \cdot \mathbf{E} = 4\pi e (Z_b n_b + n_0 - n), \tag{4}$$

where n is the electron density, n_b, \mathbf{v}_b and Z_b are the ion beam density, velocity and charge state respectively, and n_0 is the plasma ion background density. Also, the constants -e, m, and c are the electron charge, mass and speed of light, respectively. Combining the curl of Eq. (1) with Eq. (2) we obtain the equation for the generalized vortisity $\mathbf{\Theta} = \nabla \times \mathbf{v} - (e/mc)\mathbf{B}$, i.e.,

$$\frac{\partial \mathbf{\Theta}}{\partial t} = \nabla \times [\mathbf{v} \times \mathbf{\Theta}]. \tag{5}$$

For the ion beam injected into background plasma, the generalized vorticity is zero everywhere ahead of the beam, and it follows from Eq. (5) that it remains zero everywhere, or equivalently,

$$\mathbf{B} = \frac{mc}{e} \nabla \times \mathbf{v}.\tag{6}$$

Substituting Eq. (6) into the momentum equation (1), we obtain

$$\mathbf{E} = -\frac{m}{e} \left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \frac{\mathbf{v}^2}{2} \right). \tag{7}$$

Substituting Eq. (7) into Eq. (4), we obtain the expression for electron density,

$$n = n_0 + Z_b n_b + \frac{n_0}{\omega_{pe}^2} \left(\Delta \frac{v^2}{2} + \frac{\partial}{\partial t} \nabla \cdot \mathbf{v} \right), \tag{8}$$

where $\Delta \equiv (\nabla \cdot \nabla)$ is the Laplacian. Here, $\omega_{pe} = (4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency. Finally, substituting Eqs. (6)–(8) into Eq. (3), we obtain a nonlinear equation for the electron flow velocity

$$\lambda_{pe}^{2} \nabla \times \nabla \times \mathbf{v} + \frac{1}{\omega_{pe}^{2}} \left[\frac{\partial^{2}}{\partial t^{2}} \mathbf{v} + \frac{\partial}{\partial t} \nabla \frac{v^{2}}{2} + \mathbf{v} \left(\Delta \frac{v^{2}}{2} + \frac{\partial}{\partial t} \nabla \cdot \mathbf{v} \right) \right] + \mathbf{v} \left(1 + Z_{b} \frac{n_{b}}{n_{0}} \right) = Z_{b} \frac{n_{b}}{n_{0}} \mathbf{v}_{b}.(9)$$

Here, $\lambda_{pe} = c/\omega_{pe}$. It follows from Eq. (9) that for $(v_b/c)^2 \ll 1$ the steady-state solution $\mathbf{v} = \mathbf{e}_z \bar{v} \ (\partial/\partial t = 0)$ is nonrelativistic $(\bar{v}^2/c^2 \ll 1)$ and can be determined from

$$\lambda_{pe}^2 \Delta_\perp \bar{v} = \left(1 + Z_b \frac{n_b}{n_0}\right) \bar{v} - Z_b \frac{n_b}{n_0} v_b,\tag{10}$$

where $\Delta_{\perp} \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$ is a transverse Laplacian. Here, we have assumed that $n_b(\mathbf{x}_{\perp})$ and $v_b(\mathbf{x}_{\perp})$ are functions of the transverse coordinates $\mathbf{x}_{\perp} = (x, y)$. From Eq. (10), we obtain an estimate for the steady-state velocity of the background electrons, i.e.,

$$\bar{v} \sim v_b \frac{Z_b n_b / n_0}{(1 + \lambda_{pe}/a)^s + Z_b n_b / n_0} \lesssim v_b.$$
 (11)

Here, a is the radius of the injected beam, and s=1 for planar geometry, and s=2 for a cylindrically-symmetrical beam. Note that it follows from Eq. (8) that the steady-state normalized electron density $\bar{n} = n/n_0$ is given by $\bar{n} = 1 + Z_b n_b/n_0$, neglecting relativistic corrections.

Next, we express $\mathbf{v} = \mathbf{e}_z \bar{v} + \mathbf{v}'$, and derive the equation for the unstable perturbation $\mathbf{v}' \sim \exp(-i\omega t + ik_z z)$ about the steady-state equilibrium \bar{v} given by Eq. (10). Linearizing equation (9) and neglecting terms $(\bar{v}/c)^2 \ll 1$, we obtain

$$c^{2}\nabla \times \nabla \times \mathbf{v}' - \omega^{2}\mathbf{v}' - i\omega \left[\nabla(\bar{v}v_{z}') + \mathbf{e}_{z}\bar{v}\nabla \cdot \mathbf{v}'\right] + \mathbf{e}_{z}\bar{v}\Delta(\bar{v}v_{z}') + \bar{\omega}_{pe}^{2}\mathbf{v}' = 0, \tag{12}$$

where $\bar{\omega}_{pe}^2 = \omega_{pe}^2 \bar{n}$. Multiplying Eq. (12) by $(\omega \mathbf{v}')^*$, integrating over \mathbf{x}_{\perp} and taking the imaginary part, we obtain

$$Im\omega \left[c^2 \int d\mathbf{x}_{\perp} |\nabla \times \mathbf{v}'|^2 + \int d\mathbf{x}_{\perp} (|\omega|^2 + \bar{\omega}_{pe}^2) |\mathbf{v}'|^2 \right] = Im\omega \left[\int d\mathbf{x}_{\perp} |\nabla (\bar{v}v_z')|^2 \right]. \tag{13}$$

The integral on the right-hand side of Eq. (13) can be expressed as

$$\int d\mathbf{x}_{\perp} |\nabla(\bar{v}v_z')|^2 = \int d\mathbf{x}_{\perp} \bar{v}^2 \left\{ k_z^2 |\mathbf{v}'|^2 + 2k_z Im[\mathbf{v}' \times \nabla \times \mathbf{v}'^*]_z + |(\nabla \times \mathbf{v}')_{\perp}|^2 \right\} - \int d\mathbf{x}_{\perp} (\bar{v}\Delta_{\perp}\bar{v})|v_z'|^2.$$
(14)

It follows from Eqs. (10) and (11) that the last term in Eq. (14) is $(v_b/c)^2 \ll 1$ times smaller than the second term on the left hand side of Eq. (13). Hence, it follows from Eqs. (13) and (14) that the frequency, longitudinal wave-number, and polarization for unstable perturbations with $Im\omega > 0$ must satisfy the conditions

$$|\omega|^2 + \omega_{ne}^2 < k_z^2 v_m^2, \tag{15}$$

$$\int d\mathbf{x}_{\perp} |\nabla \times \mathbf{v}'|^2 < (v_m/c)^2 k_z^2 \int d\mathbf{x}_{\perp} |\mathbf{v}'|^2, \tag{16}$$

where $v_m = max(\bar{v})$. A more detailed analysis shows that the $(v_m/c)^2$ factor in Eq. (16) should be replaced by $(v_m/c)^4$. This fact together with Eq. (15) implies that the unstable perturbations are short-wavelength $[k_z^2c^2 > (c/v_m)^2(|\omega|^2 + \omega_{pe}^2) \gg |\omega^2 - \omega_{pe}^2|]$ electrostatic perturbations with $\mathbf{v}' \approx \nabla \Phi$, and that the perturbed magnetic field $\mathbf{B}' \sim \nabla \times \mathbf{v}'$ is negligibly small.

III. EIGENVALUE EQUATION

The equation for the electrostatic short-wave perturbations can be obtained by substituting $\mathbf{v}' = \nabla \Phi$ into Eq. (12). Taking the divergence, we obtain an equation for Φ

$$\nabla \cdot [(\bar{n} - \Omega^2) \nabla \Phi] = (\Omega \Delta \Omega) \Phi, \tag{17}$$

where $\Omega = (\omega - k_z \bar{v})/\omega_{pe}$. Below we specialize to the case of small beam density with $Z_b n_b/n_0 \ll 1$. In this case, it follows from Eq. (15) that $\nabla_{\perp} \Omega \gg \nabla_{\perp} \bar{n}^{1/2}$. Taking this fact into account and introducing the new variable $\phi = (\bar{n} - \Omega^2)^{1/2} \Phi$, Eq. (17) can be rewritten in the form of Schrodinger's equation

$$k_z^{-2} \Delta_{\perp} \phi = (1 - U)\phi,$$
 (18)

where the effective potential U is given by

$$U = \frac{(\nabla_{\perp} \bar{v}/\omega_{pe})^2}{(1 - \Omega^2)^2} = \frac{f}{(1 - \Omega^2)^2}.$$
 (19)

Note, that the potential U is small everywhere $(U \approx f \sim (v_b/c)^2 (Z_b n_b/n_0)^2 (a/\lambda_{pe})^s$ for $a \ll \lambda_{pe}$, and $U \approx f \sim (v_b/c)^2 (Z_b n_b/n_0)^2 (\lambda_{pe}/a)^2$ for $a \gg \lambda_{pe}$) except at a distance $d \sim |\nabla_{\perp} \bar{v}/\omega_{pe}|/|\nabla_{\perp} \Omega| \sim 1/k_z$ from the resonant points where $\Omega^2 \approx 1$. Therefore, the unstable mode growth rates are also small with $Im\omega/\omega_{pe} < f^{1/2}/2 \ll 1$.

IV. ANALYSIS OF EIGENVALUE EQUATION

For simplicity, we consider here the stability of planar flow with velocity profile $\bar{v}(x)$, which is a function of the transverse coordinate x (radial direction). We also consider only excitations with zero azimuthal velocity component $v'_y = 0$. In this case, the eigenvalue equation (18) can be written as

$$\frac{d^2\phi}{dx^2} = k_z^2 (1 - U)\phi. {20}$$

Note that Eq. (20) is equivalent in the electrostatic limit $(k_z^2 c^2 \gg |\omega^2 - \omega_{pe}^2|)$ to Eq. (20) of Ref. (Startsev et. al., 2009) if one makes the substitution $E_z = -(m/e)\omega_{pe}\Omega/(1-\Omega^2)^{1/2}\phi$. In Ref. (Startsev et. al., 2009) it was shown that Eq. (20) has no unstable solutions in the limit of large beam radius $a \gg \lambda_{pe}$. Therefore, we study here the the effects of finite beam radius on the stability of the background electron flow.

We are looking for a solution of Eq. (20) which asymptotically behaves as $\phi \sim \exp(k_z x)$ for $x \to -\infty$. For arbitrary ω this solution of Eq. (20) will behave as

$$\phi = A_{+}(\omega)e^{k_{z}x} + A_{-}(\omega)e^{-k_{z}x}, \tag{21}$$

for $x \to \infty$. The frequency of the localized mode is determined from the condition $A_+(\omega) = 0$. For a monotonic, symmetric velocity profile, $\bar{v}(|x|)$, the resonance condition where $\Omega^2 = 1$

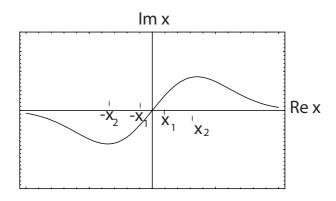


FIG. 1: Path in the complex x-plane along which the potential $|U| \ll 1$.

is satisfied at four points $x=\pm x_1(\omega)$ and $x=\pm x_2(\omega)$ $(x_2>x_1)$. Let us first consider ω such that $x_1(\omega)\gg 1/k_z$. If we analytically continue Eq. (20) into the complex x-plane, the potential U will have singularities far enough from each other that we can always go from $x=-\infty$ to $x=+\infty$ on the real x-axis along a path in the complex x-plane where $|U|\ll 1$ [see Fig.1]. Since along this path $|(dU/dx)/U| \geq 1/\lambda_{pe} \ll k_z$ we can use a quasi-classical approximation for the "wave-function" ϕ and obtain

$$\phi \approx e^{k_z x} \tag{22}$$

everywhere along this path. Hence, the coefficient $A_+ \approx 1$ and there are no localized modes. Now, let ω be such that $x_1(\omega) \sim 1/k_z \ll \lambda_{pe}$. In this case, the two singularities of the potential at x_1 and $-x_1$ are too close to each other, so that the quasi-classical approximation is not applicable near those singularities. But in this region we can approximate the velocity profile as parabolic with $\bar{v} = v_m(1 - x^2/a_*^2)$, and rewrite the eigenvalue equation (20) as

$$\frac{d^2\phi}{d\bar{x}^2} = \left[1 - \frac{\bar{x}^2}{(\bar{x}^2 - \bar{\omega})^2}\right]\phi,\tag{23}$$

where $\bar{x} = k_z x$ and $\bar{\omega} = -(k_z a_*)^2 (\omega_{pe} + \omega - k_z v_m)/(k_z v_m)$. Here $a_*^2 \approx a^2$ for $a \gg \lambda_{pe}$, and $a_*^2 \approx \sqrt{\pi} a \lambda_{pe}$ for $a \ll \lambda_{pe}$. The eigenvalues of Eq. (23) are given by

$$\bar{\omega}_0 = -0.1, \quad \bar{\omega}_n = a^+ Q^n, \quad n = 2, 4, 6, ...,$$
 (24)

$$\bar{\omega}_n = a^- Q^{n-1}, \quad n = 1, 3, 5, ...,$$
 (25)

where $a^+ = -0.125...$, $a^- = -0.00452...$ and $Q = \exp(-2\pi/\sqrt{3})$. Equation (24) gives the spectrum of symmetric modes, and Eq. (25) is the spectrum of antisymmetric modes.

The stable localized modes described by Eqs. (24) and (25) become unstable if we take into account the singularity of the potential at $x = \pm x_2$. To obtain the expression for $Im\omega$

let us multiply Eq. (17) by Φ^* and integrate it over x. Taking the imaginary part, we obtain

$$\int_0^\infty k_z dx \left[2Re\Omega \left(\frac{1}{k_z^2} \left| \frac{d\Phi}{dx} \right|^2 + |\Phi|^2 \right) - k_z^{-2} \Delta_\perp (Re\Omega) |\Phi|^2 \right] \right] = 0.$$
 (26)

For $x \sim 1/k_z$, $\phi \sim 1$, and the function Φ is given by

$$\Phi \sim \Phi_0 \equiv \left| \frac{d^2}{dx^2} \left(\frac{\bar{v}}{k_z \omega_{pe}} \right)_{x=0} \right|^{-1/2}, \tag{27}$$

and decreases exponentially as x increases. The exception is the neighborhood of the second resonance $x = x_2$, where the asymptotic behavior of Φ is given by

$$\Phi \sim \exp(-k_z x_2) \left| \frac{1}{\omega_{pe}} \frac{d\bar{v}}{dx} \right|^{-1/2} \log \eta, \quad \eta = 1 - \Omega.$$
 (28)

The neighborhood of $x \sim 1/k_z$ contributes a value of $\sim |\Phi_0|^2$ in the integrand in Eq. (26), and the contribution from the neighborhood of $x = x_2$ is given by $(\omega_p/Im\omega) \exp(-2k_zx_2)$. Hence, from Eq. (26) we obtain the estimate of the mode growth rate given by

$$Im\omega = C \left| \frac{1}{k_z} \frac{d^2 \bar{v}}{dx^2} \right|_{x=0} \exp(-2k_z x_2). \tag{29}$$

Next we estimate the dependence of the maximum growth rate on the beam radius. From the resonance conditions

$$Re\omega - k_z \bar{v}_m + \omega_{pe} \approx 0,$$
 (30)

$$Re\omega - k_z \bar{v}(x_2) - \omega_{pe} \approx 0,$$
 (31)

we obtain

$$\Psi \equiv (k_z x_2)_{min} = 2min \left[\frac{x_2/\lambda_{pe}}{|\bar{v}_m - \bar{v}(x_2)|/c} \right]. \tag{32}$$

For a large beam radius $a \gg \lambda_{pe}$, the maximum growth rate will occur at the point

$$x_2 \sim a, \quad \frac{v_m - v(x_2)}{c} \sim \frac{Z_b n_b}{n_0} \frac{v_b}{c}, \quad \Psi \sim \frac{a}{\lambda_{ne}} \frac{n_0}{Z_b n_b} \frac{c}{v_b}.$$
 (33)

For a beam with small radius $a \ll \lambda_{pe}$, we obtain

$$x_2 \sim a, \quad \frac{v_m - v(x_2)}{c} \sim \frac{Z_b n_b}{n_0} \frac{v_b}{c} \left(\frac{a}{\lambda_{pe}}\right)^{s+1}, \quad \Psi \sim \left(\frac{\lambda_{pe}}{a}\right)^s \frac{n_0}{Z_b n_b} \frac{c}{v_b},$$
 (34)

where s = 1 for planar geometry, and s = 2 for a cylindrically-symmetrical beam. Therefore, for arbitrary beam radius we can estimate the maximum growth rate as

$$\frac{Im\omega}{\omega_{pe}} \sim \frac{\exp(-2\Psi)}{\Psi^2}, \quad \Psi \sim \left(\frac{c}{v_b}\right) \left(\frac{n_0}{Z_b n_b}\right) \left[\frac{a}{\lambda_{pe}} + \left(\frac{\lambda_{pe}}{a}\right)^s\right].$$
 (35)

It follows from Eq. (35) that the strongest instability corresponds to the injection of a beam with radius $a \sim \lambda_{pe}$.

V. CONCLUSIONS

In this paper we have studied the stability of nonuniform background electron flow resulting from the injection of a long ion beam into a neutralizing plasma. We have shown that such flows can support unstable short-wavelength electrostatic perturbations with a negligible perturbed magnetic field component. The perturbations are localized near the beam center unlike the surface modes studied previously (Startsev et. al., 2009) which are localized at the beam edge and had been found to be stable. Moreover, it is found that the instability develops only if the background electron flow velocity profile $\bar{v}(r)$ satisfies the two resonance conditions $\omega - k_z \bar{V}(r_1) = -\omega_{pe}$ and $\omega - k_z \bar{v}(r_2) = \omega_{pe}$ simultaneously at different radial locations r_1 and r_2 from the beam center. The first resonance condition guarantees the existence of the body-mode excitation, whereas the second resonance condition assures that the mode is unstable. These body modes have growth rates that are exponentially small [Eq. (35)] and are largest for beams with radius of the order of the collisionless electron skin depth, $a \sim c/\omega_{pe}$.

Acknowledgments

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