## Thin Foil Transformation into Liquid Droplets due to Rayleigh-Taylor instability in NDCX-1 experiments

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In experiments at neutralized drift current experiment (NDCX-I) facility an intense ion beam heat a thin foil above the melting temperature. For NDCX-I parameters the thin film is liquefied, but it is not evaporated completely into a gas. Instead, liquefied foil is dissembled into droplets. A possible mechanism of droplet formation is Rayleigh-Taylor instability associated with liquid film acceleration due to a small difference of vapor pressures at front and back sides of the film. Both sides of the foil evaporate, but at slightly different rates. This imbalance in vapor pressure produces a net acceleration of the thin liquid film towards one side, and the Rayleigh-Taylor instability develops as a result of this acceleration. The perpendicular size of the perturbations is determined by the maximum growth rate of the Rayleigh-Taylor instability as a function of the perturbation size, and is affected by the surface tension. Droplets are then formed during the nonlinear stage of the instability.

In neutralized drift current experiment (NDCX-I) a thin target foil is heated by an ion beam pulse to the temperatures above melting point. After the target is heated above melting, experiments observe target disassembly into droplets<sup>1</sup>, as shown in Fig. 1. There could be several mechanisms of droplet formation.

- Destruction of the solid film due to build-up tension before film is liquefied.
- Formation of the droplet from the evaporated gas during expansion into a vacuum and subsequent cooling<sup>2</sup>.
- Formation of droplets due to Marangoni effect. The large gradients of temperature along the film surface cause inhomogeneous surface tension due to its dependence on temperature. Such force results in liquid stratification and droplet formation.
- For films with thickness larger than the range of ions in the film, the Rayleigh-Taylor instability due to acceleration of the liquefied film due to a difference of vapor pressures at front and back sides of the film.



Fig. 1. Beam irradiation of Pt foil led to images of what appears to be droplets and scatter light up to 500 µs after beam pulse passes through the target film on NDCX-I [1]. This is self-emission downstream from a 120 nm Pt target; image taken by Princeton Instruments PIMAX camera with 5 microsecond gate.

In the following we consider only the last effect. Estimates show that for the intensity of the ion beam the NDCX-I experiment a thin foil is liquefied by the intense ion beam pulse, but it is not transformed into a gas. Both sides of the liquefied film evaporate into a vacuum, but at slightly different rates. This imbalance in vapor pressure produces a net acceleration of the thin liquid film towards one side, and the Rayleigh-Taylor instability develops as a result of this acceleration. The perpendicular size of the perturbations is determined by the maximum growth rate as a function of the perturbation size, and is affected by the surface tension. Droplets are then formed during the nonlinear stage of the instability.

We shall make use of the Van der Waals equation-of-state (EOS), i.e.,

$$(P+n^2a)(1-nb)=nRT,$$

where *P* is the pressure, *n* is the density,  $R=kN_A$  is the gas constant. Constant *a* is a measure for the attraction between the particles and constant *b* is the volume excluded by a particle.<sup>3</sup> For this equation-of-state, the attraction energy of the atom to another atom at the minimum of the pair interaction potential is related to the temperature at the critical point, i.e.,  $\varepsilon = 81T_{cr}/32$  for Lennard-Jones potential<sup>4</sup>, where quantities are expressed in eV. The pressure of the evaporated gas is given by the Clausius–Clapeyron relation<sup>5</sup>, i.e.,

$$P_{vap} = 27P_{cr} \exp\left(-\frac{27}{8}\frac{T_{cr}}{T}\right).$$
<sup>(1)</sup>

Here,  $P_{cr}$  is the pressure at the critical point. For the Van der Waals equation-of-state,  $P_{cr}$  is given by <sup>6</sup>

$$P_{cr} = 3n_{cr}T_{cr} / 8.$$
 (2)

The density of a liquid corresponds to the point where the pressure is nearly zero (equilibrium), which gives  $n \equiv n_{fl} \approx 3n_{cr}$   $(n_{fl} \approx 1/b = 3n_{cr})$ .

The power of the long pre-pulse (I) is balanced by heat removed through the evaporation process according to the relation

$$I = 2\varepsilon \Gamma_{vap} = 2\varepsilon \frac{P_{vap}}{\sqrt{2\pi mT}}.$$
(3)

Equation (3) determines the temperature established in the liquefied film, i.e.,

$$I = 2\varepsilon\Gamma_{vap} = \frac{3}{2} \left(\frac{27}{8}\right)^2 \frac{n_{ff}T_{cr}^2}{\sqrt{2\pi mT}} \exp\left(-\frac{27}{8}\frac{Tcr}{T}\right). \tag{4}$$

For example, for gold,  $n_{fl}m = 17 g / cm^3$ ,  $T_{cr} = 7500$ K, and  $m = 197 m_p$ , where *m* is the atom mass, and  $m_p$  is the proton mass. And in order to sustain a sample at  $0.4 T_{cr} = 3000$ K requires about 500kW/cm<sup>2</sup> – typical energy density in focused NDCX-1 beam [1]. The minimum heating time required to heat a solid film to reach equilibrium between heating and cooling by gas evaporation is determined by the heat capacity of the metal, which is  $C_p = 3$ k. This gives  $\Delta tI > n\Delta x C_p T$ , where  $\Delta x$  is the width of the film, and  $\Delta t$  is the minimum time it takes to heat the sample. For a 1 µm film, this gives a minimum time of 1 µs for heating up to 3000K.

The difference in temperatures on both sides of the foil can be estimated by making use of the heat conduction equation,

$$\frac{d^2}{dx^2}\chi T = \frac{dI}{dx},$$

where dI/dx is the energy deposited into the volume by the beam stopping power, and  $\chi$  is the heat conductivity of gold 310 W/m·K [7]. The maximum difference in temperatures between both sides corresponds to the limit when the beam penetration depth is small compared with the film thickness. For 300keV potassium beam the range of beam ions in gold is 90nm<sup>8</sup>.

For NDCX-I parameters, the temperature difference is small due to the fact that the heat conduction is high. The deposited beam power is transported to both sides of the foil quickly by heat conduction, and is balanced by evaporation. Because the temperature in the foil is nearly uniform, and the vapor flux is a function of temperature given by Eq. (1), approximately one-half of the power is conducted to one side of the foil and one-half to the other. Therefore, at the edges  $x = \pm \Delta x/2$ , we have  $\frac{d}{dx} \chi T |_{x=\pm\Delta x/2} = \frac{I}{2}$ . If the penetration depth is small compared with the film thickness, then  $\chi \Delta T = \frac{1}{2}I\Delta x$ , or equivalently,

$$\chi \Delta T \mid_{x=\pm\Delta x/2} = \varepsilon \frac{27P_{cr} \exp\left(-\frac{27}{8}\frac{Tcr}{T}\right)}{\sqrt{2\pi mT}} \Delta x \,. \tag{5}$$

For a 1  $\mu$ m gold film heated by intense beam with power density I=500kW/cm<sup>2</sup>, the calculation then gives the estimate

$$\frac{\Delta T_{\rm max}}{T} \sim 2.8 \times 10^{-3} \, .$$

Substituting Eq. (5) into Eq. (1) yields that the pressure difference between the two sides is given by

$$\frac{\Delta P_{vap}}{P_{vap}} = \frac{27}{8} \frac{T_{cr}}{T} \frac{\Delta T}{T},$$

and is of order  $2.4 \times 10^{-2}$ .

The acceleration of the film as a whole is given by

$$g = \frac{\Delta P_{vap}}{\rho \Delta x}.$$
 (6)

Substituting the pressure and film thickness, 1 $\mu$ m, gives g=5.7 10<sup>6</sup> m/s<sup>2</sup> or 5.7  $\mu$ m/( $\mu$ s)<sup>2</sup>. Such a value of acceleration is fast enough to significantly distort the film on a  $\mu$ s time scale. Calculation of the maximum growth rate of the Rayleigh-Taylor instability for the thin film accelerated towards one side gives

$$\gamma \approx \sqrt{\frac{g\Delta x}{2a_c^2}},\tag{7}$$

where  $a_c = \sqrt{\frac{\alpha}{\rho g}}$  is the capillary length, when  $\Delta x < a_c$ . In the opposite limit,  $\Delta x > a_c$ , it follows<sup>9</sup> that

$$\gamma = \sqrt{\frac{2g}{3^{3/2}a_c}} \; .$$

Here,  $\alpha = CP_{cr}^{2/3}T_{cr}^{1/3}(1-T/T_{cr})$  is the surface tension given by the Guggenheim-Katayama relation [10]. Typically, the Eötvös law can be also used to obtain the value of the surface tension  $\frac{d}{dT} \left[ \alpha \left( \rho/M \right)^{-2/3} \right] = -2.12$ , where  $\alpha$  is given in milli-N/m,  $\rho$  is in g/cm<sup>3</sup>, T is the temperature in K [11] and  $\alpha (T_{cr}) = 0$ . However, application of Eötvös law to gold gives  $\alpha = 2.5$  N/m at 1338K, instead of the tabulated value of  $\alpha = 1.145$  N/m [11]. Therefore, the coefficient -2.12 has been reduced to -1. Then, the Eötvös law gives  $\alpha = 0.87$  N/m at T=3000K and  $a_c = 2.6 \mu$ m. Finally, Eq. (7) gives for the estimate of the growth rate of the Rayleigh-Taylor instability

$$\gamma \approx 0.6 \mu s^{-1} \tag{8}$$

for a  $\Delta x = 1 \ \mu m$  film and  $a_c \sim 2.6 \ \mu m$ . Therefore, target can disable into droplets on time scale of few microseconds corresponding to development of the nonlinear stage of the Rayleigh-Taylor instability.

In summary, it is proposed that a likely scenario for droplet formation in NDCX-I for targets with a thickness larger than the range of ions in the film (~100nm for NDCX-I parameters) is a result of the Rayleigh-Taylor instability.

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