Centroid and Envelope Dynamics of High-Intensity Charged-Particle Beams in an External Focusing Lattice and Oscillating Wobbler

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The centroid and envelope dynamics of a high-intensity charged-particle beam are investigated as a beam smoothing technique to achieve uniform illumination over a suitably chosen region of the target for applications to ion-beam-driven high energy density physics and heavy ion fusion. The motion of the beam centroid projected onto the target follows a smooth pattern to achieve the desired illumination, for improved stability properties during the beam-target interaction. The centroid dynamics is controlled by an oscillating "wobbler," a set of electrically biased plates driven by rf voltage.

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Beam dynamics is often studied in terms of envelope and centroid motions [1–5]. For example, unstable breathing modes can be described by envelope instabilities [1,6], and the two-stream electron cloud instability [7,8] and beam-beam interactions [9] are effectively modeled by following the centroid dynamics. Envelope dynamics is also employed to design beam focusing systems [10,11], while the purpose of studying centroid dynamics in most cases is to suppress instability or minimize the oscillation of the beam centroid around the design orbit [12]. As a general remark, the dynamics of the beam centroid has not been extensively explored for practical applications.

Recently, the dynamics of the beam centroid has been investigated as a possible beam smoothing technique [13–16] to achieve a uniform illumination over a suitably chosen region of the target for applications to ion-beam-driven high energy density physics and heavy ion fusion. The basic idea is to induce an oscillatory motion of the centroid for each transverse slice of the beam such that the centroids of different slices strike different locations on the target. The motion of the centroid projected onto the target is designed to follow a smooth pattern in order to achieve the desired uniform illumination over a suitably chosen region, e.g., an annular region, for significantly improved stability properties during the target implosion phase [14,17]. The centroid dynamics is actively controlled by the deflection force imposed by a set of biased electrical plates, which are called "wobblers," because of the wobbling motion that they induce in the beam centroid motion. The bias voltage on the wobbler plates oscillates with time in order to deliver different beam slices to different locations on the target (see Fig. 1). In laser-driven inertial confinement fusion research, uniformity of laser illumination is also critically important, and sophisticated smoothing systems using distributed phase-plate technology have been developed [18]. The wobbler system for high-intensity beams described here is analogous to these smoothing systems for laser beams.

From the point of view of the beam dynamics, the motions of the centroid and envelope represent different degrees of freedom. If the self-generated space-charge force is not strong, then the centroid dynamics and the envelope dynamics are decoupled. In this case, the centroid dynamics is described by the dynamical equations for a charged particle moving in the external focusing lattice and wobbler fields. For heavy ion fusion and high energy density physics applications, the beam intensity is high, and the effects of the self-generated space-charge force must be included. It is therefore necessary to determine the governing equations for the centroid dynamics for high-intensity beams, and ask whether the centroid dynamics is coupled to the beam envelope dynamics relative to the centroid motion. The purpose of this Letter is to address these important questions regarding the centroid and envelope dynamics of high-intensity beams in an external focusing lattice and wobbler fields.

Our theoretical study is based on the nonlinear Vlasov-Maxwell equations for high-intensity beams [19]. Two different approaches are adopted. The first approach is to

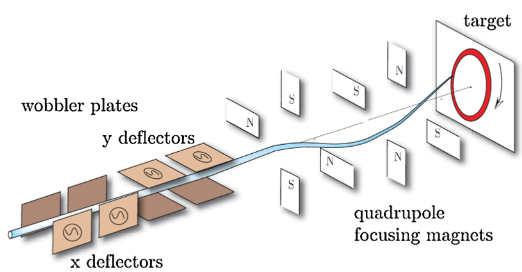

FIG. 1 (color). Quadrupole focusing lattice and wobbler system. The motion of the centroid projected onto the target follows a smooth pattern in order to achieve uniform illumination over a suitably chosen region of the target.

derive a set of rate equations for the centroid, and the root-mean-square (rms) envelope and emittance, by taking appropriate moments of the Vlasov-Maxwell equations. The second approach is to construct a generalized self-consistent Kapchinskij-Vladimirskij (KV) solution of the Vlasov-Maxwell equations including the envelope dynamics as well as the centroid dynamics. Using these two models, we will show that the wobbler deflection force acts only on the centroid motion, and that the envelope dynamics is independent of the wobbler fields. Furthermore, if the conducting wall is far away from the beam, then the envelope dynamics and the centroid dynamics are completely decoupled even when the space-charge force is strong. In a broader sense, this systematic study and conclusion are of general importance for high-intensity beam dynamics, beyond the wobbler technique discussed here. The decoupling of the envelope and centroid motions in the presence of space charge has been assumed for approximately 50 years in calculating the modification to resonances by space charge in circular accelerators [1]. However, the envelope dynamics and the centroid dynamics will be coupled through the self-field potential if the conducting wall is nearby.

In a quadrupole focusing lattice with wobbler fields, the transverse dynamics of a particle in the laboratory-frame coordinates (x, y) is determined from [19]

$$\begin{aligned} x'' &= -\kappa_x(s)x - \frac{\partial\psi}{\partial x} + F_x(s), \\ y'' &= -\kappa_y(s)y - \frac{\partial\psi}{\partial y} + F_y(s), \end{aligned} \tag{1}$$

where $\psi = e\phi/\gamma^3 m\beta^2 c^2$ is the normalized self-field potential, $\kappa_x(s) = \kappa_q(s)$ and $\kappa_y(s) = -\kappa_q(s)$ are the focusing strengths of the quadrupole lattice, and $F_x(s)$ and $F_y(s)$ are the transverse deflection forces due to the wobblers. The nonlinear Vlasov-Maxwell equations for the beam distribution function $f(s, x, y, v_x, v_y)$ and self-field potential ψ are [19]

$$\frac{\partial f}{\partial s} + v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} - \left(\kappa_x x + \frac{\partial\psi}{\partial x} - F_x\right)\frac{\partial f}{\partial v_x} - \left(\kappa_y y + \frac{\partial\psi}{\partial y} - F_y\right)\frac{\partial f}{\partial v_y} = 0, \tag{2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = -\frac{2\pi K_b}{N_b}\int f dv_x dv_y, \tag{3}$$

where $N_b = \int f dv_x dv_y dx dy$ is the line density of the beam particles, and $K_b = 2N_b e^2/\gamma^3 m\beta^2 c^2$ is the self-field perveance. To derive the rms envelope equations and the centroid equations [1–5], we start from the rate equation for a phase-space moment of the Vlasov equation. Let $\chi(x, y, v_x, v_y, s)$ be any phase-space function, then the χ moment of f is defined as $\langle\chi\rangle \equiv (\int \chi f dx dy dv_x dv_y)/N_b$. From Eq. (2), we obtain [19] the rate equation for $\langle\chi\rangle$

$$\frac{d\langle\chi\rangle}{ds} = \left\langle \frac{\partial\chi}{\partial s} + v_x\frac{\partial\chi}{\partial x} + v_y\frac{\partial\chi}{\partial y} - \left(\kappa_x x + \frac{\partial\psi}{\partial x} - F_x\right)\frac{\partial\chi}{\partial v_x} - \left(\kappa_y y + \frac{\partial\psi}{\partial y} - F_y\right)\frac{\partial\chi}{\partial v_y}\right\rangle. \tag{4}$$

The transverse displacement of the beam centroid is defined by the first moment of f, i.e., $\mu \equiv \langle x\rangle$, $\nu \equiv \langle y\rangle$. Applying Eq. (4), we obtain $\mu' = \langle x\rangle' = \langle v_x\rangle$ and $\nu' = \langle y\rangle' = \langle v_y\rangle$. Letting $\chi = v_x$ and $\chi = v_y$ in Eq. (4), we obtain the dynamical equations for the centroid motion

$$\mu'' = \langle v_x\rangle' = -\kappa_x\mu + F_x - \left\langle\frac{\partial\psi}{\partial x}\right\rangle, \tag{5}$$

$$\nu'' = \langle v_y\rangle' = -\kappa_y\nu + F_y - \left\langle\frac{\partial\psi}{\partial y}\right\rangle. \tag{6}$$

The rms envelope dimensions (a, b) and transverse emittances $(\varepsilon_x, \varepsilon_y)$ are defined relative to the centroid by

$$a \equiv \sqrt{\langle x-\mu\rangle^2}, \qquad b \equiv \sqrt{\langle y-\nu\rangle^2},$$
$$\varepsilon_x \equiv 2\sqrt{a^2\langle(v_x-\mu')^2\rangle - \langle(v_x-\mu')(x-\mu)\rangle^2},$$
$$\varepsilon_y \equiv 2\sqrt{b^2\langle(v_y-\nu')^2\rangle - \langle(v_y-\nu')(y-\nu)\rangle^2}.$$

From the rate equations for $\chi = (x-\mu)^2$, $\chi = (v_x-\mu')(x-\mu)$, and $\chi = (v_x-\mu')^2$, we obtain the following dynamical equations for a and ε_x:

$$a'' + \kappa_x a = \frac{\varepsilon_x^2}{4a^3} - \frac{1}{a}\left\langle\frac{\partial\psi}{\partial x}(x-\mu)\right\rangle, \tag{7}$$

$$\frac{d}{ds}\left(\frac{\varepsilon_x^2}{8}\right) = \frac{d}{ds}\left(\frac{a^2}{2}\right)\left\langle\frac{\partial\psi}{\partial x}(x-\mu)\right\rangle - a^2\left\langle\frac{\partial\psi}{\partial x}(v_x-\mu')\right\rangle. \tag{8}$$

Similarly, the dynamical equations for b and ε_y are given by

$$b'' + \kappa_x b = \frac{\varepsilon_y^2}{4b^3} - \frac{1}{b}\left\langle\frac{\partial\psi}{\partial y}(y-\nu)\right\rangle, \tag{9}$$

$$\frac{d}{ds}\left(\frac{\varepsilon_y^2}{8}\right) = \frac{d}{ds}\left(\frac{b^2}{2}\right)\left\langle\frac{\partial\psi}{\partial y}(y-\nu)\right\rangle - b^2\left\langle\frac{\partial\psi}{\partial y}(v_y-\nu')\right\rangle. \tag{10}$$

The evolution of the centroid dynamics, the rms envelope dimensions, and the transverse emittances are determined from Eqs. (5)–(10). From Eqs. (7)–(10), it is clear that the deflection force imposed by the wobbler fields does not directly affect the envelope dynamics and emittances. Furthermore, if the conducting wall is far away from the beam, or if image-charge effects are negligible, then it can be shown that the self-field terms in Eqs. (5) and (6) vanish, and the self-field potential ψ in Eqs. (7)–(10) is a function of $(x-\mu, y-\nu)$ only, which indicates that the self-field force does not affect the centroid dynamics, and the evolution of the envelope dimensions and emittances is independent of the centroid motion. In this case, there is a complete decoupling between the centroid dynamics and the dynamics of the envelope dimensions and emittances. The centroid motion is affected only by the focusing lattice and wobbler fields, and the envelope dimensions and emittances evolve as if there were no wobbler fields and no centroid dynamics. This is an ideal situation for the envisioned applications of the beam wobbling technique, because the wobbler system can be designed to generate the desired centroid motion on the target without considering the potentially deleterious effects on the envelope and emittance.

However, if the conducting wall is not far removed from the beam, then the dynamics of the centroid, the envelope dimensions and emittances are coupled through the self-field force. To determine the self-field force on the beam centroid, we note that in Eqs. (5) and (6),

$$-\left(\left\langle\frac{\partial\psi}{\partial x}\right\rangle,\left\langle\frac{\partial\psi}{\partial y}\right\rangle\right)=\frac{N_b}{2\pi K_b}\int_{\text{wall}}(\nabla\psi\nabla\psi-|\nabla\psi|^2\mathbf{I})\cdot d\mathbf{s}, \tag{11}$$

where $\mathbf{I}$ is the unit tensor, and the surface integral is over the conducting wall. The self-field force on the centroid motion is determined by the self-field on the conducting wall. As the conducting wall approaches infinity, the self-field force vanishes. For the self-field force terms in Eqs. (7)–(10), ψ will depend on $(x-\mu,\ y-\nu)$ as well as $(\mu,\ \nu)$ if the conducting wall is nearby, and the centroid dynamics will affect the dynamics of the envelope dimensions and emittances. This effect should be minimized in the design of wobbler systems. The image-charge effect has been previously analyzed in Ref. [3], and the equations employed in CIRCE [4] show that the equations become decoupled when the pipe radius is set to infinity.

Assuming that the conducting wall is far away from the beam, then in the coordinate system centered at the centroid, $X=x-\mu$, $Y=y-\nu$, we find that the envelope equations and the emittance equations are exactly the same as those in the laboratory coordinate system in the absence of centroid dynamics. Therefore, known results for the latter case can be applied directly to Eqs. (7)–(10). A particularly important result is for the case where the beams have fixed-shape density profiles $n(X,Y,s)=N_bS(X^2/2a^2+Y^2/2b^2)/2\pi ab$, where S is the density shape function. It can then be shown [19] that Eqs. (7) and (9) reduce exactly to

$$a''+\kappa_x a=\frac{\varepsilon_x^2}{4a^3}-\frac{K_b}{2(a+b)},\qquad b''+\kappa_x a=\frac{\varepsilon_y^2}{4b^3}-\frac{K_b}{2(a+b)}. \tag{12}$$

The similarity between the cases with and without centroid dynamics suggests that a self-consistent KV solution to the nonlinear Vlasov-Maxwell equations may exist for high-intensity beams including the centroid dynamics in an external focusing lattice and wobbler fields. We now show that this is indeed true. To construct the self-consistent solution of the nonlinear Vlasov-Maxwell equations, we adopt a model in which the self-field force is assumed to be linear in the centroid frame, i.e., $\psi=-K_b(X^2/\bar{a}^2+Y^2/\bar{b}^2)/(\bar{a}+\bar{b})$. Here, $\bar{a}$ and $\bar{b}$ are the envelope dimensions in the centroid frame that will be determined from Eq. (15). It will be clear later that $\bar{a}$ and $\bar{b}$ are related to the rms envelope dimensions a and b through $\bar{a}=\sqrt{2}a$ and $\bar{b}=\sqrt{2}b$. Let the centroid motion satisfy

$$\mu''+\kappa_x\mu-F_x=0,\qquad \nu''+\kappa_y\nu-F_y=0, \tag{13}$$

then it follows that X and Y evolve according to

$$\begin{aligned}X''+\left[\kappa_x-\frac{2K_b}{\bar{a}(\bar{a}+\bar{b})}\right]X=0,\\ Y''+\left[\kappa_y-\frac{2K_b}{\bar{b}(\bar{a}+\bar{b})}\right]Y=0.\end{aligned} \tag{14}$$

Since Eq. (14) is linear in X and Y, it admits the Courant-Snyder invariants for the X and Y motions, i.e.,

$$A_X=\frac{\varepsilon_x^2X^2}{\bar{a}^2}+\varepsilon_x^2(\bar{a}X'-X\bar{a}')^2,$$
$$A_Y=\frac{\varepsilon_y^2Y^2}{\bar{b}^2}+\varepsilon_y^2(\bar{b}Y'-Y\bar{b}')^2,$$

where ε_x and ε_y are constants corresponding to the conserved transverse emittances, and $\bar{a}$ and $\bar{b}$ are determined from the envelope equations

$$\bar{a}''+\kappa_x\bar{a}-\frac{2K_b}{(\bar{a}+\bar{b})}=\frac{\varepsilon_x^2}{\bar{a}^3},\qquad \bar{b}''+\kappa_y\bar{b}-\frac{2K_b}{(\bar{a}+\bar{b})}=\frac{\varepsilon_y^2}{\bar{b}^3}. \tag{15}$$

Therefore, it can be shown that the choice of distribution function [19]

$$f=\frac{N_b}{\pi^2\varepsilon_x\varepsilon_y}\delta\left(\frac{A_X}{\varepsilon_x}+\frac{A_Y}{\varepsilon_y}-1\right) \tag{16}$$

is an exact solution of the Vlasov equation (2). To verify that the distribution function f given by Eq. (16) generates the linear self-field force assumed, we calculate the density profile to be spatially uniform inside of the elliptical cross-section beam, i.e., $n(X,Y,s)=\int f dv_x dv_y=N_b/\pi\bar{a}\bar{b}$ when $X^2/a^2+Y^2/b^2\le 1$, and $n(X,Y,s)=0$ when $X^2/a^2+Y^2/b^2>1$, which indeed generates the initially assumed self-field potential upon solving Poisson's equation (3). Note that the KV distribution does not follow directly from the moment equations for the envelope and centroid because the moment equations do not specify the distribution function, and finding a distribution function that solves the Vlasov-Maxwell equations is generally nontrivial. Since the KV distribution is not particularly physical, it serves primarily as a simplified theoretical model for the wobbler dynamics. Combined with the moment equations, it gives a leading-order description of the wobbler dynamics. A KV solution for axisymmetric (solenoidal) focusing without wobbler fields is given in Ref. [20]. The KV solution to the nonlinear Vlasov-

Maxwell equations considered in this Letter corresponds (exactly) to the case where the beam has a flattop density profile. For more general choices of distribution function corresponding to beams with density profiles that are not flattop, we expect that the rms envelope equations and the associated centroid equations derived by taking appropriate moments of the Vlasov-Maxwell equations remain a good approximation, particularly if the change in beam