

# STUDIES OF THE BEHAVIOR OF MODIFIED-DISTRIBUTION-FUNCTION BEAMS ON THE PRINCETON PAUL TRAP SIMULATOR EXPERIMENT (PTSX)\*

E. P. Gilson<sup>†</sup>, R. C. Davidson, M. Dorf, P. C. Efthimion, R. Majeski, E. A. Startsev  
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey, 08543, USA

## Abstract

The Paul Trap Simulator Experiment (PTSX) is a compact laboratory Paul trap that simulates a long, thin charged-particle bunch coasting through a kilometers-long magnetic alternating-gradient (AG) transport system by putting the physicist in the frame-of-reference of the beam. Results are presented from experiments in which the axial distribution function is modified by lowering the axial confinement barrier to allow particles in the tail of the axial distribution function to escape. Measurements of the axial energy distribution and the transverse density profile are taken to determine the effects of the modified distribution function on the charge bunch. It is observed that the reduced axial-trapping potential leads to an increase of the transverse effective temperature.

## INTRODUCTION

The Paul Trap Simulator Experiment (PTSX) is a compact and flexible laboratory facility that simulates the propagation of intense charged particle beams over thousands of lattice periods through magnetic alternating-gradient (AG) quadrupole transport systems [1, 2, 3, 4, 5]. The simulation makes use of the isomorphism between the transverse equations of motion for particles in the two systems [1, 6, 7]. In the work described in this paper, the PTSX facility has been used to modify the axial distribution function of the trapped particles to investigate the coupling between the axial and transverse degrees of freedom.

The data suggest that the effective axial temperature of the trapped ions  $kT_{\parallel}$  is comparable to the initial effective transverse temperature  $kT_{\perp}$  and is about 0.1 eV. Further, the data demonstrate that  $kT_{\perp}$  increases when particles with large axial energy are allowed to escape because those particles that remain trapped experience transverse electric fields deep within the axial-trapping-electrode region that broaden the plasma's transverse density distribution.

The PTSX device is a linear Paul trap confining a one-component plasma of particles with charge  $e_b$ , where the  $e_b \vec{E}_{\perp}^{ext}$  force that the PTSX electrodes exert on the trapped particles is analogous to the  $e_b v_z \times \vec{B}_{\perp}^{ext}$  force that the AG system exerts on the beam particles in the beam frame provided that long, coasting beams that are thin relative to the AG system magnet spacing are considered. The amplitude and frequency of the voltage waveform applied to

the PTSX electrodes correspond to the quadrupole magnet strength and lattice spacing in the AG system. Further, the self-field forces in both systems can be described by scalar potentials that obey Poisson's equation.

## PTSX

The PTSX device is a linear Paul trap constructed from a 2.8 m-long,  $r_w = 10$  cm-radius cylinder (Fig. 1). The cylinder is divided into two 40 cm-long end cylinders and a  $2L = 2$  m-long central cylinder. All cylinders are azimuthally divided into four  $90^\circ$  segments so that when an oscillating voltage  $V_0(t)$  is applied with alternating polarity on adjacent segments, the resulting oscillating transverse quadrupole electric field exerts a ponderomotive force that confines the plasma radially. To trap the plasma axially, the two end cylinders are biased to a constant voltage  $\hat{V}$ . Voltage waveforms with amplitudes up to 400 V and frequencies up to 100 kHz can be used. The trapping voltage is nominally  $\hat{V} = 36$  V. The vacuum pressure of  $5 \times 10^{-9}$  Torr prevents neutral collisions from playing an important role in determining the plasma behavior.

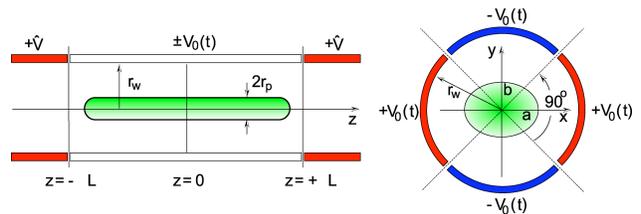


Figure 1: The PTSX device consists of three cylindrical electrodes with radius  $r_w = 0.1$  m, each divided into four  $90^\circ$  sectors. An oscillating voltage  $\pm V_0(t)$  confines the charge bunch in the transverse plane to a radius  $r_p$ . Static voltages  $+\hat{V}$  on the end electrodes confine the ions axially within a length  $2L = 2$  m.

The 1.5 cm-diameter aluminosilicate cesium emitter injects 3.1 V singly-charged cesium ions into the machine when a 1 V bias is applied between the emitter and an acceleration grid. The ion source is situated inside of one of the 40 cm-long cylinders, and to inject a pure cesium ion plasma into the trap, the segments on this 40 cm-long cylinder are temporarily set to oscillate with the voltage  $\pm V_0(t)$ . The injection time is set to be 1.7 ms in order to allow cesium ions to only just fill the trap.

After trapping the plasma for up to 300 ms, the 40 cm-

\* This research was supported by the U.S. Department of Energy.

<sup>†</sup> egilson@pppl.gov

long cylinder on the opposite end of the PTSX device from the ion source is set to oscillate with voltage  $\pm V_0(t)$ , and the plasma streams out of the trap. Part of the exiting plasma is collected on a moveable 5 mm-diameter collector disk. The radial density profile is computed using the measured radial charge profile and knowledge of the area of the collector and the length of the plasma column [8]. Since the plasma ions can take several milliseconds to leave the trap, the measurements are necessarily averaged over hundreds of lattice periods.

Near the axis, the potential is quadrupolar and the average smooth-focusing frequency [9] of particles' transverse oscillations can be expressed for an applied voltage  $V(t) = V_0 \max \sin(2\pi ft)$  as [1, 9]

$$\omega_q = \frac{8e_b V_0 \max}{m_b r_w^2 \pi f} \xi, \quad (1)$$

where  $m_b = 133$  amu for  $\text{Cs}^+$  ions. The factor  $\xi$  depends on the shape of the voltage waveform and  $\xi = 1/2\sqrt{2\pi}$  for the sinusoidal waveform used herein. The smooth-focusing vacuum phase advance  $\sigma_v^{sf}$  is given by  $\sigma_v^{sf} = \omega_q/f$  [3, 9]. In order for the particles to be confined radially, the normalized beam intensity  $s \equiv \omega_p^2(0)/2\omega_q^2$  must be less than unity, where  $\omega_p(0) = [n_b(0)e_b^2/m_b\epsilon_0]^{1/2}$  is the on-axis plasma frequency. The limit of low  $s$  corresponds to the emittance-dominated regime, while  $s \rightarrow 1$  corresponds to the space-charge-dominated regime. For a flat-top density profile, the normalized beam intensity parameter  $s$  is related to the depressed tune  $\nu/\nu_o$  as  $\nu/\nu_o = \sqrt{1-s}$ .

When  $kT_\perp$  approaches 0 ( $s \rightarrow 1$ ), the plasma radial density profile is nearly uniform. In the case of low space-charge density ( $s \rightarrow 0$ ), the radial density profile is nearly Gaussian. In all cases, under quasi-steady-state conditions, a global radial force balance equation can be expressed as [9]

$$m_b \omega_q^2 R_b^2 = 2kT_\perp + \frac{N_b e_b^2}{4\pi\epsilon_0}. \quad (2)$$

In the analysis of PTSX results, the mean-squared radius  $R_b^2$  and the line density  $N_b$  are calculated as moments of the measured plasma density profiles;  $kT_\perp$  is inferred from Eq. (2).

## EVAPORATIVE COOLING

“Evaporative cooling” occurs when the axial trapping potential is lowered to allow particles with sufficiently large axial energy to escape. By counting the number of particles that escape as a function of the value of the reduced axial trapping potential,  $kT_\parallel$  can be inferred. The coupling between the transverse and longitudinal degrees of freedom is investigated by measuring the average transverse density profile after allowing particles with the most longitudinal energy to escape.

The applied axial-trapping voltage on the PTSX end electrodes is typically larger than 30 V. Ions that are injected into PTSX with an axially-directed kinetic energy

of 3.1 eV will be well trapped. The axial-trapping voltage on the upstream end electrodes can be reduced for a short time, however. During this time, the ion source is biased so that no ions can enter the trap. Ions within the trap that are sufficiently energetic can escape over the reduced barrier. After the axial-trapping voltage is restored to  $> 30$  V, the plasma is held, and then finally dumped onto the collector disk to measure the amount of charge that remains trapped. Although the end electrodes are 40 cm long and 10 cm in radius, the axial-trapping barrier is only 87% of the applied voltage because of the 0 V-biased ion source grid located 10 cm into the end electrodes (Fig. 2).

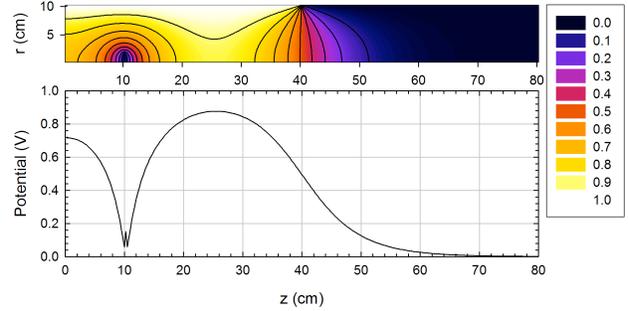


Figure 2: Poisson's equation is solved numerically for the 40-cm-long end electrode at unit voltage, the 0 V deceleration grid at  $z = 10$  cm, and the first 40 cm of 2-m-long main confinement electrodes at 0 V (top –  $r - z$  color contour plot). The presence of the deceleration grid causes the axial-trapping barrier height to be only 87% of the applied voltage (bottom – potential along  $r = 0$ ).

Figure 3 shows the on-axis measured charge on the collector as a function of the height of the reduced upstream axial-trapping barrier. The measured charge is normalized to unity at large axial-trapping barrier height where all of the particles remain trapped. The reduced barrier is applied for only one round-trip transit time of the ions (2 ms), and correspondingly, the data show that when the barrier height is reduced to zero, no charge remains trapped. It is assumed that the axial velocity distribution is a drifting Maxwellian distribution with a mean velocity corresponding to a kinetic energy of 3.1 eV, and a width given by  $kT_\parallel$ . Integration of this velocity distribution gives the number of particles that remain trapped, and this is plotted in Figure 3 for three temperatures:  $kT = 0.05$  eV, 0.10 eV, and 0.20 eV. The model with  $kT = 0.1$  eV is the most consistent with the data. Note that the on-axis charge is a reasonable proxy for the total number of trapped particles.

## TRANSVERSE HEATING

An axial-trapping barrier height of 3.6 V was then chosen so that approximately 30% of the particles, those with the most axial energy, are released. The coupling of transverse and longitudinal degrees of freedom is explored by allowing the plasma to relax after the evaporative cool-

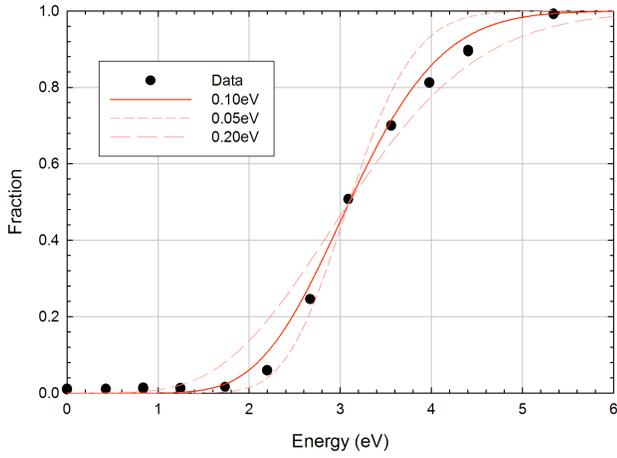


Figure 3: The measured on-axis charge that remains trapped (black dots) drops to zero as the axial-trapping barrier is reduced to zero. The rate of change in the on-axis charge is consistent with an axial temperature of 0.1 eV when the axial distribution is assumed to be a drifting Maxwellian with a mean velocity corresponding to 3.1 eV (solid red line).

ing, and then measuring the average transverse density profile. For a plasma that experiences no evaporative cooling,  $kT_{\perp} \sim 0.17$  eV (Fig. 4). If the evaporative cooling of the axial distribution occurs between  $t = 7$  ms and 10 ms,  $kT_{\perp} = 0.30$  eV, but remains relatively steady with time thereafter (Fig. 4). When the cooling is applied from  $t = 7$  ms until the end of the plasma confinement, it is observed that  $kT_{\perp}$  continues to grow with time, despite the fact that all particles with axial energies greater than 3.6 eV have left the system by  $t = 10$  ms (Fig. 4).

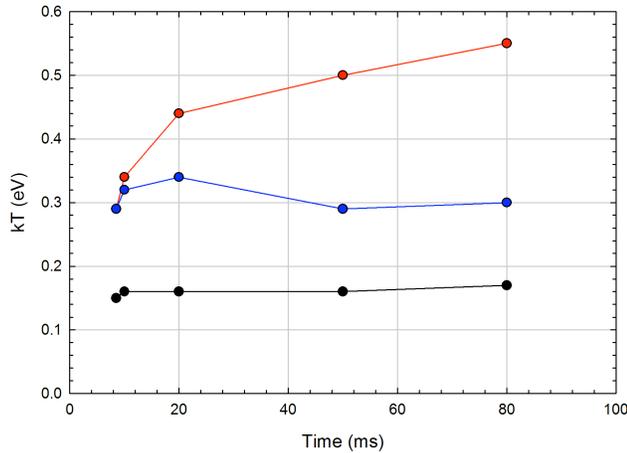


Figure 4: For a plasma that experiences no evaporative cooling,  $kT_{\perp} = 0.17$  eV (black), but  $kT_{\perp} = 0.30$  eV for a plasma where evaporative cooling occurs between  $t = 7$  ms and 10 ms (blue). If the cooling is applied permanently at  $t = 7$  ms, then  $kT_{\perp}$  continues to grow with time (red).

The data suggest that any cooling of  $kT_{\perp}$  by coupling

between the transverse distribution and the cooled longitudinal distribution may be masked by a larger effect that raises  $kT_{\perp}$  when the axial-trapping barrier is lowered. In fact, particles that penetrate deep into the end electrode region experience radial electric fields that are different than when the axial-trapping voltage is large, and particles do not penetrate deeply into the end electrode region (see Fig. 1). These electric fields lead to radial transport and an increase in  $kT_{\perp}$ . Experiments were performed in which the cooling is applied from  $t = 7$  ms until the end of the plasma confinement, but the axial-trapping barrier height is kept higher than 5.2 V where 99% of particles remain trapped. For barrier heights of 43.5 V and 8.7 V, the measured values of  $kT_{\perp}$  are 0.18 eV and 0.19 eV, respectively. However, for barrier heights of 7.0 eV and 5.2 eV, the measured values of  $kT_{\perp}$  are 0.24 eV and 0.39 eV, respectively, showing significant transverse heating even though more than 99% of particles remain trapped, and therefore there is no evaporative cooling.

## CONCLUSIONS

The axial temperature  $kT_{\parallel}$  of plasmas trapped in PTSX is approximately 0.1 eV and is similar to  $kT_{\perp}$ . Truncating the axial distribution by lowering the axial-trapping barrier height to release energetic ions does not ultimately lead to a cooler  $kT_{\perp}$ . Instead,  $kT_{\perp}$  increases due to the electric fields that trapped ions experience when penetrating far into the trapping electrode region.

## ACKNOWLEDGEMENTS

This research was supported by the U.S. Department of Energy.

## REFERENCES

- [1] R. C. Davidson, H. Qin, and G. Shvets, *Phys. Plasmas* **7**, 1020 (2000).
- [2] M. Reiser, *Theory and Design of Charged Particle Beams*, Wiley, New York, 1994.
- [3] E. P. Gilson, R. C. Davidson, P. C. Efthimion and R. Majeski, *Phys. Rev. Lett.* **92**, 155002 (2004).
- [4] E. P. Gilson, M. Chung, R. C. Davidson, M. Dorf, P. C. Efthimion, R. Majeski, *Phys. Plasmas* **13**, 056705 (2006).
- [5] M. Chung, E. P. Gilson, M. Dorf, R. C. Davidson, P. C. Efthimion, R. Majeski, *Phys. Rev. ST Accel. Beams* **10**, 064202 (2007).
- [6] H. Okamoto, H. Tanaka, *Nucl. Instrum. and Methods A* **437**, 178 (1999).
- [7] N. Kjærgaard, K. Mølhave, M. Drewsen, *Phys. Rev. E* **66**, 015401 (2002).
- [8] E. P. Gilson, R. C. Davidson, P. C. Efthimion, R. Majeski, E. A. Startsev, *American Institute of Physics Conference Proceedings* **692**, 211 (2003).
- [9] R. C. Davidson, H. Qin, *Physics of Intense Charged Particle Beams in High Intensity Accelerators*, World Scientific, Singapore, 2001.