Study of Drift Compression for Heavy Ion Beams

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Motivation

- In the currently envisioned configurations for heavy ion fusion (HIF), it is necessary to longitudinally compress the beam bunches by a large factor after the acceleration phase and before the beam particles are focused onto the fusion target.
  - In order to obtain enough fusion energy gain, the peak current for each beam is required to be order 10^3A, and the bunch length to be as short as 0.5m.
  - To deliver the beam particles at the required energy, it is both expensive and technically difficult to accelerate short bunches at high current.
- The objective of drift compression is to compress a long beam bunch by imposing a negative longitudinal velocity tilt over the length of the beam in the beam frame.

Drift Compression for Heavy Ion Fusion Driver

- Assume a Cs^+ beam for HIF driver with $A = 133$, $q = 1$, $(\gamma - 1)mc^2 = 2.5GeV$, $z_{bf} = 0.60$m, and $< I >= 2500$A.

- The goal of drift compression is:
  - Length $z_b \longrightarrow \times \dfrac{1}{16}$. Perveance $K \longrightarrow \times 16$.

- Allowable changes of other system parameters:
  - Velocity tilt $|v_{zb}| \longrightarrow \leq 0.01$.
  - Beam radius $a \longrightarrow \times 2$.
  - Half lattice period $L \longrightarrow \times \dfrac{1}{2}$.
  - Filling factor $\eta \longrightarrow \times 4$. $\eta B' \longrightarrow \times 4$.

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Important Questions

- Longitudinal Dynamics:
  - What is the dynamics of $z_b(s)$?
  - How long is the beam line? ($s_f = 421.5$m)
  - How large initial velocity tilt can we afford? ($v_{zb0} = -0.0227$)
  - Space charge effect? (strong and helpful)
  - Stability? (stable without longitudinal focusing by envelope equation)
- Transverse Dynamics:
  - Non-periodic lattice design, $L(s)$, $B'(s)$, $\eta(s)$, $\kappa(s)$, $K(s)$.
  - Non-periodic envelope, matched solutions? adiabatically-matched solutions?

Outline

- Longitudinal Dynamics:
  - 1D fluid model.
  - Self-similar solutions.
  - Longitudinal envelope equation.
  - Drift compression design.
  - Pulse shaping
- Transverse Dynamics:
  - Non-periodic lattice design.
  - Adiabatically-matched solutions of the transverse envelope equations in a non-periodic lattice.

Longitudinal Dynamics – 1D fluid model

- One dimensional fluid model in the beam frame for
  - $\lambda(t,z)$: line density,
  - $v_z(t,z)$: longitudinal velocity,
  - $p_z(t,z)$: longitudinal pressure.
- g-factor model for electric field.

$$eE_z = -\frac{ge^2}{\gamma^2}\frac{\partial\lambda}{\partial z}, \tag{1}$$

$$g = 2\ln\frac{r_w}{r_b}. \tag{2}$$

- Take g and r_b as constants for present purpose.
- External focusing: $-\kappa_z z$.

Longitudinal Dynamics – 1D fluid model

⇨ In the beam frame:

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \quad \text{(continuity)}, \tag{3}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + \frac{e^2 g}{m\gamma^5}\frac{\partial \lambda}{\partial z} + \frac{\kappa_z z}{m\gamma^3} + \frac{r_b^2}{m\gamma^3 \lambda}\frac{\partial p_z}{\partial z} = 0 \text{ (momentum)}, \tag{4}$$

$$\frac{\partial p_z}{\partial t} + v_z \frac{\partial p_z}{\partial z} + 3p_z \frac{\partial v_z}{\partial z} = 0 \text{ (energy)}. \tag{5}$$

⇨ Eqs. (3) , (4), and (5) form a nonlinear hyperbolic PDE system. If neglecting κ_z and p_z, Eqs. (3) and (4) have the same form as the shallow-water equations.

⇨ Eq. (5) is equivalent to

$$\frac{d}{dt}\left(\frac{p_z}{\lambda^3}\right) = 0. \tag{6}$$

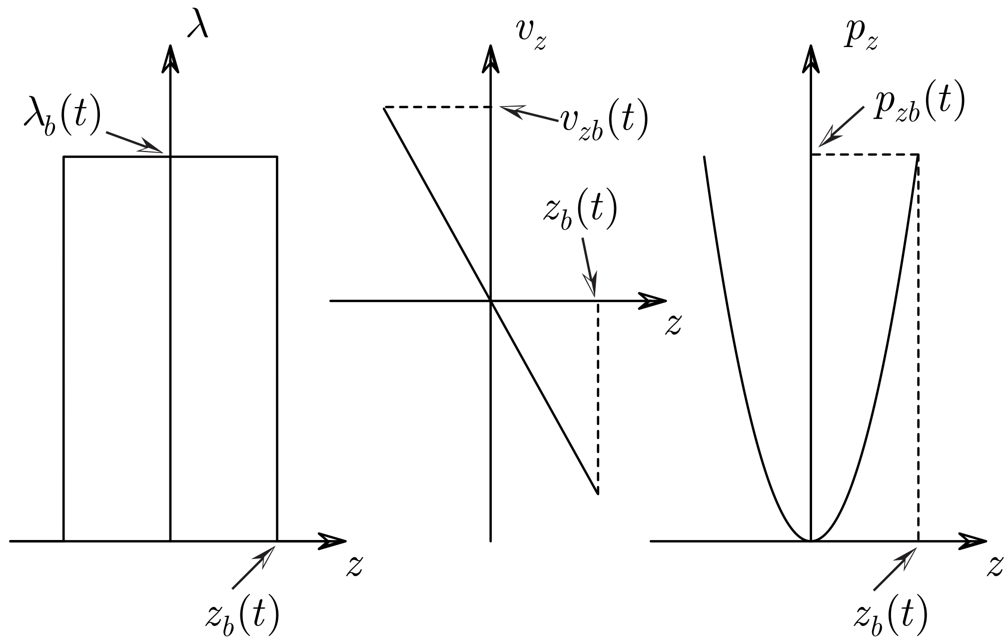
⇨ Self-similar drift compression schemes preserve the geometric shape of the bunched beam, as well as the density profile, the pressure profile, and the velocity distribution. The nonlinear PDE system, Eqs. (3), (4), and (5), admits at least two self-similar drift compression solutions.

Linear self-similar drift compression solution

⇨

$$\lambda(t,z) = \lambda_b(t),\ v_z(t,z) = -v_{zb}(t)\frac{z}{z_b(t)}, \tag{7}$$

$$p_z(t,z) = p_{zb}(t)\frac{z^2}{z_b^2(t)},\ \frac{dz_b(t)}{dt} = -v_{zb}(t). \tag{8}$$



Linear self-similar drift compression solution:

⇨ From the continuity equation (3), we obtain

$$\frac{1}{\lambda_b}\frac{d\lambda_b}{dt} + \frac{1}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b\lambda_b = const. = N_b/2\,, \tag{9}$$

⇨ From the energy equation (5) , we obtain

$$z_b^3 p_{zb} = const. = W\,. \tag{10}$$

⇨ Similarly, for the momentum equation (4), the z-dependence drops out as well, giving

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2c^2} z_b + \frac{\varepsilon_l^2}{z_b^3} = 0\,, \tag{11}$$

where $\varepsilon_l \equiv \left(2r_b^2 W/m\gamma^3\beta^2c^2N_b\right)^{1/2}$.

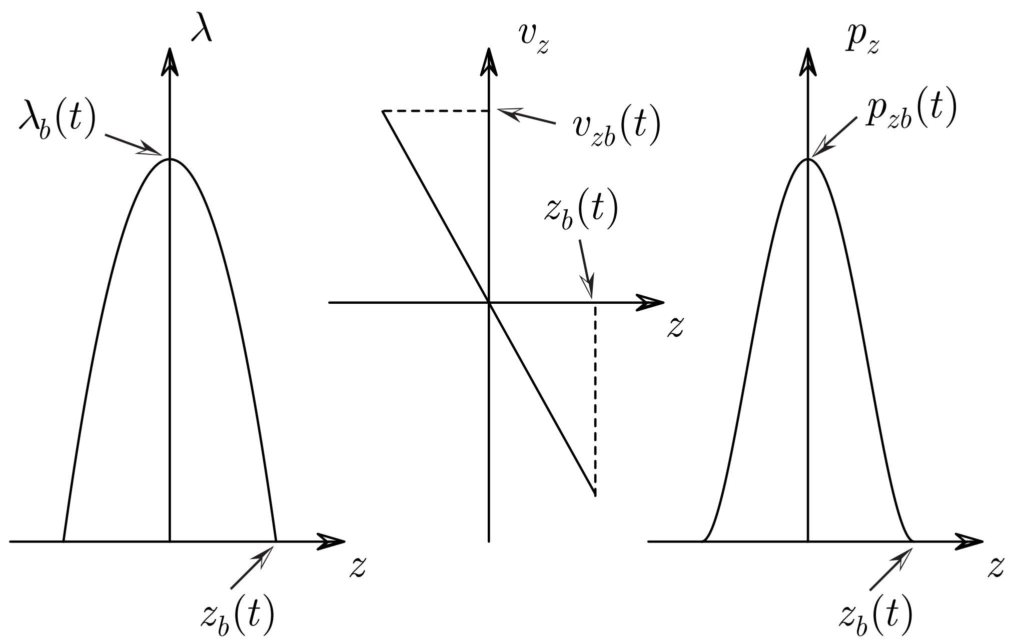
⇨ Equations (9), (10) and (11) describe the dynamics of the time-dependent variables $\lambda_b(t)$, $z_b(t)$, and $p_{zb}(t)$.

⇨ Equation (11) predicts a dramatic compression scenario where the beam longitudinally "implodes" because of the singularity of the (focusing) pressure term in Eq. (11) as $z_b \to 0$.

Parabolic Self-Similar Solution

$$\lambda(t,z) = \lambda_b(t)\left(1 - \frac{z^2}{z_b^2(t)}\right), \; v_z(t,z) = -v_{zb}(t)\frac{z}{z_b(t)}, \tag{12}$$

$$p_z(t,z) = p_{zb}(t)\left(1 - \frac{z^2}{z_b^2(t)}\right)^2, \; \frac{dz_b(t)}{dt} = -v_{zb}(t). \tag{13}$$



Parabolic Self-Similar Solution

⇨ Substituting Eqs. (12) and (13) into Eqs. (3) and (5), we find that the z-dependence drops out, and

$$\frac{d\lambda_b}{dt} - \frac{v_{zb}}{z_b}\lambda_b = 0, \tag{14}$$

$$\frac{dp_{zb}}{dt} - 3\frac{v_{zb}}{z_b}p_{zb} = 0. \tag{15}$$

⇨ Remarkably, but not surprisingly, for the momentum equation (4), the z-dependence also drops out, giving

$$-\frac{dv_{zb}}{dt} - \frac{e^2 g}{m\gamma^5}\frac{2\lambda_b}{z_b} + \frac{\kappa_z z_b}{m\gamma^3} - \frac{4r_b^2 p_{zb}}{m\gamma^3\lambda_b z_b} = 0 \tag{16}$$

⇨ Eqs. (13) – (16) form a coupled ordinary differential equation (ODE) system. Most remarkably, these equations recover the longitudinal envelope equation. From Eqs. (13), (15), and (14), we obtain

$$\frac{1}{\lambda_b}\frac{d\lambda_b}{dt} + \frac{1}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b\lambda_b = const. = \frac{3}{4}N_b\,, \tag{17}$$

$$\frac{1}{p_{zb}}\frac{dp_{zb}}{dt} + \frac{3}{z_b}\frac{dz_b}{dt} = 0 \Longrightarrow z_b^3 p_{zb} = const. = W\,. \tag{18}$$

Longitudinal Envelope Equation

- Substituting Eqs. (17), (18) and (13) into Eq. (16), we obtain

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2 c^2} z_b - K_l \frac{1}{z_b^2} - \varepsilon_l^2 \frac{1}{z_b^3} = 0, \tag{19}$$

where $s = \beta ct$, $K_l \equiv 3N_b e^2 g / 2m\gamma^5\beta^2 c^2$ is the effective longitudinal self-field perveance, and $\varepsilon_l \equiv (4r_b^2 W / m\gamma^3\beta^2 c^2)^{1/2}$ is the longitudinal emittance.

- The longitudinal envelope equation can be integrated once to give

$$(z_{b0}'^2 - z_{bf}'^2) = 2K_l \left(\frac{1}{z_{bf}} - \frac{1}{z_{b0}}\right) + \varepsilon_l^2 \left(\frac{1}{z_{bf}^2} - \frac{1}{z_{b0}^2}\right), \tag{20}$$

where $z_{b0} = z_b(s = 0)$, $z_{bf} = z_b(s = s_f)$, $z_{b0}' = dz_b/ds(s = 0)$, and $z_{bf}' = dz_b/ds(s = s_f)$.

- Given $(z_{bf}, z_{b0}, K_l, \varepsilon_l)$, we want (v_{zb0}, v_{zbf}, s_f) to be as small as possible. But
  - Smaller $v_{bz0} \Longleftrightarrow$ Larger s_f.
  - Smaller $v_{bzf} \Longleftrightarrow$ Larger s_f.

  Need to study the trade-off.

Longitudinal Dynamics

- In the drift compression scheme considered in this paper, we take $\varepsilon_l = 7.7 \times 10^{-6}$m, $K_l = 1.3 \times 10^{-4}$ m, corresponding to an average final current $\langle I_f \rangle = 2500$A, $z_{bf} = 0.6$m.

- If we require $|v_{bzf}| \leq 0.01$, for the given beam parameters, $|v_{bz0}| \leq 0.0227$.

- The beam path length required for drift compression can be expressed as

$$s_f = -\int_{z_{b0}}^{z_{bf}} \frac{dz_b}{\sqrt{z_{b0}'^2 - 2K_l(\frac{1}{z_b} - \frac{1}{z_{b0}}) - \varepsilon_l^2(\frac{1}{z_b^2} - \frac{1}{z_{b0}^2})}} = 421.5m. \qquad (21)$$

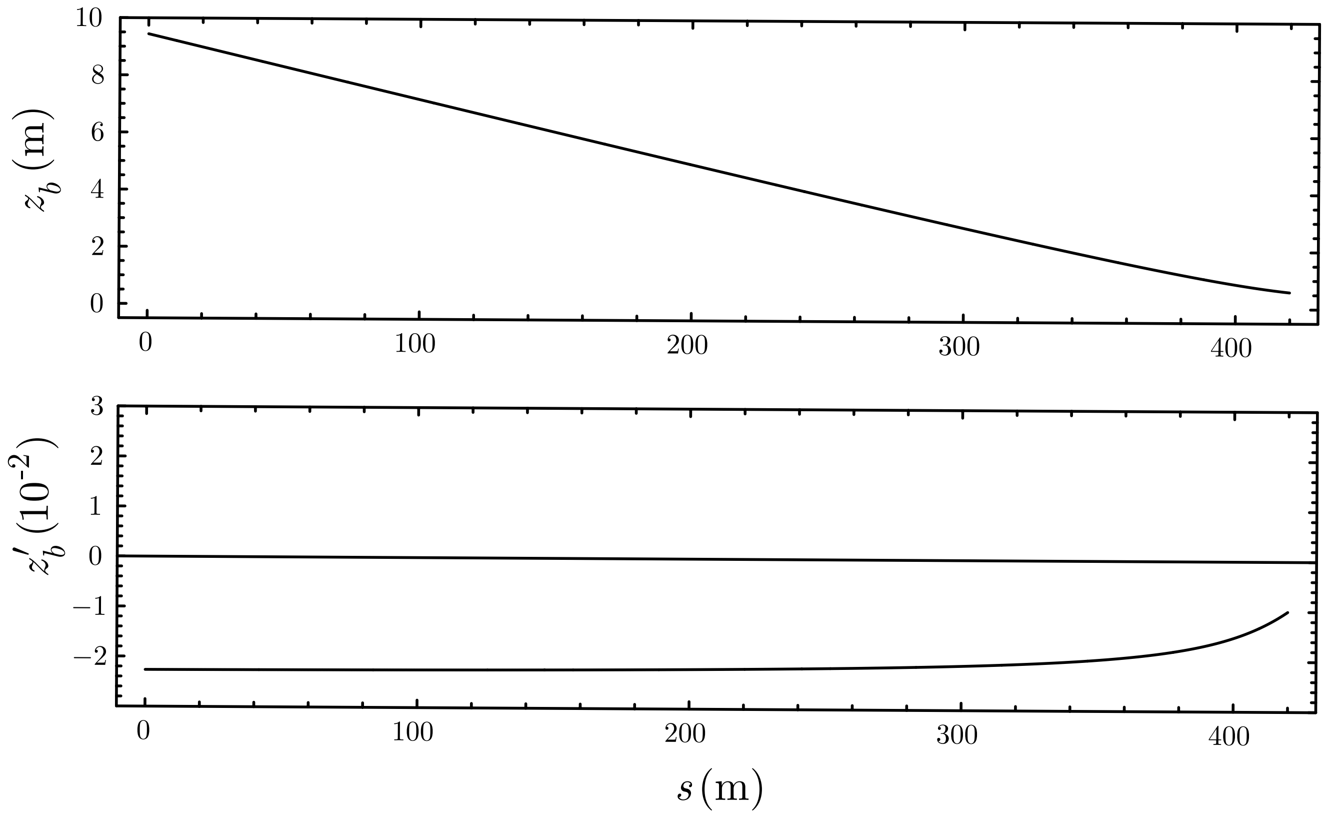
- If only consider free streaming with $v_{z0} = -0.0227$, we have

$$s_f = -\int_{z_{b0}}^{z_{bf}} \frac{dz}{v_{bz0}} = 392.3m. \qquad (22)$$

- With the help of space charge, the initial velocity tilt reduces from 0.0227 to 0.01. The required beam line is only 10% longer compared with the free streaming case.

Longitudinal Dynamics

⇨ The longitudinal envelope equation can be solved numerically.



Longitudinal Pulse Shaping

⇨ The parabolic self-similar drift compression solution requires the initial beam pulse shape to be parabolic.

⇨ Need to shape the beam pulse into a parabolic form before imposing a velocity tilt.

⇨ Need to solve the pulse shaping problem in general — finding the initial velocity distribution $V(z) \equiv v_z(t=0,z)$ such that a given initial pulse shape $\Lambda(z) \equiv \lambda(t=0,z)$ evolves into a given final pulse shape $\Lambda_T(z) \equiv \lambda(t=T,z)$ at time $t=T$.

⇨ Choose the following normalized variables:

$$\overline{v}_z = \frac{v_z}{\beta c},\ \overline{z} = \frac{z}{z_{b0}},\ \overline{\lambda} = \frac{\lambda}{\lambda_{b0}},\ \overline{t} = \frac{t\beta c}{z_{b0}}, \tag{23}$$

where z_{b0} is the initial beam half-length, and λ_{b0} is the initial beam line density at the beam center $(z=0)$.

Longitudinal Pulse Shaping

⇨ In the normalized variables, the one-dimensional fluid equations, neglecting pressure effects and external focusing, are given by

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \ , \tag{24}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + \overline{K}_l \frac{\partial \lambda}{\partial z} = 0 \ , \tag{25}$$

where $\overline{K}_l \equiv \lambda_{b0} e^2 g / m\gamma^5 \beta^2 c^2$ is the normalized longitudinal perveance.

⇨ $\overline{K}_l$ will be treated as a small parameter.

⇨ To order lowest order,

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0 \ , \tag{26}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} = 0 \ . \tag{27}$$

Longitudinal Pulse Shaping

⇨ Equations (26) and (27) can solved by integrating along characteristics. On the characteristics defined by

$$C: \quad \frac{dz}{dt} = v_z, \tag{28}$$

Equations (26) and (27) are

$$\frac{d\lambda}{dt} = -\lambda \frac{\partial v_z}{\partial z}, \tag{29}$$

$$\frac{dv_z}{dt} = 0 \ . \tag{30}$$

⇨ Because $dv_z/dt = 0$ on C, the family of characteristics C are straight lines in the (t, z) plan, which can be represented as

$$C: \ z = \xi + V(\xi)t \, , \tag{31}$$

where

$$V(\xi) \equiv v_z(t = 0, \xi) \, . \tag{32}$$

Longitudinal Pulse Shaping

- The solution for $v_z(t,z)$ can be formally written as

$$v_z(t,z) = V(\xi(t,z))\,, \tag{33}$$

where $\xi(t,z)$ as a function of t and z is determined from Eq. (31).

- From Eqs. (33) and (31), four useful identities can be derived, *i.e.*,

$$\frac{\partial \xi}{\partial z} = \frac{1}{1+V'(\xi)t}\,, \tag{34}$$

$$\frac{\partial \xi}{\partial t} = \frac{-V(\xi)}{1+V'(\xi)t}\,, \tag{35}$$

$$\frac{\partial v_z}{\partial z} = \frac{V'(\xi)}{1+V'(\xi)t}\,, \tag{36}$$

$$\frac{\partial v_z}{\partial t} = \frac{-V(\xi)V'(\xi)}{1+V'(\xi)t}\,. \tag{37}$$

Longitudinal Pulse Shaping

⇨ From Eqs. (29) and (36), we obtain

$$\frac{d\ln\lambda}{dt} = \frac{-V'(\xi)}{1+V'(\xi)t} \quad \text{on } C \ . \tag{38}$$

⇨ Since ξ is a constant on C, Eq. (38) can be integrated to give

$$\begin{aligned} \ln\lambda &= \ln\lambda(t=0,\xi) + \int_0^t \frac{-V'(\xi)}{1+V'(\xi)t} dt \\ &= \ln\Lambda(\xi) + \ln[1+V'(\xi)t] \, , \end{aligned} \tag{39}$$

where $\Lambda(z) \equiv \lambda(t=0,z)$ is the initial line density profile. The solution to Eq. (38) for $\lambda(t,z)$ is

$$\lambda(t,z) = \frac{\Lambda(\xi)}{1+V'(\xi)t} \, . \tag{40}$$

Longitudinal Pulse Shaping

- For the pulse shaping problem, the final line density profile $\Lambda_T(z) \equiv \lambda(t = T, z)$ is specified. We therefore obtain

$$\Lambda_T(z) = \Lambda_T[\xi + V(\xi)T] = \frac{\Lambda(\xi)}{1 + V'(\xi)T}, \tag{41}$$

which can be viewed as an ordinary differential equation for $V(\xi)$.

- It can be simplified using the variable $U(\xi)$ defined by

$$U(\xi) \equiv \xi + V(\xi)T \ . \tag{42}$$

In terms of $U(\xi)$, Eq. (41) becomes

$$\Lambda_T(U)dU = \Lambda(\xi)d\xi \, . \tag{43}$$

- Finally, $U(\xi)$ is determined by solving Eq. (43) for the given functional forms $\Lambda_T(z)$ and $\Lambda(z)$. $V(\xi)$ is simply related to $U(\xi)$ by

$$V(\xi) = \frac{U(\xi) - \xi}{T} \, . \tag{44}$$

Example: Pulse Shaping without Compression

- Consider two examples with the following symmetries and boundary conditions,

$$v_z(t,-z) = -v_z(t,z)\,,\ \lambda(t,-z) = \lambda(t,z)\,, \tag{45}$$

$$V(\xi = 0) = 0\,,\ U(\xi = 0) = 0\,. \tag{46}$$

- **Example 1—Pulse Shaping Without Compression:**

$$\Lambda(z) = \begin{cases} 1 - z^m\,, & 0 \le z \le 1\,, \\ 0\,, & 1 < z\,, \\ \Lambda(-z)\,, & z < 0\,, \end{cases} \tag{47}$$

$$\Lambda_T(z) = \begin{cases} (1 - z^n)\dfrac{m(n+1)}{n(m+1)}\,, & 0 \le z \le 1\,, \\ 0\,, & 1 < z\,, \\ \Lambda(-z)\,, & z < 0\,. \end{cases} \tag{48}$$

Example: Pulse Shaping without Compression

- Equation (43) can integrated to give

$$\left[U(\xi) - \frac{U(\xi)^{n+1}}{n+1}\right] \frac{m(n+1)}{n(m+1)} = \xi - \frac{\xi^{m+1}}{m+1} . \tag{49}$$

- The parabolic self-similar drift compression solution corresponds to $n = 2$. In this case, there are three solutions for $U(\xi)$. The solution satisfying the right boundary condition is

$$U(\xi) = -\frac{1 - i\sqrt{3} + \sqrt[3]{-2}p^2}{\sqrt[3]{4}p} , \tag{50}$$

where

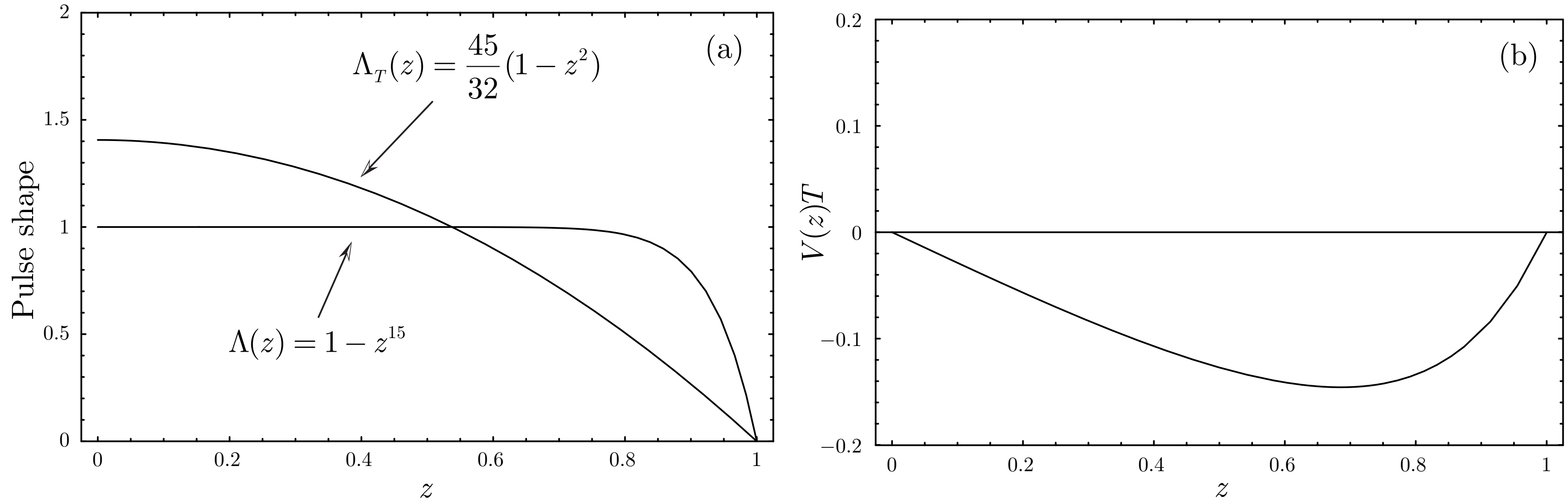
$$p = \sqrt[3]{-3a + \sqrt{-4 + 9a^2}} , \tag{51}$$

$$a = \frac{2(m+1)}{3m}(\xi - \frac{\xi^{m+1}}{m+1}) . \tag{52}$$

- For large value of $m \gg 1$, $\Lambda(z)$ has a flat-top shape with a fast fall-off near the ends of the pulse.

Example: Pulse Shaping without Compression

⇨ Initial pulse shape $\Lambda(z) = 1 - z^{15}$ and final pulse shape $\Lambda_T(z) = (45/32)(1 - z^2)$ are plotted in (a). The initial velocity $V(z)$ given by Eq. (44) is plotted in (b).



⇨ **Example 2—Pulse Shaping With Compression:**

$$\Lambda(z) = \begin{cases} 1 - z^m\,, & 0 \leq z \leq 1\,, \\ 0\,, & 1 < z\,, \\ \Lambda(-z)\,, & z < 0\,, \end{cases} \tag{53}$$

$$\Lambda_T(z) = \begin{cases} [1 - (\alpha z)^n]\dfrac{\alpha m(n+1)}{n(m+1)}\,, & 0 \leq z \leq \dfrac{1}{\alpha}\,, \\ 0\,, & \dfrac{1}{\alpha} < z\,, \\ \Lambda(-z)\,, & z < 0\,, \end{cases} \tag{54}$$

where $\alpha > 1$ is the compression factor.

⇨ Equation (43) can be integrated to give

$$\left[\alpha U(\xi) - \frac{(\alpha U(\xi))^{n+1}}{n+1}\right]\frac{m(n+1)}{n(m+1)} = \xi - \frac{\xi^{m+1}}{m+1}\,, \tag{55}$$

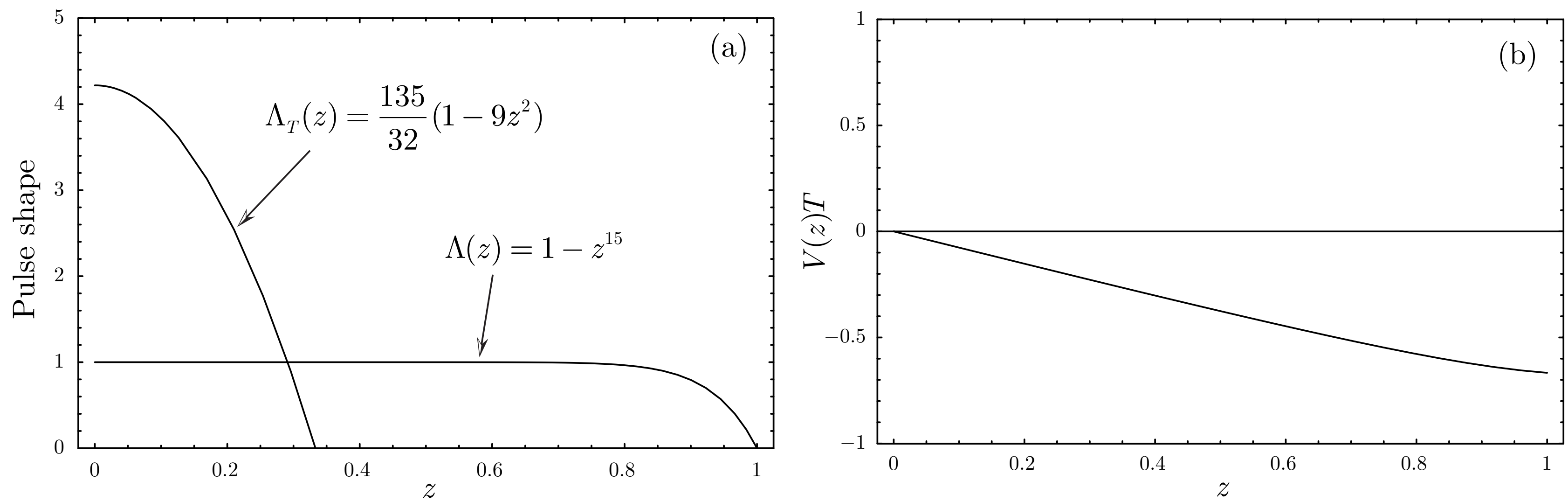
which is identical to Eq. (49) if $\alpha U(\xi)$ is replaced by $U(\xi)$. It is easy to verify that $\alpha U(\xi = 1) = 1$ and therefore

$$V(\xi = 1) = \frac{(1/\alpha - 1)}{T}\,. \tag{56}$$

Example: Pulse Shaping with Compression

- For the case of a beam being shaped but not compressed, $\alpha = 1$ and $V(\xi = 1) = 0$. When $\alpha > 1$, the beam is simultaneously being shaped and compressed, and $V(\xi = 1) < 0$.

- Initial pulse shape $\Lambda(z) = 1 - z^{15}$ and final pulse shape $\Lambda_T(z) = (135/32)(1 - 9z^2)$ are plotted in (a). The initial velocity $V(z)$ given by Eq. (44) is plotted in (b).



1st Order Space-Charge Correction

⇨ We now carry out the analysis to $O(\overline{K}_l)$. Let

$$\lambda(t,z) = \lambda_0(t,z) + \lambda_1(t,z)\,, \tag{57}$$

$$v_z(t,z) = v_{z0}(t,z) + v_{z1}(t,z)\,. \tag{58}$$

⇨ To $O(\overline{K}_l)$, Eqs. (24) and (25) can be expressed as

$$\left(\frac{d}{dt}\right)_0 \lambda_1 = \frac{\partial \lambda_1}{\partial t} + v_{z0}\frac{\partial \lambda_1}{\partial z} = -\lambda_1 \frac{\partial v_{z0}}{\partial z} - \frac{\partial}{\partial z}(\lambda_0 v_{z1})\,, \tag{59}$$

$$\left(\frac{d}{dt}\right)_0 v_{z1} = \frac{\partial v_{z1}}{\partial t} + v_{z0}\frac{\partial v_{z1}}{\partial z} = -v_{z1}\frac{\partial v_{z0}}{\partial z} - \overline{K}_l \frac{\partial \lambda_0}{\partial z}\,. \tag{60}$$

⇨ Using the method of variational coefficients, the solution to Eq. (60) is found to be

$$v_{z1} = \frac{1}{1+V_0'(\xi)t}\left\{V_1(\xi) - \overline{K}_l \frac{\partial}{\partial \xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\ln[1+V_0'(\xi)t]\right]\right\}\,. \tag{61}$$

⇨ By the same procedure, Eq. (59) can be integrated to give

$$\lambda_1 = \frac{\Lambda_1(\xi)}{1+V_0'(\xi)t} - \frac{1}{1+V_0'(\xi)t}\frac{\partial}{\partial\xi}\left\{\frac{\Lambda_0(\xi)V_1(\xi)t}{1+V_0'(\xi)t}\right. \tag{62}$$
$$- \overline{K}_l\Lambda_0(\xi)\frac{\partial}{\partial\xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\right]\frac{V_0'(\xi)t-\ln[1+V_0'(\xi)t]}{[1+V_0'(\xi)t]^2}$$
$$\left. - \overline{K}_l\frac{\Lambda_0^2(\xi)}{V_0'(\xi)}V_0''(\xi)\frac{t^2}{[1+V_0'(\xi)t]^2}\right\}.$$

⇨ At time $t = T$, we obtain

$$\Lambda_T(z) = \lambda_0(t=T,z) + \lambda_1(t=T,z). \tag{63}$$

Since $\Lambda_T(z)$ and $\Lambda(z)$ are prescribed in the pulse shaping problem, we take $\Lambda_{T1}(z) = 0$ and $\Lambda_1(z) = 0$. This results in

$$V_1(\xi) = \overline{K}_l\frac{\partial}{\partial\xi}\left[\frac{\Lambda_0(\xi)}{V_0'(\xi)}\right]\frac{V_0'(\xi)-\ln[1+V_0'(\xi)T]/T}{1+V_0'(\xi)T} \tag{64}$$
$$+ \overline{K}_l\frac{\Lambda_0(\xi)}{V_0'(\xi)}V_0''(\xi)\frac{T}{1+V_0'(\xi)T} + c'.$$

Transverse Dynamics in Non-Periodic Lattice

- Transverse envelope equations:

$$\begin{aligned} \frac{d^2a}{ds^2} + \kappa_q a - \frac{2K(s)}{a+b} - \frac{\epsilon_x^2}{a^3} &= 0, \\ \frac{d^2b}{ds^2} - \kappa_q b - \frac{2K(s)}{a+b} - \frac{\epsilon_x^2}{b^3} &= 0. \end{aligned} \tag{65}$$

- $K(s)$ is non-periodic due to the longitudinal compression.

- κ_x and κ_y need to be non-periodic to reduce the expansion of the beam radius.

- Since the quadrupole lattice is not periodic, the concept of a "matched" beam is not well defined.

- However, if the the non-periodicity is small, that is, if the quadrupole lattice changes slowly along the beam path, we can seek an "adiabatically"-matched beam which, by definition, is locally matched everywhere.

Transverse Dynamics and Lattice Design

- Goal:
  - Constant vacuum phase advance $\sigma_v = \pi/5 \longrightarrow \eta B' L^2 = const.$
  - Length $z_b \longrightarrow \times \dfrac{1}{16}$. Perveance $K \longrightarrow \times 16$.
  - Velocity tilt $|v_{bz}| \longrightarrow \leq 0.01$.
  - Beam radius $a \longrightarrow \times 2$.
  - Half lattice period $L \longrightarrow \times \dfrac{1}{2}$.
  - Filling factor $\eta \longrightarrow \times 4$. $\eta B' \longrightarrow \times 4$.
- How do K, L, η, B', a, and b depend on s?
  - $K(s)$ is given by the longitudinal dynamics.
  - $L(s), \eta(s)$, and $B'(s)$ are determined by requirements such as constant vacuum phase advance.
  - $a(s)$ and $b(s)$ are determined by the transverse envelope equations.

Non-periodic Lattice Design

- A lattice which keeps both the vacuum phase advance and depressed phase advance constant is less likely to induce beam mismatch.

- Vacuum phase advance σ_v and depressed phase advance σ are given by

$$2(1-\cos\sigma_v) = (1-\frac{2\eta}{3})\eta^2\left(\frac{B'}{[B\rho]}\right)^2 L^4, \tag{66}$$

$$\sigma^2 = 2(1-\cos\sigma_v) - K\left(\frac{2L}{\langle a\rangle}\right)^2. \tag{67}$$

- Assuming $\eta \ll 1$, we obtain

$$\eta^2(\frac{B'}{[B\rho]})^2 L^4 = const.,\ \ K(\frac{2L}{\langle a\rangle})^2 = const., \tag{68}$$

for constant vacuum phase advance and constant depressed phase advance.

- It is under-determined. As one possible choice, let

$$L = L_0\exp(-\ln 2\frac{s}{s_f}), \qquad \eta = \eta_0\exp(2\ln 2\frac{s}{s_f}), \qquad B' = const. \tag{69}$$

Non-periodic Lattice Design

⇨ Let the lattice lengths are $L_0,\ L_1,\ ...,\ L_N = L_f$,

$$
\begin{aligned}
&L_1 = L_0 \exp(-\ln 2 \frac{2L_0}{s_f}), \\
&L_2 = L_0 \exp(-\ln 2 \frac{2(L_0 + L_1)}{s_f}), \\
&\ldots\ldots \\
&L_i = L_0 \exp(-\ln 2 \frac{2\sum_0^{i-1} L_i}{s_f}), \\
&2(L_1 + L_2 + ... + L_N) = S_f.
\end{aligned} \tag{70}
$$

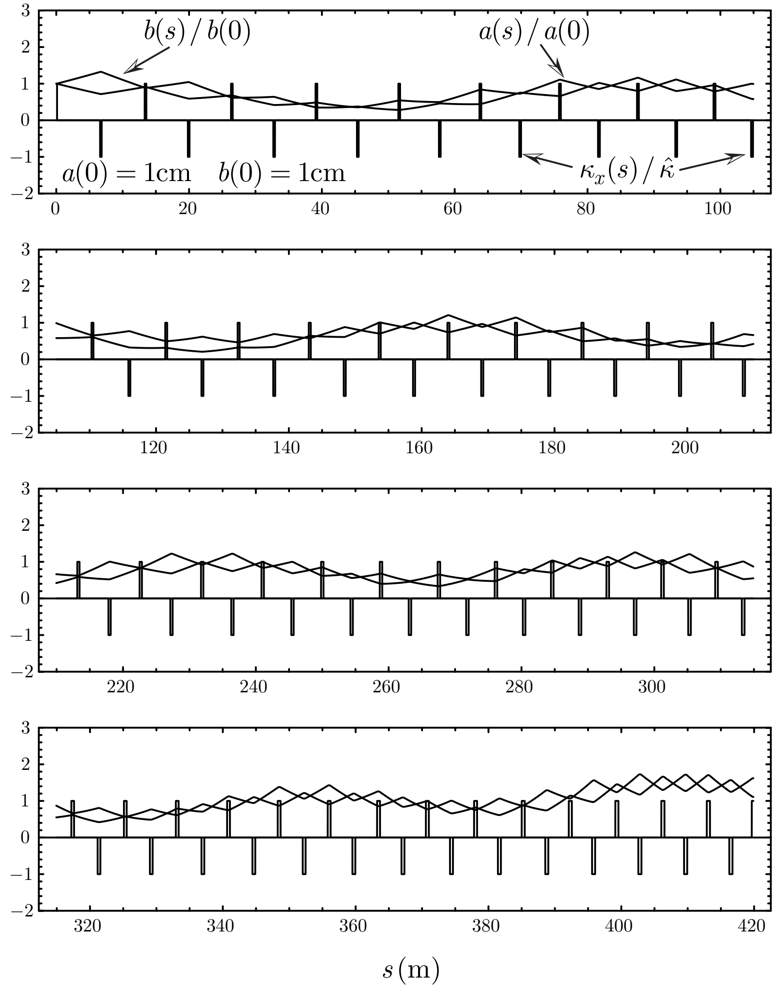
⇨ For $L_f = 3.36$m, $L_0 = 6.72$m, and $s_f = 421.5$m, calculation gives $N = 45$.

Adiabatically-Matched Solution

- Currently, there are no well-defined rules to determine *a priori* which solution is adiabatically-matched.
- In general, satisfactory results can be obtained by using an intuitive trial-and-error approach.
- A recently derived equation for the average beam envelope in non-periodic lattices will provide a systematic understanding of the adiabatically-matched solutions
- For an adiabatically-matched solution,
  - The envelope is locally matched and contains no oscillations other than the local envelope oscillations.
  - On the global scale, the beam radius increases monotonically.

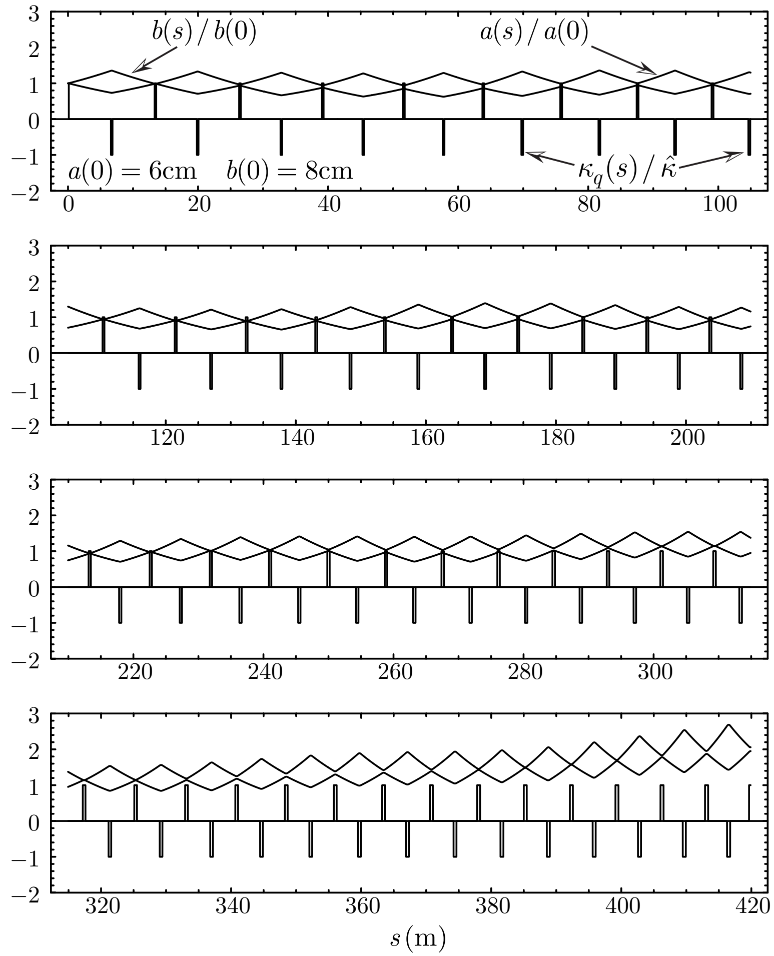
Example of Non-adiabatically-Matched Solution

⇨ The solutions shown are not adiabatically-matched because the envelope oscillations have low frequency components



An adiabatically-Matched Solution

⇨ The solutions shown are adiabatically-matched. The average beam size increases by a factor of 2, which agrees with the design assumption.



Basic Design Parameters

- At the end of drift compression:
  - Length $z_{bf} = 10ns \times \beta c = 0.5937\text{m}$. Perveance $K = 1.470 \times 10^{-4}\text{m}$.
  - Longitudinal perveance $K_l = 1.309 \times 10^{-4}\text{m}$.
  - Velocity tilt $|v_{zbf}| \leq 0.01$.
  - Beam radius $a \sim 12\text{cm}$, $b \sim 16\text{cm}$.
  - Half lattice period $L_f = 3.36\text{m}$.
  - Filling factor $\eta_f = 0.144$. $\eta_f B_f' = 4.567\text{T/m}$. Phase advance $\sigma_v = \pi/5$.
- At the beginning of drift compression:
  - Length $z_{b0} = 0ns \times \beta c = 9.5\text{m}$. Perveance $K = 9.18 \times 10^{-6}\text{m}$.
  - Longitudinal perveance $K_l = 1.309 \times 10^{-4}\text{m}$.
  - Velocity tilt $v_{zb0} \leq 0.0227$.
  - Beam radius $a = 6\text{cm}$, $b = 8\text{cm}$.
  - Half lattice period $L_0 = 6.72\text{m}$.
  - Filling factor $\eta_0 = 0.036$. $\eta_f B_f' = 1.14\text{T/m}$. Phase advance $\sigma_v = \pi/5$.

Conclusions

- ⇨ The longitudinal dynamics of drift compression and pulse shaping have been studied using a one-dimensional warm-fluid model.

- ⇨ It was found that at least two self-similar drift compression solutions exist for the one-dimensional warm-fluid equations: the linear self-similar drift compression solution, and the parabolic self-similar drift compression solution.

- ⇨ Detailed analysis showed that the latter solution has several desirable features and is a good candidate for practical drift compression schemes.

- ⇨ The pulse shaping problem is solved perturbatively in the weak space-charge limit, such that an arbitrary pulse shape produced after the acceleration phase can be shaped into those required by the self-similar drift compression solutions.

- ⇨ A non-periodic quadrupole lattice configuration has been designed for a beam undergoing drift compression with fixed vacuum phase advance and depressed phase advance.

- ⇨ An adiabatically-matched solution was found for the transverse envelope equations in the non-periodic lattice.

Important Physics Issues for Future Study

- Other self-similar drift compression solutions may exist. A systematic method to discover families of self-similar drift compression solutions is needed.
- Pulse shaping problem in strong space-charge region and over the entire beam path.
- Coupling between longitudinal and transverse dynamics.
- Stability (sensitivity) using fluid and kinetic models.
- Emittance growth during the compression.
- Longitudinal "shock" formation during the compression.

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