Recent Results From The Paul Trap Simulator Experiment

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The Paul Trap Simulator Experiment (PTSX) is a compact laboratory facility whose purpose is to simulate the nonlinear dynamics of intense charged particle beam propagation over large distances through an alternating-gradient transport system. The simulation is possible because the quadrupole electric fields of the cylindrical Paul trap exert radial forces on the charged particles that are analogous to the radial forces that a periodic focusing quadrupole magnetic field exert on the beam particles in the beam frame. Initial experiments clearly demonstrate the loss of confinement when the vacuum phase advance σ_v of the system exceeds 90 degrees. Recent experiments show that PTSX is able to successfully trap plasmas of moderate intensity for thousands of equivalent lattice periods.

* Research supported by the U.S. Department of Energy

Introduction

- Purpose: simulate nonlinear dynamics of intense beam propagation over large distances through alternating-gradient transport systems.

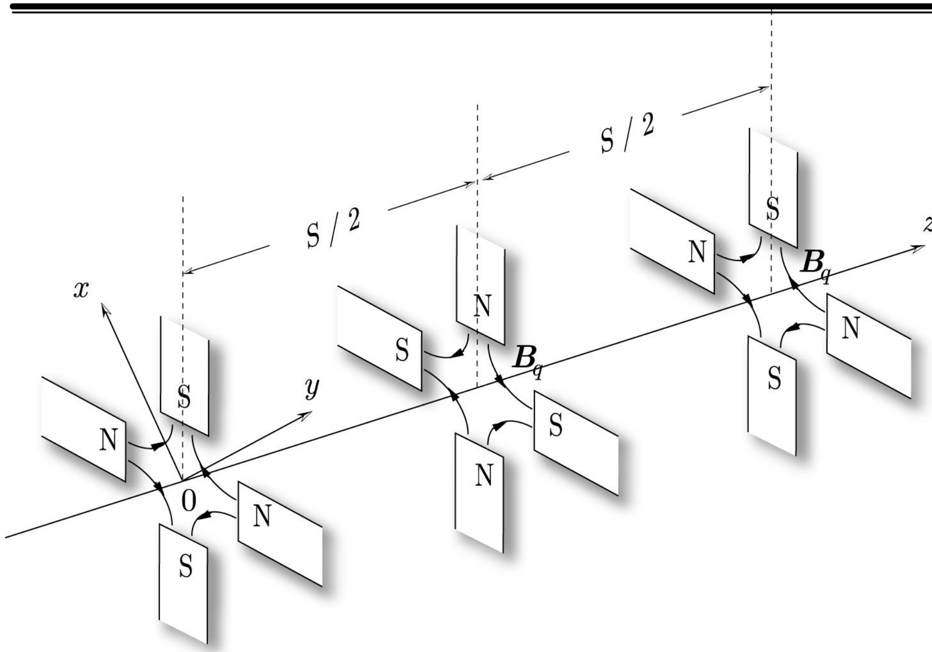
    Okamoto and Tanaka, Nucl. Instrum. Methods A **437**, 178 (1999). Davidson, et al., Phys. Plasmas **7**, 1020 (2000).

- Applications: heavy ion fusion, spallation neutron sources, and nuclear waste treatment.

- PTSX will explore physics issues such as:

    - Beam mismatch and envelope instabilities,
    - Collective wave excitations,
    - Chaotic particle dynamics and production of halo particles,
    - Mechanisms for emittance growth,
    - Compression techniques,
    - Effects of distribution function on stability properties.

Alternating Gradient Focusing Systems



$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -k_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

- Consider a thin ($r_b << S$) intense nonneutral ion beam (ion charge = $+Z_b e$, rest mass = m_b) propagating in the z-direction through a periodic focusing quadrupole field with average axial momentum $g_b m_b \boldsymbol{b}_b c$, and axial periodicity length S.
- Here, r_b is the characteristic beam radius, $V_b = \boldsymbol{b}_b c$ is the average axial velocity, and $(g_b\text{-}1)m_b c^2$ is the directed kinetic energy, where $g_b = (1\text{-}\boldsymbol{b}_b{}^2)^{-1/2}$ is the relativistic mass factor.
- The particle motion in the beam frame is assumed to be nonrelativistic.

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Theoretical Model and Assumptions

- Introduce the scaled "time" variable

$$s = \boldsymbol{b}_b ct$$

and the (dimensionless) transverse velocities

$$x' = \frac{dx}{ds} \quad \text{and} \quad y' = \frac{dy}{ds}$$

- The beam particles propagate in the *z*-direction through an alternating-gradient quadrupole field

$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(s)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

with lattice coupling coefficient defined by

$$k_q(s) \equiv \frac{Z_b e B_q'(s)}{g_b m_b \boldsymbol{b}_b c^2}$$

Here, $B_q'(s) \equiv \left(\partial B_x^q / \partial y\right)_{(0,0)} = \left(\partial B_y^q / \partial x\right)_{(0,0)}$ and $x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y$ is the transverse displacement of a particle from the beam axis.

- Here,

$$k_q(s+S) = k_q(s)$$

where $S = const.$ is the axial periodicity length.

Theoretical Model and Assumptions

- Neglecting the axial velocity spread, and approximating $v_z \cong \boldsymbol{b}_b c$, the applied transverse focusing force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -k_q(s)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

over the transverse dimensions of the beam ($r_b << S$).

- The (dimensionless) self-field potential experienced by a beam ion is

$$y\,(x,y,s) = \frac{Z_b e}{g_b m_b \boldsymbol{b}_b^2 c^2}\left[f\,(x,y,s) - \boldsymbol{b}_b A_z(x,y,s)\right]$$

where $f\,(x, y, s)$ is the space-charge potential, and $A_z(x,y,s) \cong \boldsymbol{b}_b f\,(x,y,s)$ is the axial component of the vector potential.

- The corresponding self-field force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{self}(\boldsymbol{x}) = -\left[\frac{\partial y}{\partial x}\hat{\boldsymbol{e}}_x + \frac{\partial y}{\partial y}\hat{\boldsymbol{e}}_y\right]$$

Theoretical Model and Assumptions

- Transverse particle orbits $x(s)$ and $y(s)$ in the laboratory frame are determined from

$$\frac{d^2}{ds^2}x(s)+k_q(s)x(s)=-\frac{\partial}{\partial x}y\,(x,y,s)$$

$$\frac{d^2}{ds^2}y(s)-k_q(s)y(s)=-\frac{\partial}{\partial y}y\,(x,y,s)$$

- The characteristic axial wavelength l_q of transverse particle oscillations induced by a quadrupole field with amplitude $\hat{k}_q$ is

$$l_q \sim \frac{2p}{\sqrt{\hat{k}_q}}$$

- The dimensionless small parameter e assumed in the present analysis is

$$e \sim \left(\frac{S}{l_q}\right)^2 \sim \frac{\hat{k}_q S^2}{(2p)^2} < 1$$

which is proportional to the strength of the applied focusing field.

Theoretical Model and Assumptions

- The laboratory-frame Hamiltonian $\hat{H}(x, y, x', y', s)$ for single-particle motion in the transverse phase space (x, y, x', y') is

$$\hat{H}(x, y, x', y', s) = \frac{1}{2}(x'^2 + y'^2) + \frac{1}{2}k_q(s)(x^2 - y^2) + y\,(x, y, s)$$

- The Vlasov equation describing the nonlinear evolution of the distribution function $f_b(x, y, x', y', s)$ in laboratory-frame variables is given by

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(k_q(s)x + \frac{\partial y}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-k_q(s)y + \frac{\partial y}{\partial y}\right)\frac{\partial}{\partial y'}\right\} f_b = 0$$

where $y\,(x, y, s) = e_b f^s(x, y, s) / g_b^3 m_b b_b^2 c^2$ is the dimensionless self-field potential.

Theoretical Model and Assumptions

- The self-field potential y (x, y, s) is determined self-consistently in terms of the distribution function $f_b\,(x, y, x', y', s)$ from

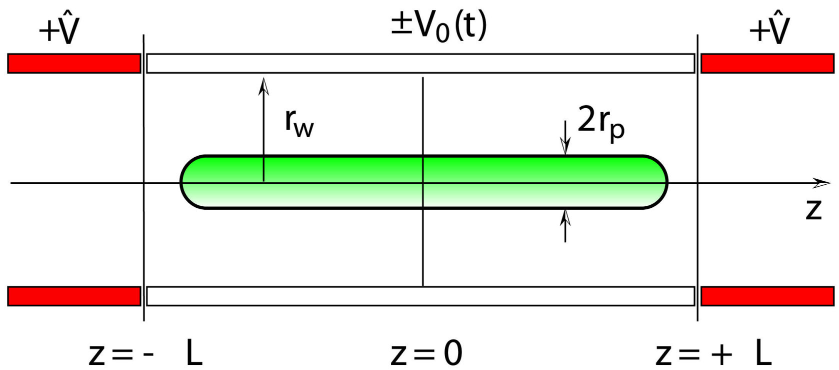
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)y = -\frac{2p\,K_b}{N_b}\int dx'dy'\, f_b$$

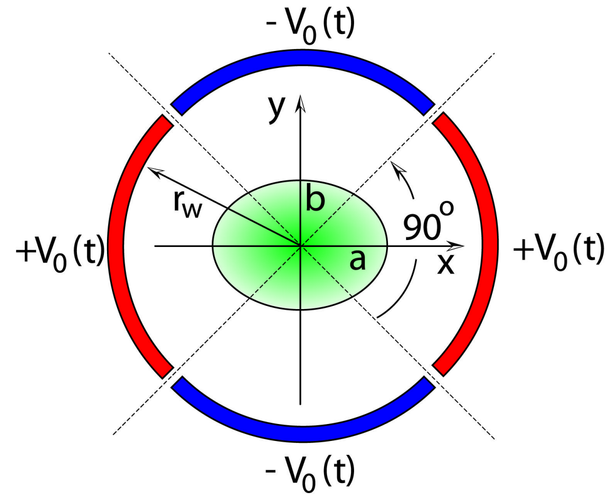
- Here, $n_b(x,y,s)=\int dx'dy'\, f_b(x,y,x',y',s)$ is the number density of the beam ions, and the constants K_b and N_b are the self-field perveance and the number of beam ions per unit axial length, respectively, defined by

$$K_b=\frac{2N_b Z_b^2 e^2}{g_b^3 m_b \mathbf{b}_b^2 c^2}=const.$$

$$N_b=\int dx\,dy\,dx'dy'\, f_b(x,y,x',y',s)=const.$$

Paul Trap Simulator Configuration





Plasma column length	2 m	Maximum wall voltage	400 V
Wall electrode radius	10 cm	End electrode voltage	400 V
Plasma column radius	1 cm	Voltage oscillation frequency	100 kHz

- The injection electrodes oscillate with the voltage $\pm V_0(t)$ during ion injection.
- The 40 cm long electrodes at the far end of the trap are held at a constant voltage during injection to prevent ions from leaving the far end of the trap.
- The dump electrodes oscillate with the voltage $\pm V_0(t)$ during ion dumping.

Transverse Hamiltonian for Particle Motion in a Paul Trap

Transverse Hamiltonian (dimensional units) for a long charge bunch in a Paul trap with time periodic wall voltages $V_0(t+T)=V_0(t)$ is given by

$$H_\perp(x,y,\dot{x},\dot{y},s)=\frac{1}{2}m_b(\dot{x}^2+\dot{y}^2)+e_b f_{ap}(x,y,t)+e_b f^s(x,y,t)$$

where the applied potential $(0\le r\le r_w)$

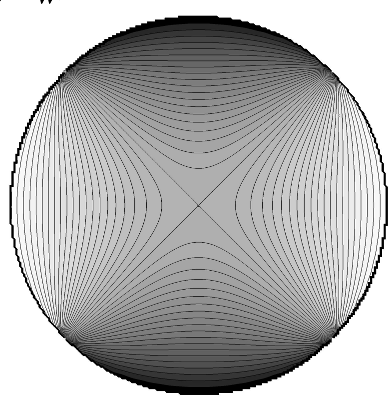
$$f_{ap}(x,y,t)=\frac{4V_0(t)}{p}\sum_{\ell=1}^{\infty}\frac{\sin(\ell p/2)}{\ell}\left(\frac{r}{r_w}\right)^{2\ell}\cos(2\ell q)$$

can be approximated by (for $r_p << r_w$)

$$e_b f_{ap}(x,y,t)=\frac{1}{2}k_q(t)(x^2-y^2),\ \text{where}\ k_q(t)=\frac{8e_b V_0(t)}{m_b p\, r_w^2}$$

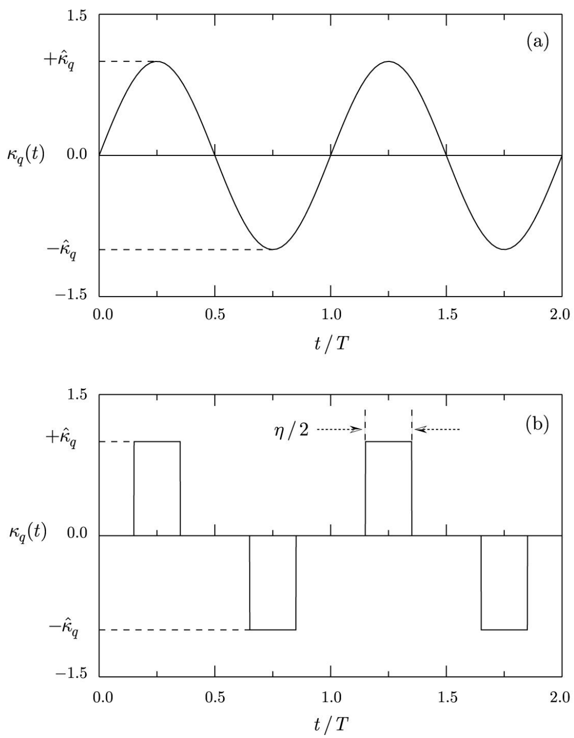
with corrections of order $(r_p/r_w)^4$.

$f_{ap}(x,y,t_0)$

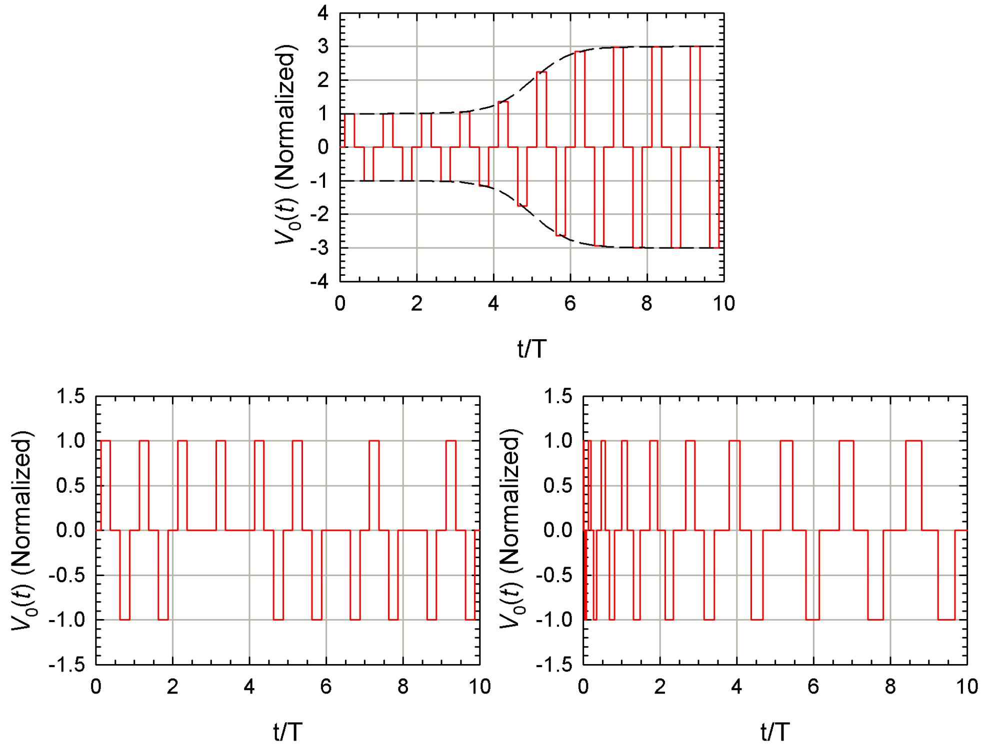


Waveform Examples

$$\kappa_q(t) = \hat{\kappa}_q \sin\left(\frac{2\pi t}{T}\right)$$



$$\kappa_q(t) = \hat{\kappa}_q \begin{cases} 1, & (0.25-\eta/4) < t/T < (0.25+\eta/4) \\ -1, & (0.75-\eta/4) < t/T < (0.75+\eta/4) \\ 0, & \text{otherwise} \end{cases}$$



- “Carrier” waveform is arbitrary.
- Individual electrodes will eventually be allowed to have different waveforms.

Constraints on Parameters

The average transverse focusing frequency is given by,

$$w_q = \frac{8e_b V_{0\,\max}}{m_b p\, r_w^2 f} x$$

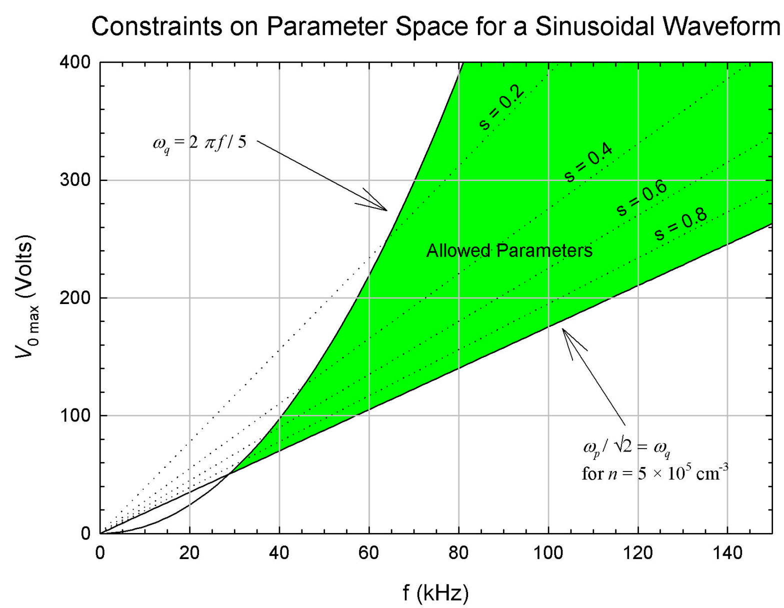
where $x = \dfrac{1}{2\sqrt{2p}}$ for a sinusoidal waveform, and

$$x = \frac{h\sqrt{3-2h}}{4\sqrt{3}}$$ for a periodic step-function waveform with fill factor h.

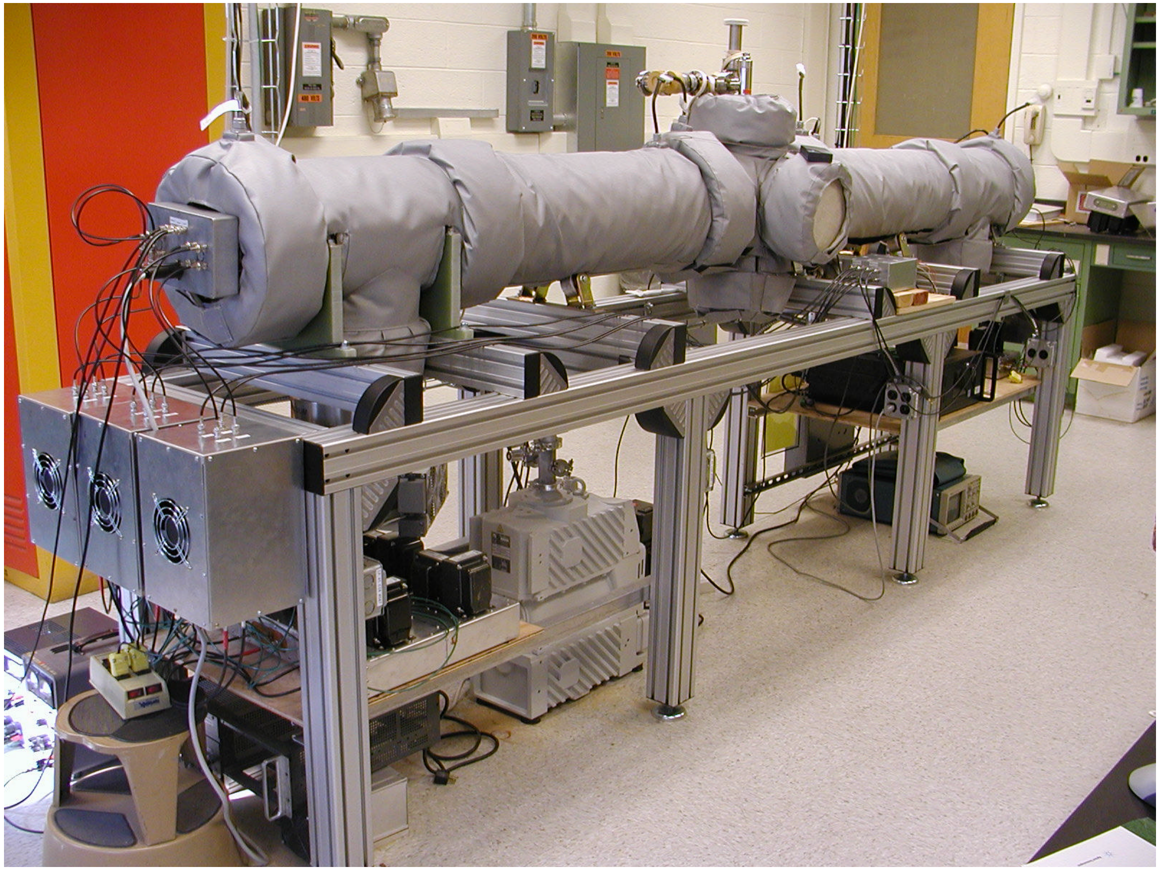
There are primary constraints on the normalized intensity parameter s and the vacuum phase advance s.

$$s \equiv \frac{w_p^2}{2w_q^2} < 1$$

$$s_v = \frac{w_q}{f} < \frac{2p}{5}$$



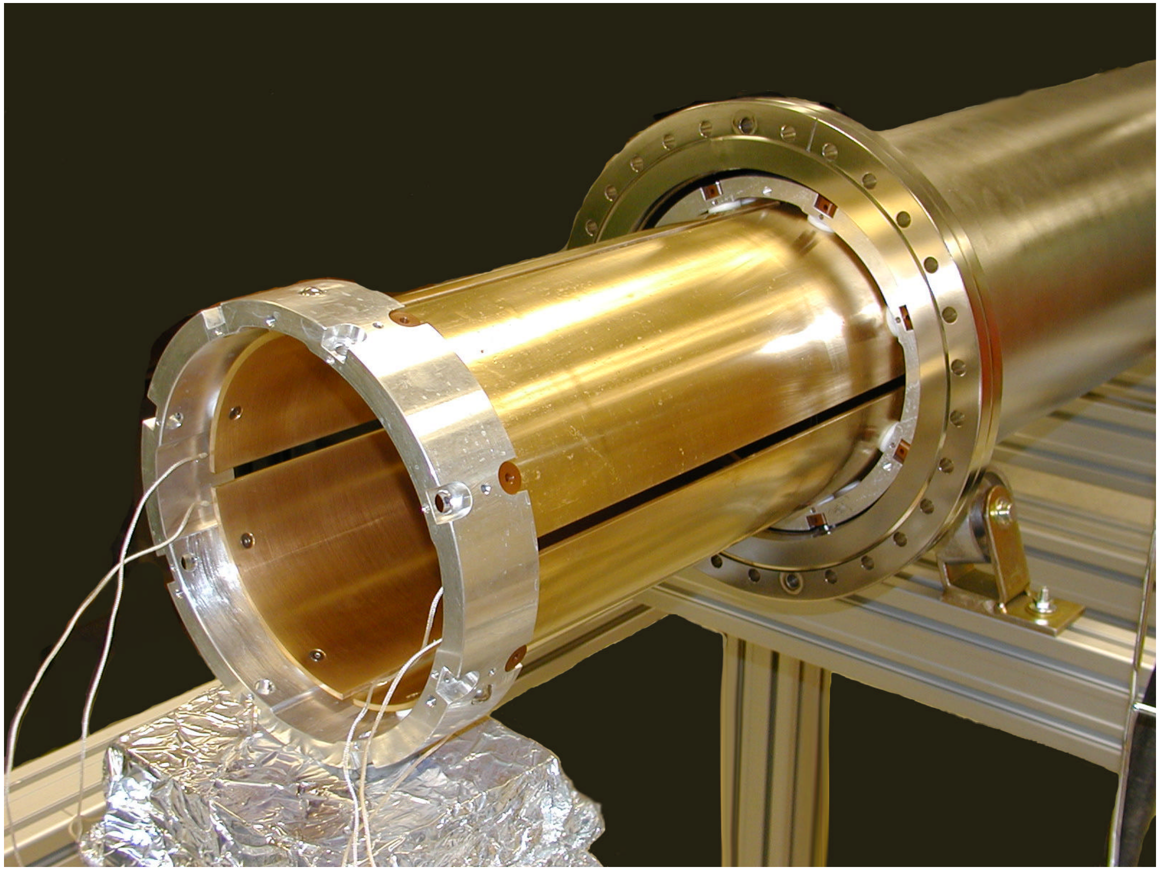
PTSX Apparatus



- Laboratory preparation, procurement, assembly, bakeout, and pumpdown of PTSX vacuum chamber to 5.25×10^{-10} Torr (May, 2002).

Paul Trap Simulator Experiment vacuum chamber.

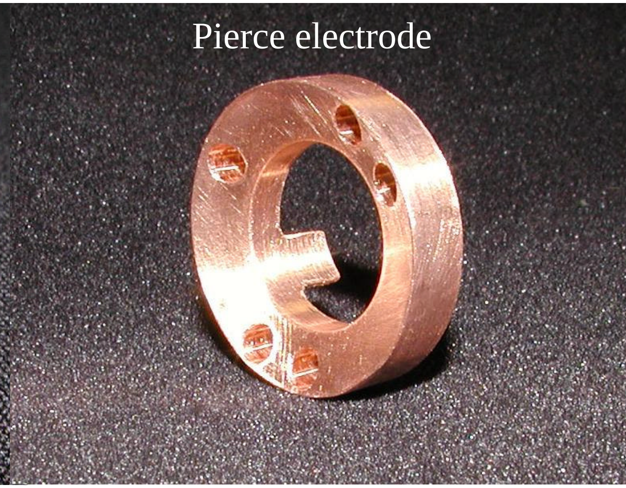
PTSX Dump Electrodes

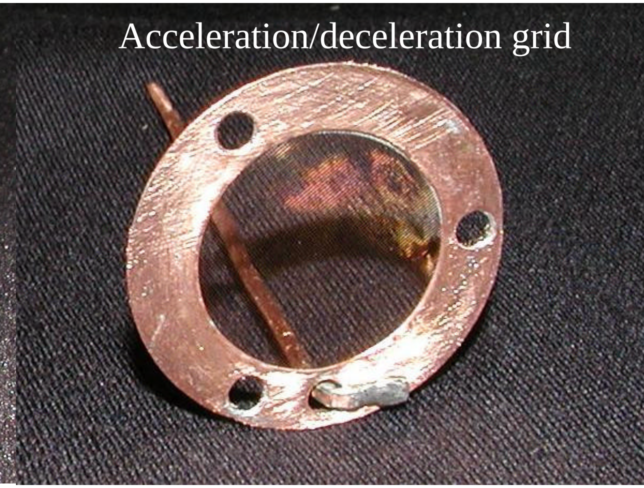


- 8 inch diameter stainless steel gold-plated electrodes are supported by aluminum rings, teflon, and vespel spacers.

Paul Trap Simulator Experiment electrodes.

PTSX Ion Source



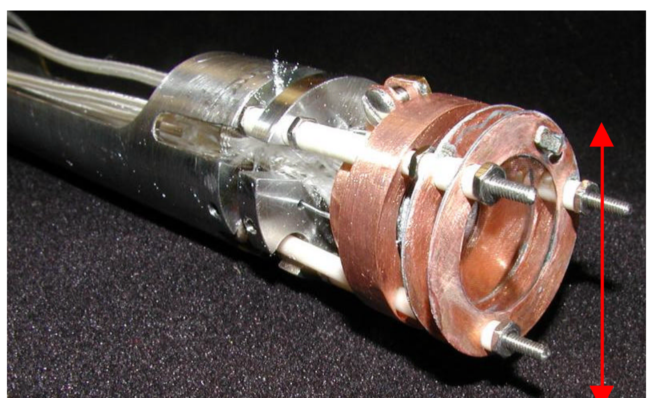


11 A

1100 C

67.5 opening angle

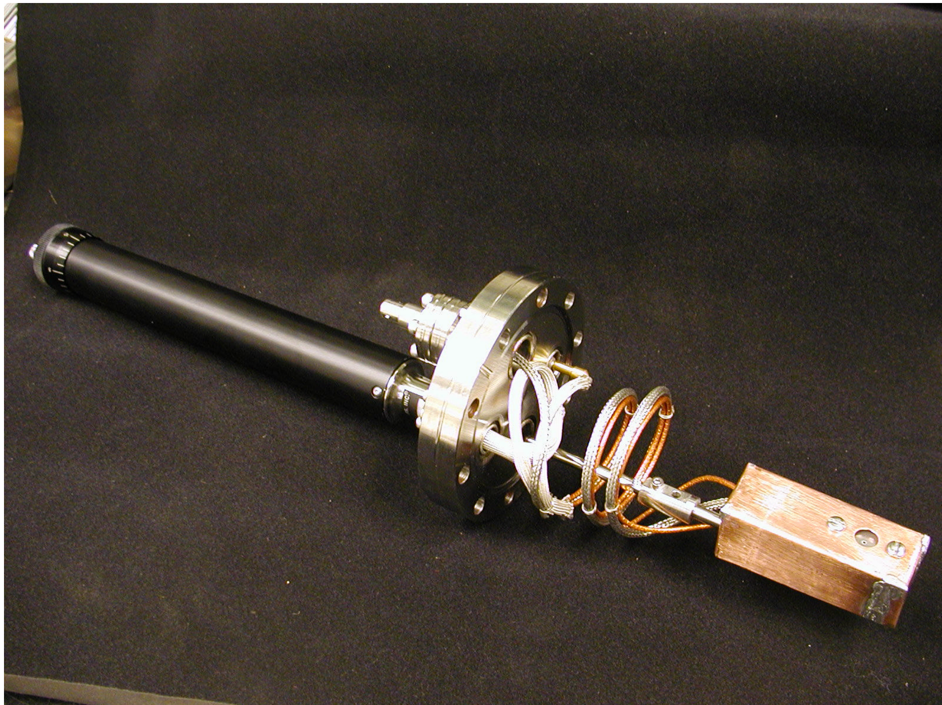
85% transparent electroformed copper mesh



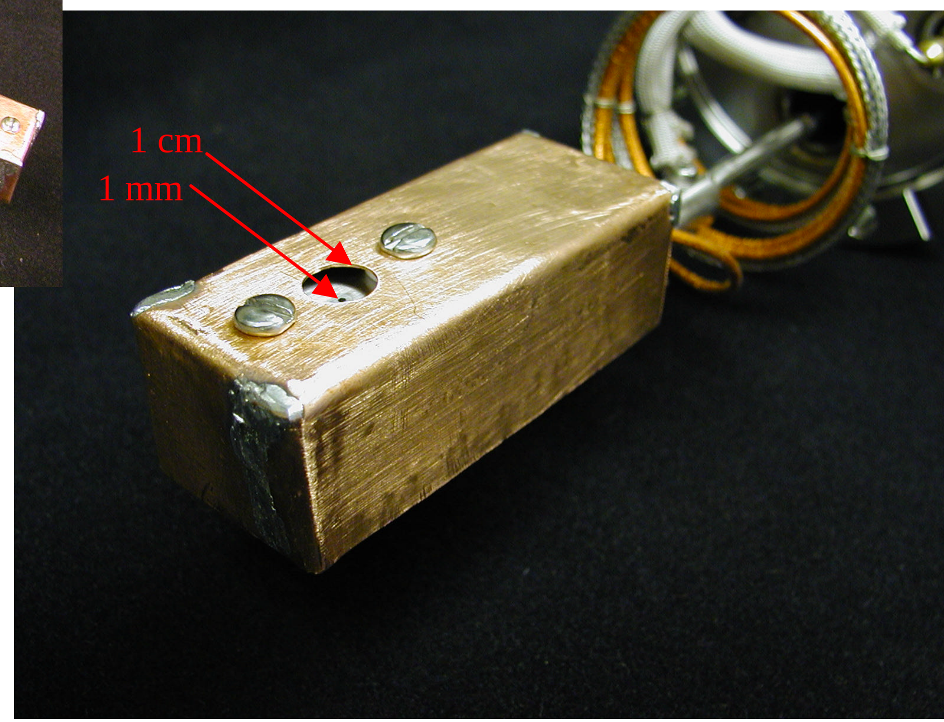
1.25 in

- Aluminosilicate cesium source produces up to 30 μA of ion current when a 200 V acceleration voltage is used.

PTSX Faraday Cup Diagnostic

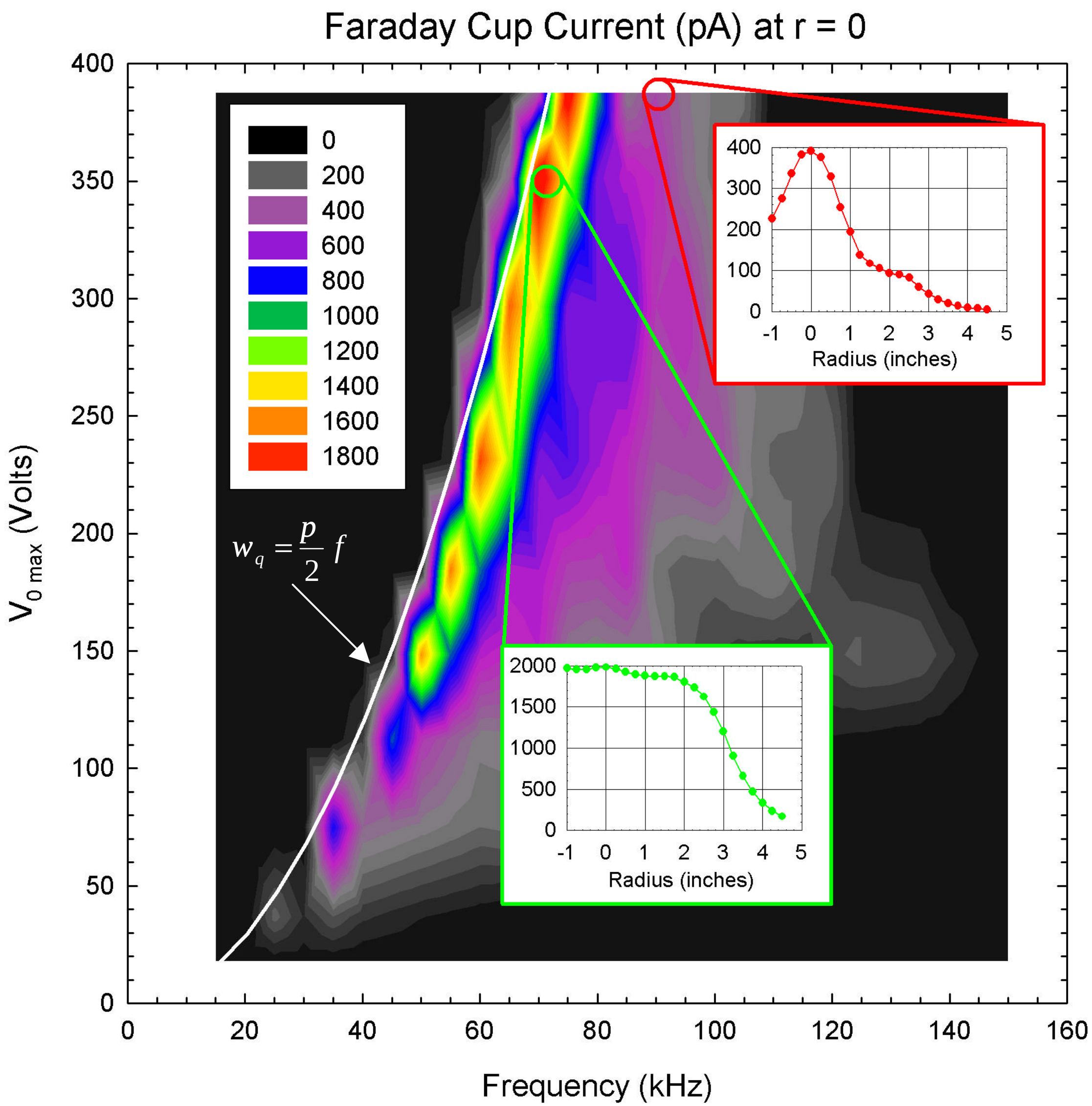


- Faraday cup with sensitive electrometer allows 20 fC resolution.
- Linear motion feedthrough with 6" stroke allows measurement of radial density dependence. 1 mm diameter aperture gives fine spatial resolution.



- Copper shield has been modified to reduce impact of stray ions.

Instability of Single Particle Orbits



- Experiment - stream Cs^+ ions from source to collector without axial trapping of the plasma.

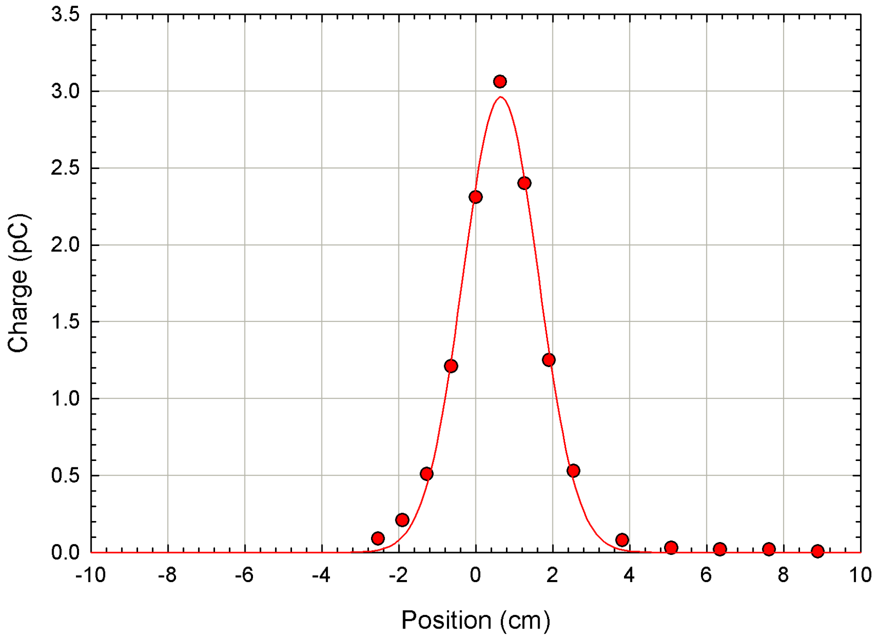
Electrode parameters:

- $V_0(t) = V_{0\ max} \sin(2p\, f\, t)$
- $V_{0\ max} = 387.5$ V
- $f = 90$ kHz

Ion source parameters:

- $V_{accel} = -183.3$ V
- $V_{decel} = -5.0$ V

Radial Profile of Trapped Ions



The charge $Q(r)$ is collected through the 1 cm aperture is averaged over 2000 plasma shots.

The density is calculated as,

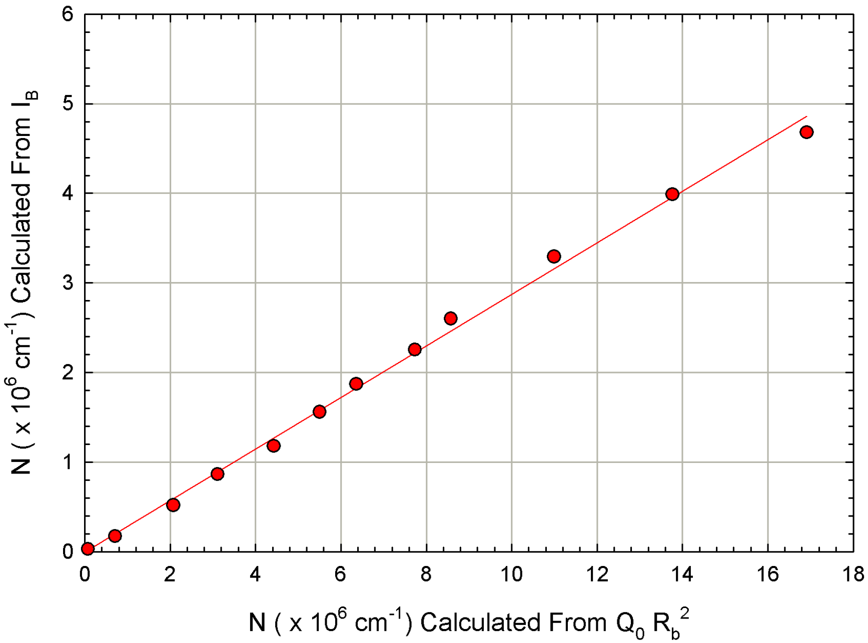
$$n(r) = \frac{Q(r)}{p r_{aperture}^2 l_{plasma}}$$

A Gaussian fit yields 3 parameters:

Q_0	amplitude
R_b	rms radius
r_0	offset

• $V_{emitter}$ = 3 V	• V_0 = 235 V	• t_{inject} = 5 ms
• V_{accel} = 2.x V	• f = 75 kHz	• t_{hold} = 1 ms
• V_{decel} = 2 V	• σ_v = 49°	• t_{dump} = 10 ms
• E_b = 3 eV	• $w_q = 6.5 \times 10^4\ s^{-1}$	• $n(r_0) = 1.2 \times 10^5\ cm^{-3}$
• I_b = 5 nA	• R_b = 1.4 cm	• s = 0.13

Comparison of Techniques for Measuring Line Density



$$0.8\,\text{nA} < I_b < 135\,\text{nA}$$

The line density $N_b = \int n(r) 2pr\, dr$ can be measured in two ways,

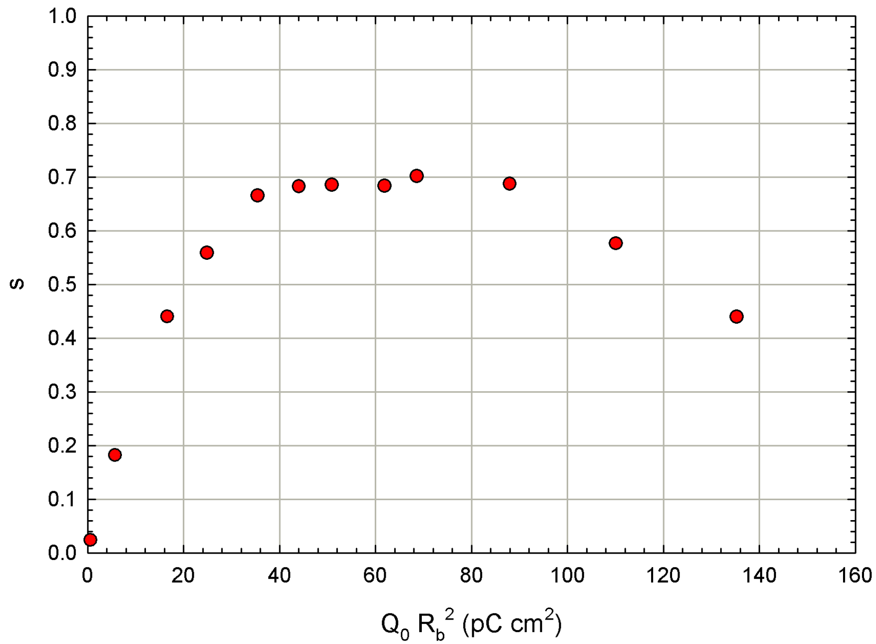
$$N_b = 5.2 \times 10^7 \frac{I_b(mA)}{\sqrt{E_b(\text{eV})}} \quad \text{cm}^{-1}$$

$$N_b = Q_0 p R_b^2 \frac{1}{p r_{aperture}^2 l_{plasma}}$$

l_{plasma} is taken to be 200 cm.

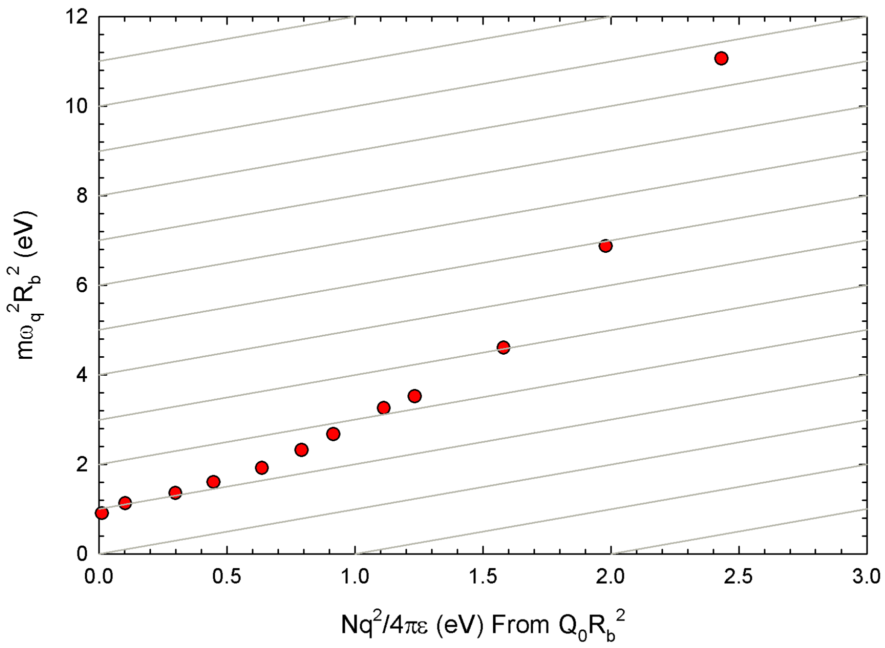
$r_{aperture} = 0.5$ cm.

Accessible Range of *s*



Varying I_b over the range 0.8 nA to 135 nA allows PTSX to trap plasmas with *s* up to 0.7.

Scaling of R_b^2 with Line Density

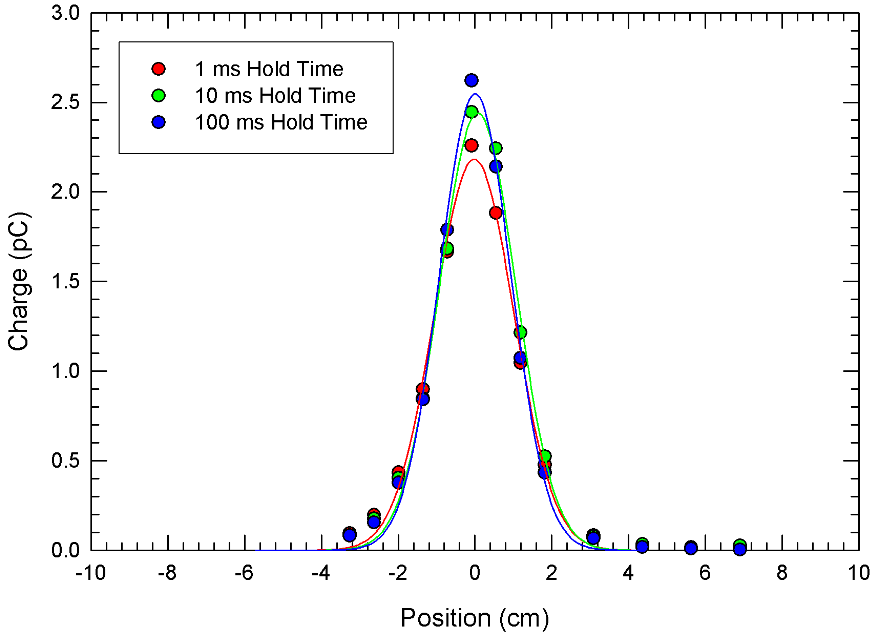


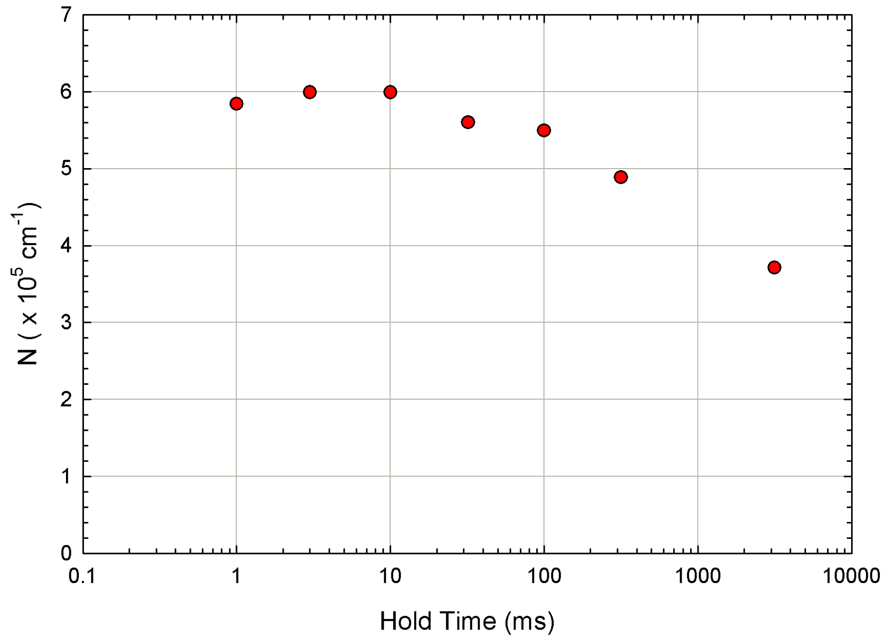
The equilibrium mean-squared-radius is determined by balancing the confining force against the thermal pressure and the space-charge force.

$$m w_q^2 R_b^2 = 2kT + \frac{N_b q^2}{4 p e_o}$$

At larger N, the increasing mismatch between the ion source and the trapped plasma likely leads to plasma heating. $kT = 0.5$ eV as N approaches zero.

Lifetime of Trapped Plasma

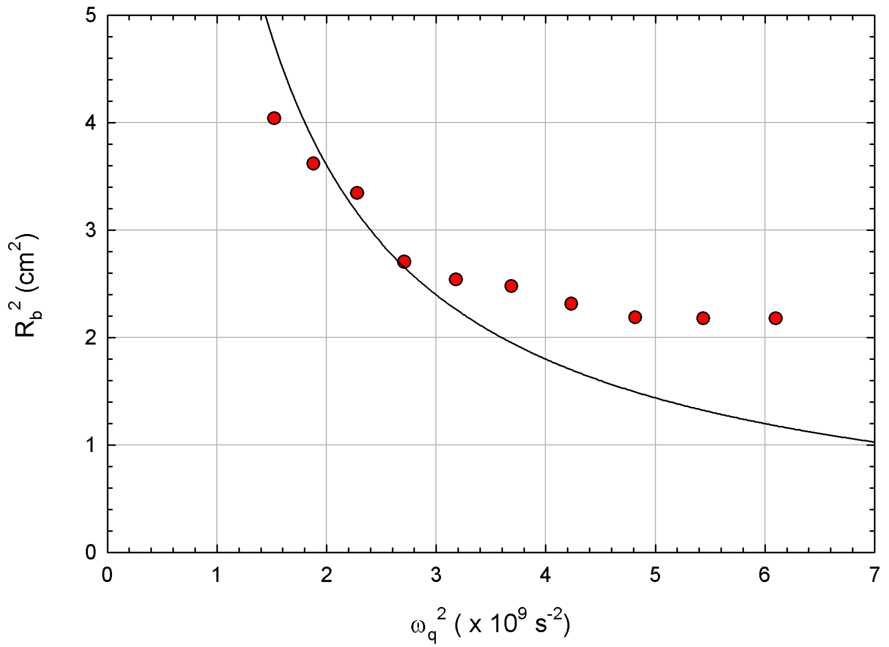




At *f* = 75 kHz, a lifetime of 100 ms corresponds to 7,500 periods.

If the spatial period of an alternating-gradient transport system is 1 m, the PTSX simulation would correspond to a 7.5 km beamline.

Dependence of R_b^2 on w_q^2



$$R_b^2 = \frac{2kT}{mw_q^2}$$ for an emittance-dominated beam.

R_b^2 saturates at ~ 2 cm^2, possibly because of the finite size of the ion source.

Conclusions

PTSX is a versatile facility in which to simulate collective processes and the transverse dynamics of intense charged particle beam propagation over large distances through an alternating-gradient magnetic quadrupole focusing system using a compact laboratory Paul trap.