Landau Damping and Anomalous Skin Effect in Low-pressure Gas Discharges: Self-consistent Treatment of collisionless Heating

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Motivation

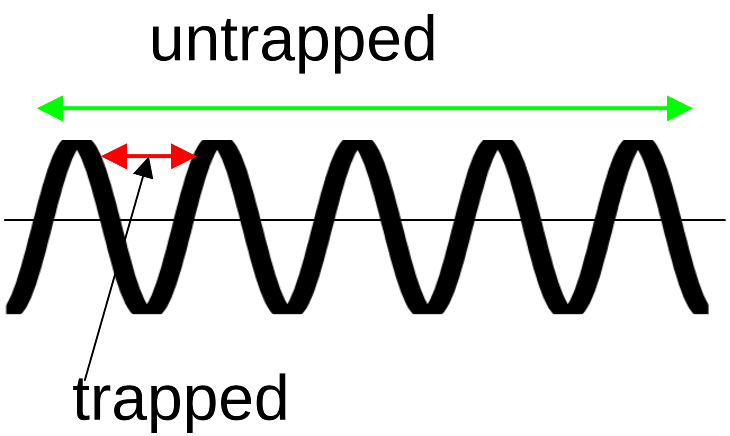
- Low pressure rf discharges routinely operate at conditions where collisionless heating dominates.
- A lot of diagnostics (n_e , E, B, φ, EDF).
- Self-consistent simulations are possible and needed.

Plasma Parameters

- Plasma density $n = 10^9 - 10^{13}$ cm^{-3}.
- Gas pressure = few mTorr.
- Small degree of ionization < 10^{-4}.
- Electron temperature T_e = few eV.
- Ion temperature T_i = 0.03 eV.
- Spatial scale = mm- m.

Nonlinear Landau Damping of a Single Wave

Nonlinear Oscillation in a Potential Well

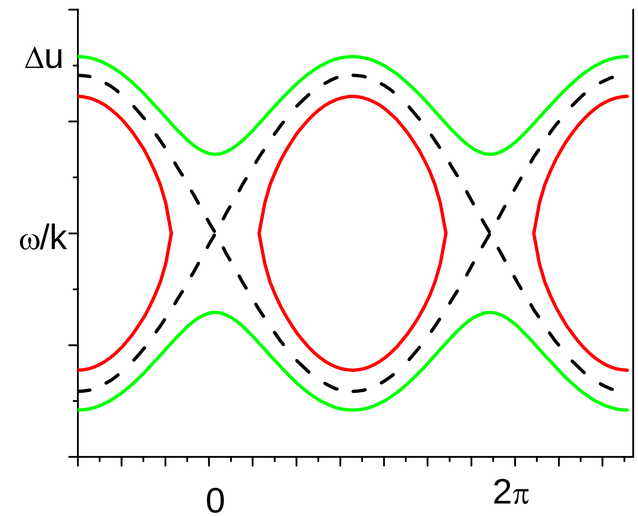


Resonance $u \simeq \omega / k$

$$H(u,x) = \frac{m}{2}\left(u - \frac{\omega}{k}\right)^2 - e\phi_0 \cos kx$$

$$\Delta u \equiv \sqrt{\frac{2e\phi_0}{m}}$$

$$\tau \equiv 1/k\Delta u$$

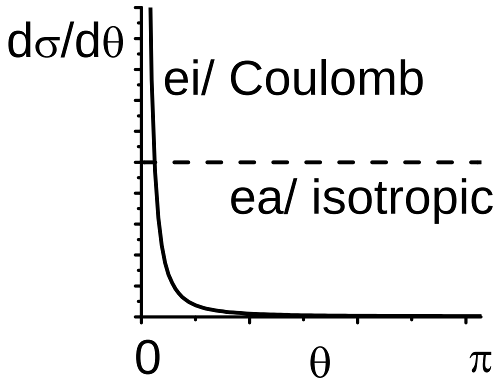


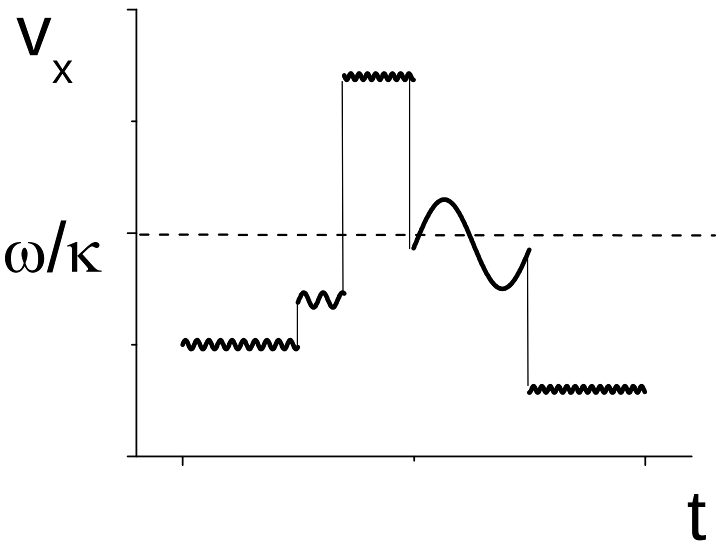
Linear theory if $\left|\omega - ku\right|\tau >> 1$

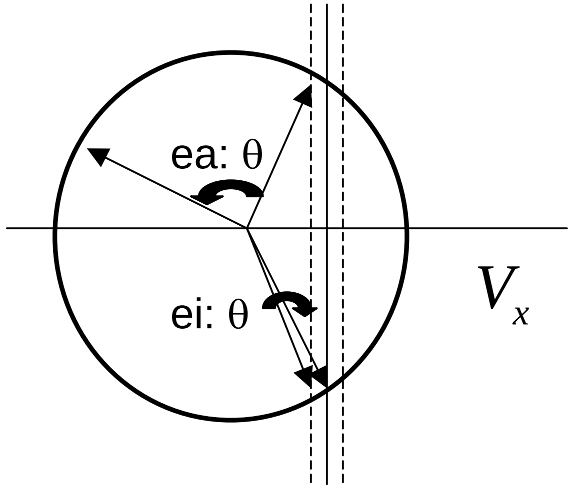
$$\tilde{u} = \frac{ek\phi_0}{m(\omega - ku)}\exp(ikx - i\omega t)$$

Oscillation velocity is limited ! $\left|\tilde{u}\right| < \Delta u$

Nonlinear Oscillation and Collisions

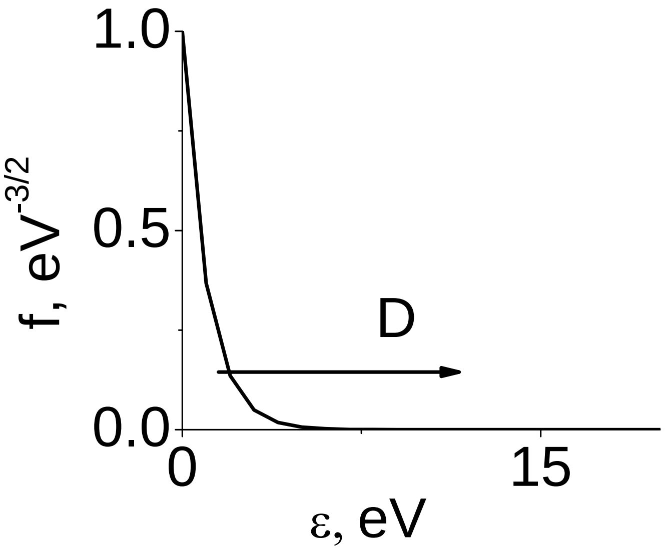


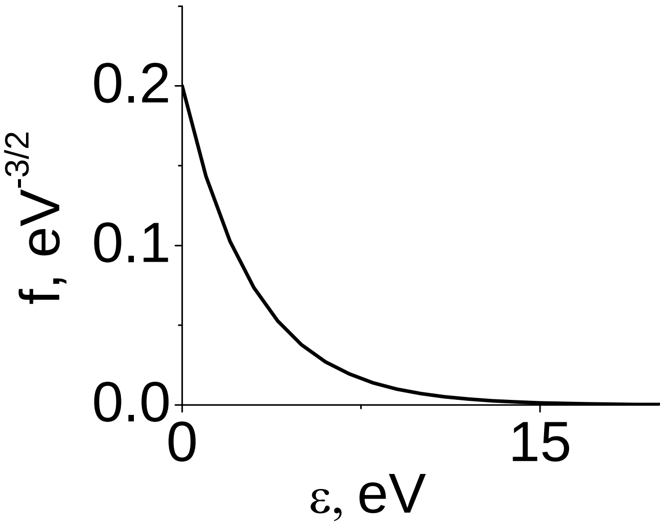




Electron dynamics is a combination of rare collisions and oscillations in the wave.

Diffusion in Energy Corresponds to Heating





$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\sqrt{\varepsilon}\partial\varepsilon}\left(\sqrt{\varepsilon}D_{\varepsilon}(\varepsilon)\frac{\partial f_0}{\partial\varepsilon}\right) \qquad P = -\int_0^{\infty}\sqrt{\varepsilon}D_{\varepsilon}(\varepsilon)\frac{\partial f_0}{\partial\varepsilon}d\varepsilon$$

Electron distribution function and absorbed power are functions of *D*

Diffusion in Energy Qualitative Discussion

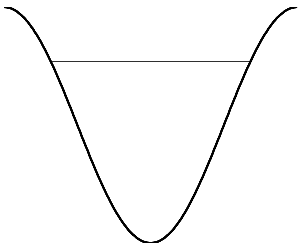
- Random collisions => diffusion in energy.
- D=frequency * (energy change)^2 averaged over velocity direction.

$$\Delta\varepsilon = mu\tilde{u} = \frac{euk\phi_0}{(\omega - ku)} \qquad D(\varepsilon) = \int_{-V}^{V} \frac{du}{V} \frac{\left(euk\phi_0\right)^2 \nu}{(\omega - ku)^2}$$

$$mV^2/2 = \varepsilon$$

Resonance residence time is limited by collisions or by bounce in the potential well $\dfrac{1}{|\omega - ku|} < \dfrac{1}{\nu}, \tau$

Diffusion in Energy Qualitative Discussion

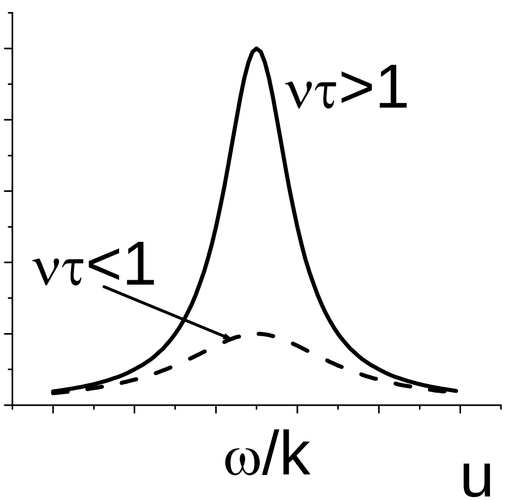


$$\Delta u \equiv \sqrt{\frac{2e\phi_0}{m}}$$

$$\tau \equiv 1/k\Delta u$$

$$\frac{1}{(\omega-ku)^2} \rightarrow \frac{1}{(\omega-ku)^2+\nu^2+\tau^{-2}}$$

$$D(\varepsilon)=\int_{-V}^{V}\frac{du}{V}\frac{\left(euk\phi_0\right)^2\nu}{(\omega-ku)^2+\nu^2+\tau^{-2}}$$



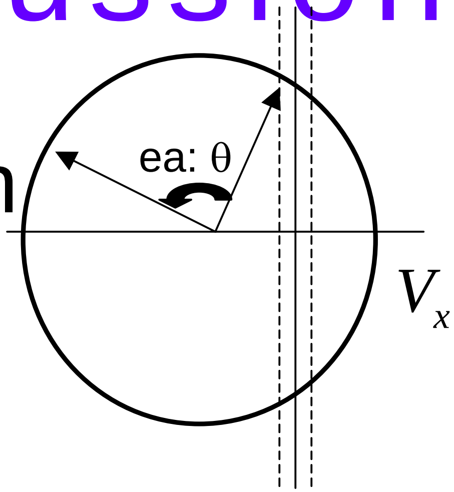
$$\frac{\nu}{(\omega-ku)^2+\nu^2}=\pi\delta(\omega-ku) \qquad \nu\tau >> 1$$

$$\frac{\nu}{(\omega-ku)^2+\tau^{-2}}=(\nu\tau)\pi\delta(\omega-ku) \qquad \nu\tau << 1$$

Collisionless heating depends on collisions!

Diffusion in Energy Quantitative Discussion

- Solving Boltzmann equation with collision and nonlinear bouncing

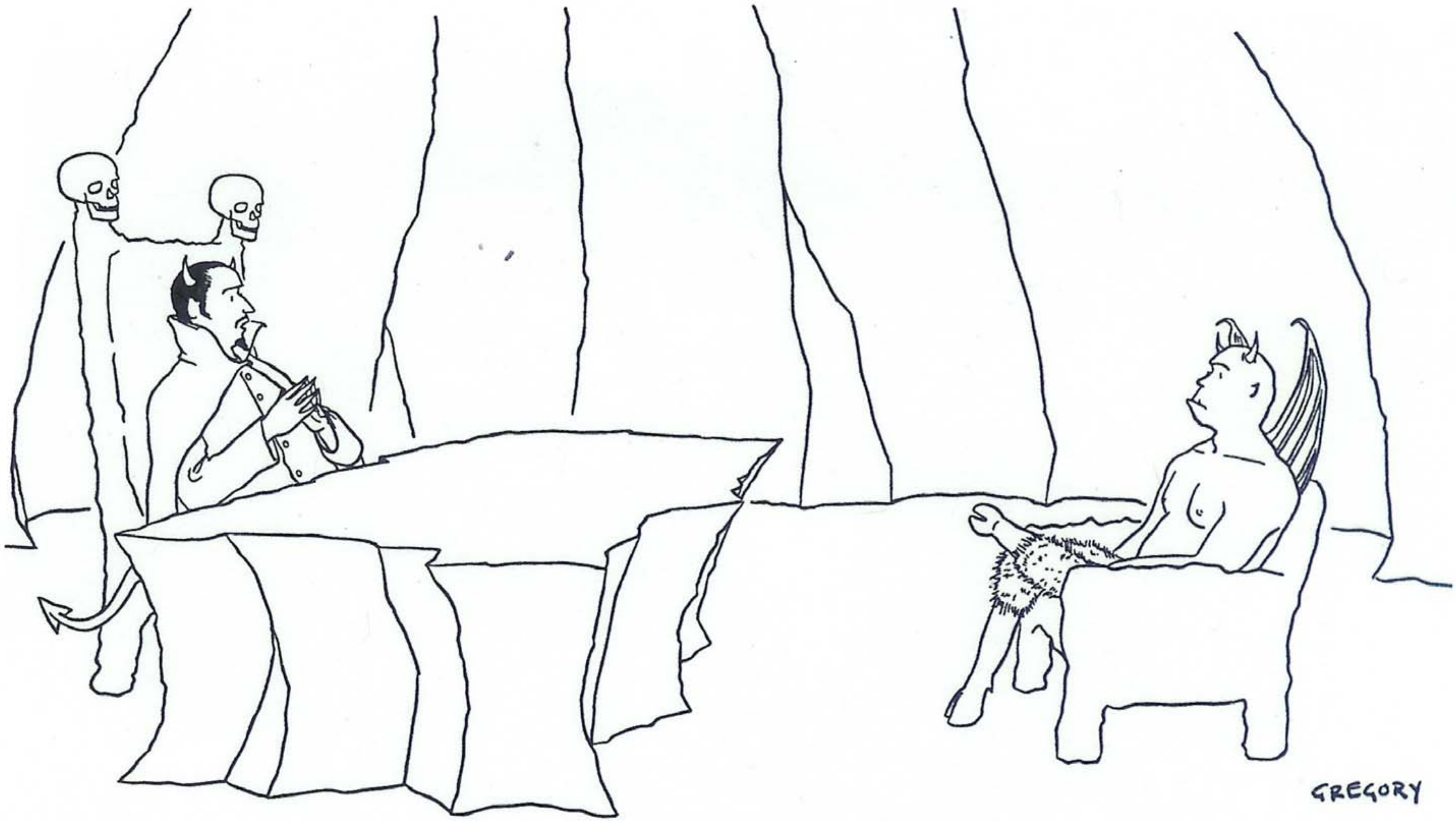


$$f = f_0(\varepsilon) + f_1 e^{-i\omega t}$$

$$-i\omega f_1 + v_x \frac{\partial f_1}{\partial x} - \frac{eE}{m}\frac{\partial f_1}{\partial v_x} = \frac{eEv_x}{m}\frac{\partial f_0}{\partial \varepsilon} - \nu f_1,$$

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\sqrt{\varepsilon}\partial \varepsilon}\left(\sqrt{\varepsilon} D_\varepsilon(\varepsilon)\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$D_\varepsilon = \frac{\nu}{2}\left\langle \int_0^\infty d\tau \nu e^{-\nu(t-\tau)}\left[\varepsilon(t) - \varepsilon(t-\tau)\right]^2 \right\rangle_\theta,$$

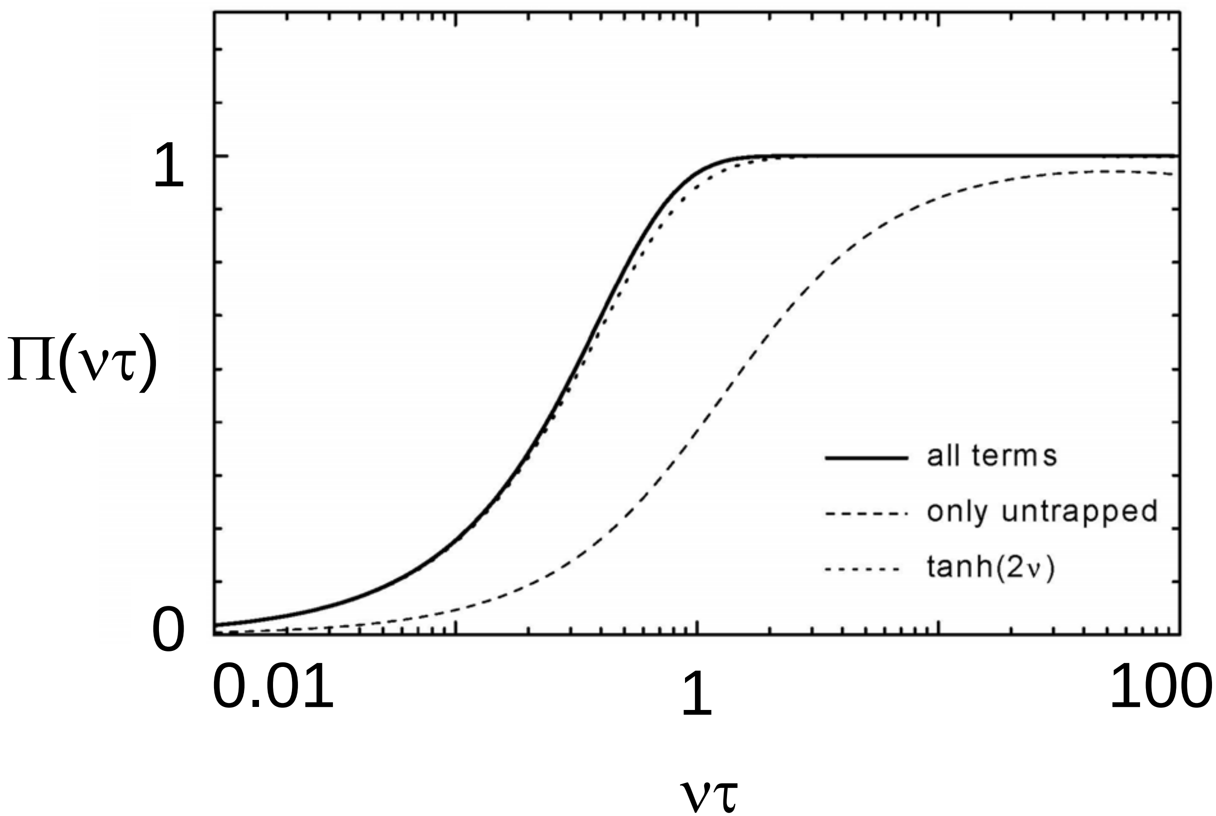


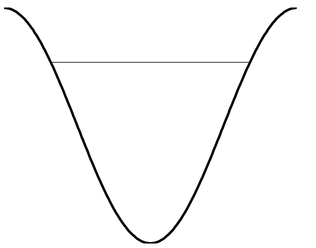
"I need someone well versed in the art of torture—do you know PowerPoint?"

Analytical Solution for Nonlinear Damping Decrement Accounting for Collisions

$$\gamma_{nl} = \gamma_L \Pi(\nu\tau)$$

I.Kaganovich, PRL 1999

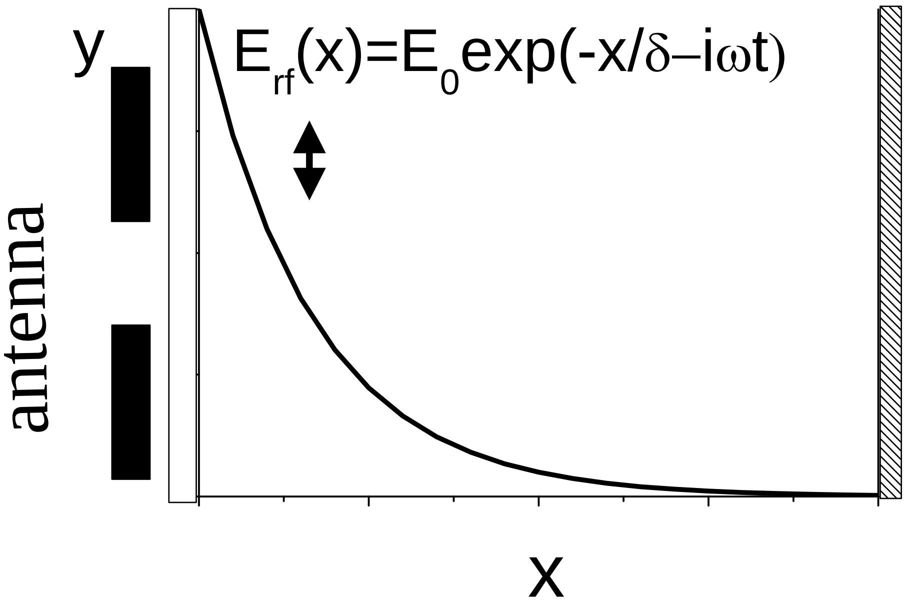




$$\Delta u \equiv \sqrt{\frac{2e\phi_0}{m}}$$

$$\tau \equiv 1/k\Delta u$$

Anomalous Skin Effect



$$\frac{dE}{dx}=\frac{i\omega}{c}B;\quad \frac{dB}{dx}=-\frac{4\pi}{c}j$$

$$\omega \ll \omega_p \qquad j=\frac{e^2nE}{-im\omega}$$

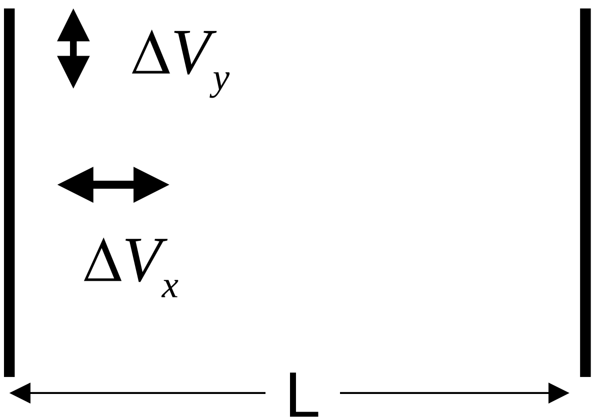
$$\delta=\frac{c}{\omega_p}$$ Normal skin effect

$$\frac{V_T}{\delta}\gg\omega \quad j_{anom}\ll\frac{e^2nE}{-im\omega} \quad \delta_{anom}\gg\frac{c}{\omega_p}$$ Anomalous skin effect

Linear theory $E_y \Rightarrow \Delta V_y$ ↕ nonlinear theory $E+[VB]\Rightarrow \Delta V_x$ ↔

$$mV_y-\frac{e}{c}A_y=0 \Rightarrow \Delta V_y=0$$

Collisionless Heating in Slab Geometry



bounce resonances

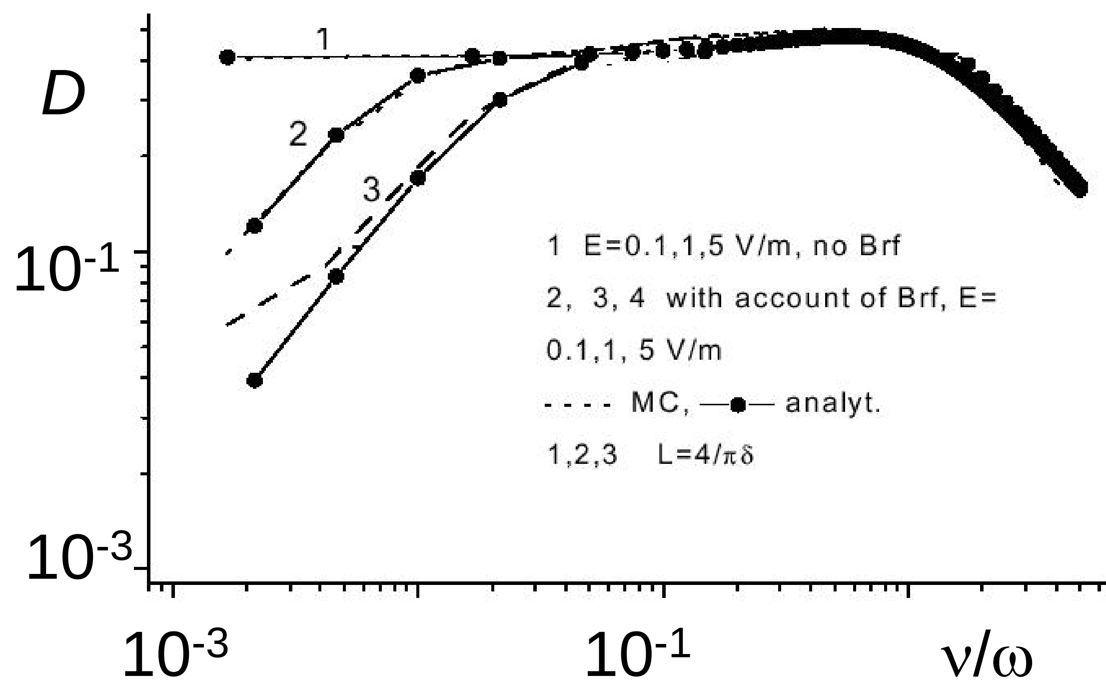
$$\omega T_b = \omega 2L / V_x = 2\pi n$$

ΔV_x Change resonance

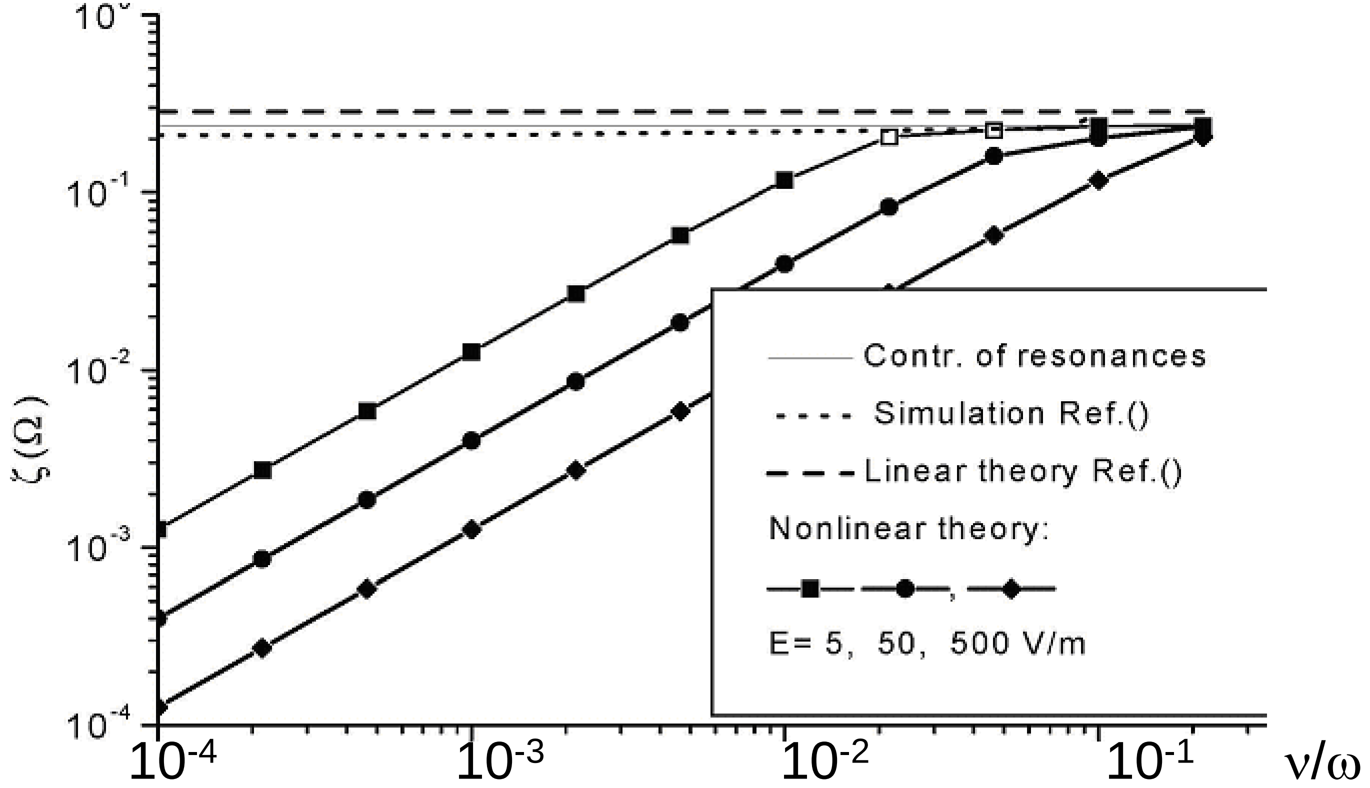
ΔV_y No change in resonance

$$MC : D = < \Delta\varepsilon^2 / \Delta t >$$

$$Theory : D = D_{ql} \tanh(\nu\tau)$$



Importance of Nonlinear Effects for Calculation of Surface Impedance



- The real part of surface impedance in ohm. The plasma parameters are $n=10^{11}$ cm^{-3}, T_e=5ev, l=4cm.

Conclusions

- The electron Boltzmann kinetic equation has been solved analytically for nonlinear Landau damping problem for any value of collision frequency.

  $\gamma_{nl} = \gamma_l \tanh(\nu\tau_r)$.

- The efficiency of the collisionless heating is described by the diffusion coefficient $D = D_{ql} \tanh(\nu\tau_r)$.

Self-consistent System of Equations for Kinetic Description of Low-pressure Discharges Accounting for nonlocal and collisionless Electron Dynamics

For more info:

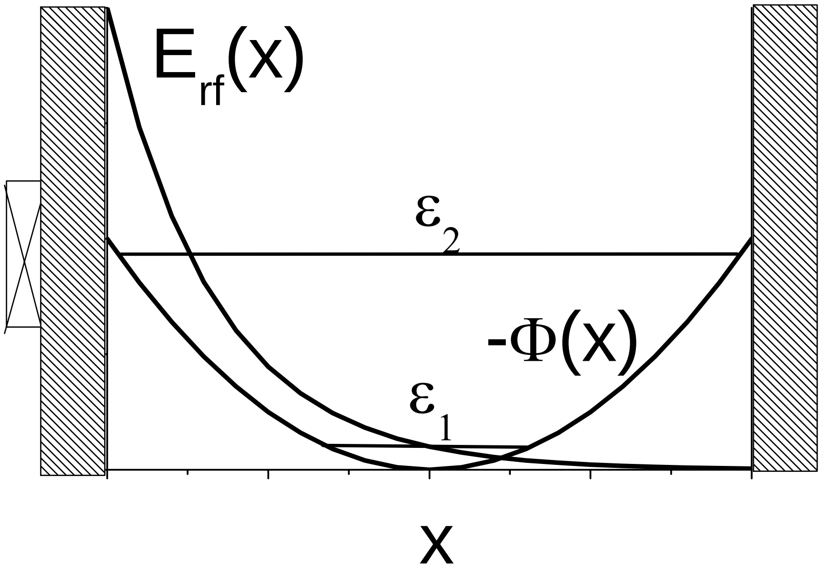
Phys. Rev.E **68**, 026411 (2003);

Plasma Sources Sci. Technol. **12**, 170 & 302 (2003),

Overview

- Calculate nonlocal conductivity in nonuniform plasma.
- Find a nonMaxwellian electron energy distribution function driven by collisionless heating of resonant electrons.
- What to expect: self-consistent system for kinetic treatment of collisionless and nonlocal phenomena in inductive discharge.

Inductive Discharge



The electron energy distribution is given by

$$-\frac{d}{d\varepsilon}D_{\varepsilon}\frac{df_0}{d\varepsilon}=S^*(f_0)\,,$$

The transverse rf electric field is given by

$$\frac{d^2E_y}{dx^2}+\frac{\omega^2}{c^2}E_y=-\frac{4\pi i\omega}{c^2}[j(x)+I\delta(x)]$$

Nonlocal Conductivity

$$J_y(x) = \frac{e^2 n_{e0}}{m}\left(\int_0^x G(x,x')E_y(x')dx' + \int_x^L G(x',x)E_y(x')dx'\right)$$

$$G(x',x) = 2\int_0^\infty \frac{\cosh(\Phi(x_1^*,x))\cosh(\Phi(x',x_2^*))}{\sinh(\Phi(x_1^*,x_2^*))}\frac{\Gamma(\varepsilon)}{\left(v^2+2e\left|\varphi(x)-\varphi(x')\right|/m\right)^{1/2}}dv,$$

$$\Phi(x_0,x) = \int_{x_0}^x \frac{i\omega+\nu}{\sqrt{2e(\varepsilon-\varphi(x'))/m}}dx', \quad \Gamma(\varepsilon) = \int_\varepsilon^\infty f_0(\varepsilon')d\varepsilon'.$$

Nonlocal conductivity is a function of the EEDF f_0 and the plasma potential $\varphi(x)$.

Energy Diffusion Coefficient

$$-\frac{d}{d\varepsilon}(D_\varepsilon+\overline{D_{ee}})\frac{df_0}{d\varepsilon}-\frac{d}{d\varepsilon}\overline{V_{ee}}f_0=\sum_k\left[\overline{\nu_k^*(u+\varepsilon_k^*)\frac{\sqrt{(u+\varepsilon_k^*)}}{\sqrt{u}}f_0(\varepsilon+\varepsilon_k^*)-\overline{\nu_k^*}f_0}\right],$$

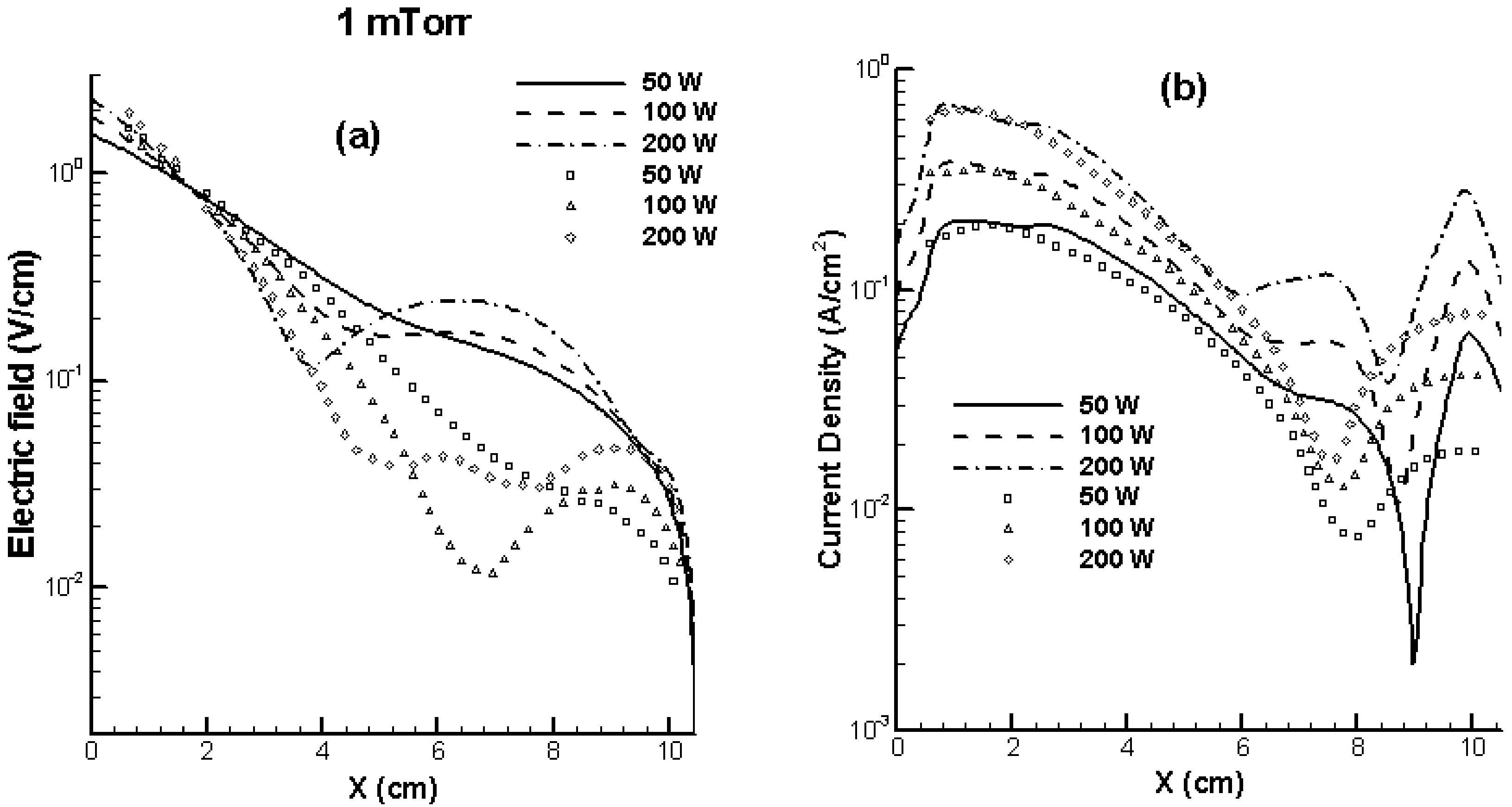
$$D_\varepsilon=\frac{\pi e^2}{4m^2}\sum_{n=-\infty}^{\infty}\int_0^\varepsilon d\varepsilon_x\left|E_{yn}(\varepsilon_x)\right|^2\frac{\varepsilon-\varepsilon_x}{\Omega_b(\varepsilon_x)}\frac{\nu}{[\Omega_b(\varepsilon_x)n-\omega]^2+\nu^2}$$

$$E_{yn}(\varepsilon_x)=\frac{\Omega_b(\varepsilon_x)}{\pi}\int_0^L\frac{E_y(x)\cos(n\theta(x))}{|v_x|}dx.$$

D_{ee}, V_{ee} are from the electron-electron collision integral, ν_k^* is inelastic collision frequency, upper bar denotes space averaging with constant energy.

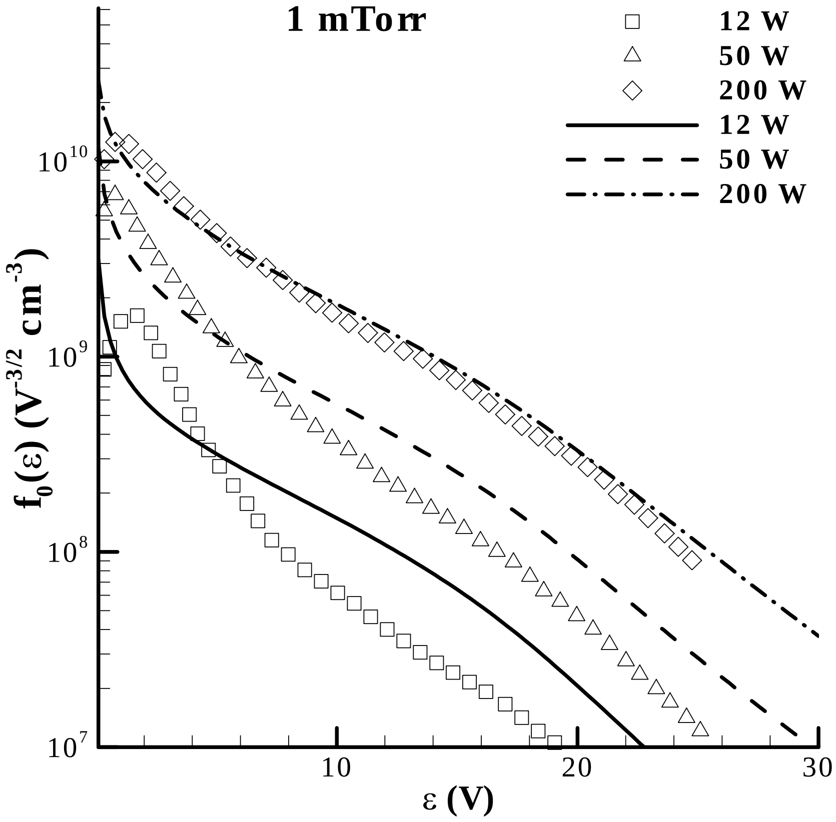
Energy diffusion coefficient is function of the rf electric field E_y and the plasma potential $\varphi(x)$.

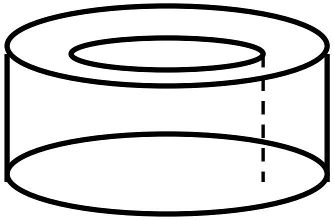
Comparison With Experiment



Comparison between experimental data [V. A. Godyak and R. B. Piejak, *J. Appl. Phys*. **82**, 5944 (1997).] and simulation predictions using a non-local model (a) RF electric field and (b) the current density profiles for a argon pressure of 1 mTorr.

Comparison With Experiment

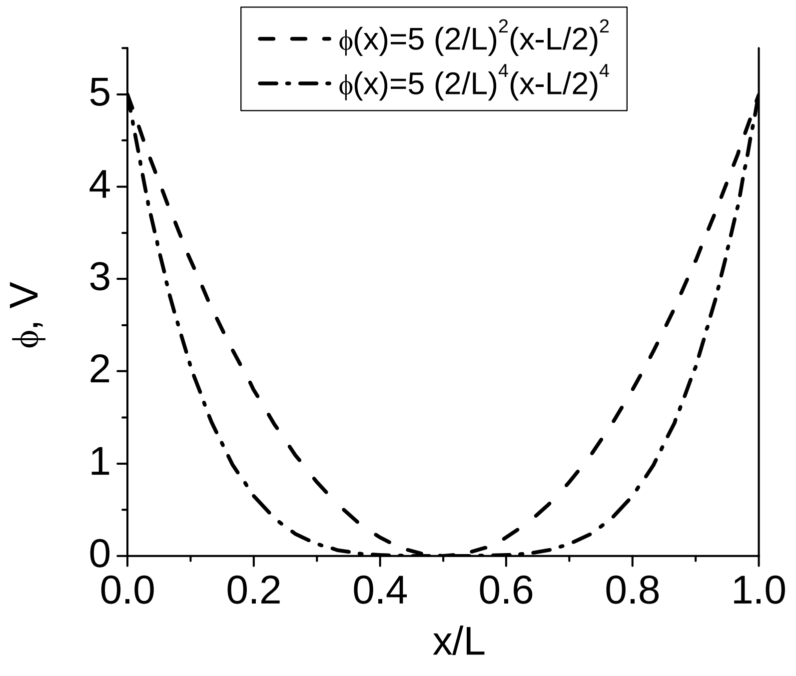


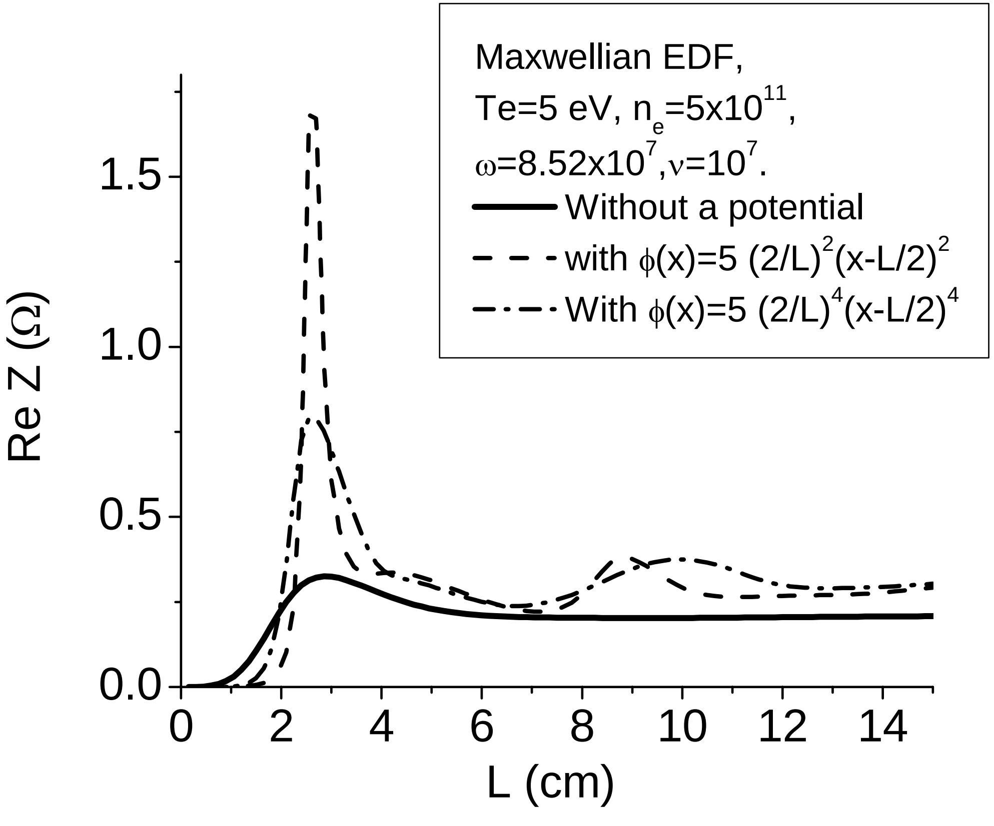


R=10cm,L=10cm, antenna R=4cm

Comparison between simulated (lines) and experimental (symbols) EEDFs for 1 mTorr. Data are taken from V. A. Godyak and V. I. Kolobov, *phys. Rev. Lett*., **81**, 369 (1998).

Influence of Plasma Potential on rf Heating



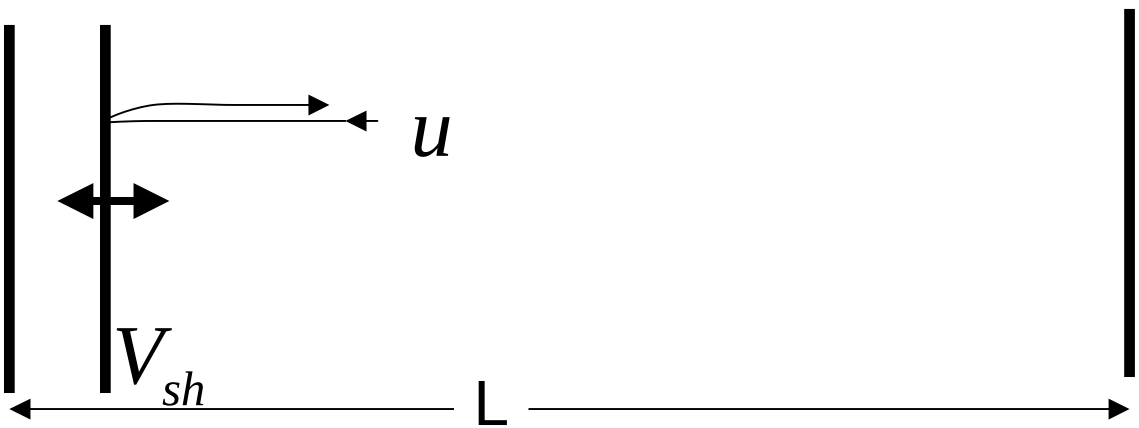


Surface impedance for different plasma profiles.

Conclusion

- The self-consistent system of equations is derived for description of collisionless heating and anomalous skin effect in nonuniform plasmas.
- The robust kinetic code was developed for fast modeling of discharges, which predicts nonMaxwellian electron energy distribution functions in rf discharges.

Stochastic Heating



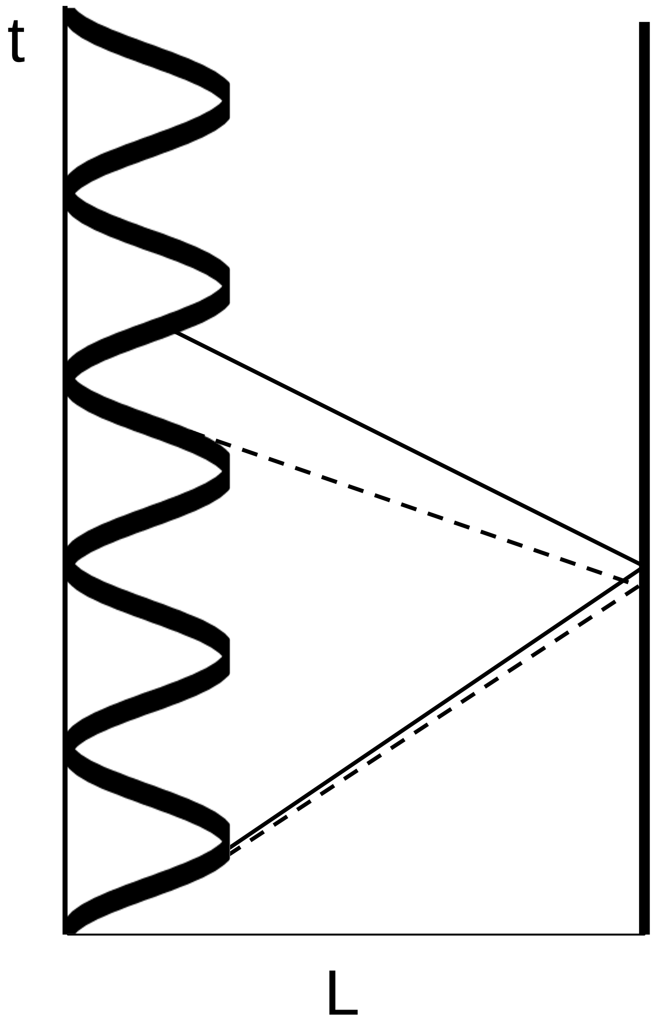
Thermodynamics: equilibrium with hot wall $T_w \to \infty, (M = \infty)$

$\Rightarrow$ Electrons are always heated

Necessary condition: subsequent collisions with sheath are random/ independent.

1. Dynamic chaos
2. Randomization due to collisions

Stochastic heating as Fermi acceleration



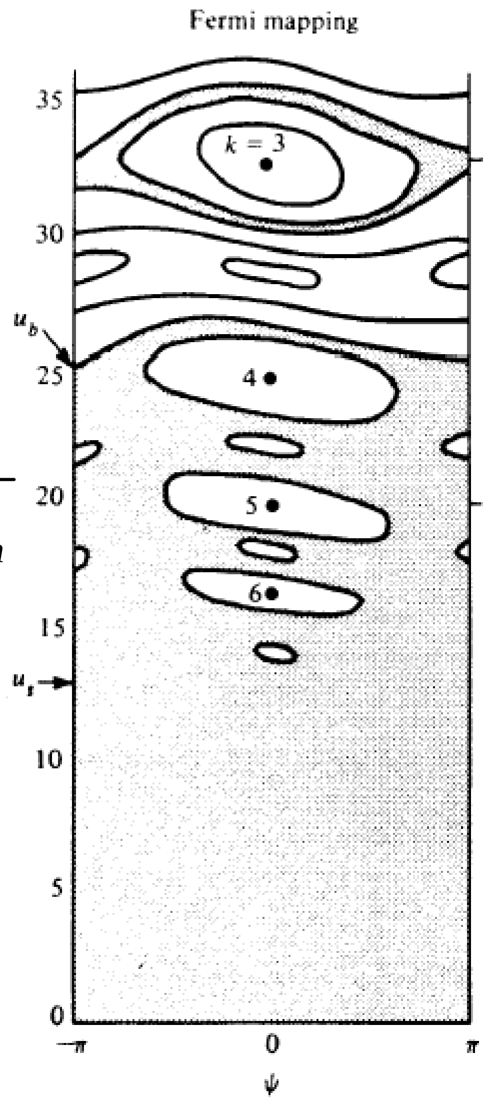
$$\omega >> u / 2L$$

Dynamic chaos: if phase change due to velocity kick >> 1

$$\Delta\phi = \Delta\omega\tau = \Delta\frac{\omega 2L}{u} = \frac{\omega 2L\Delta u}{u^2}$$

$$\omega > \frac{u}{2L}\frac{u}{\Delta u}$$

Fermi mapping



$$\frac{u}{2V_{sh}}$$

$$\omega 2L / u = 2\pi k$$

$$\Delta\phi(\Delta u) < 1$$

$$\Delta\phi(\Delta u) > 1$$

$$\omega L / V_{sh} = 200\pi$$

Resonances

k integer

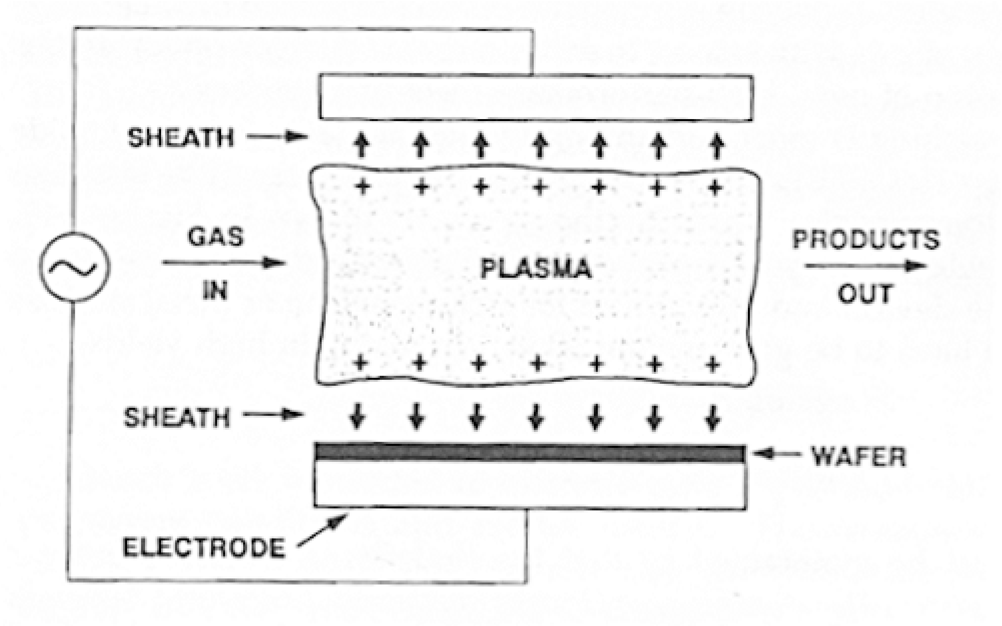
Regular motion
 k~1

Stochastic sea
 k>>1

Paradox

- For $\Delta\phi(\Delta u) < 1$ if there is no collisions there is no dissipation. However, collisionless heating exists.
- *Collisions* need to be accounted in *collisionless* heating!
- Role of collisions and non-linear effects as randomisation processes need to be investegated.

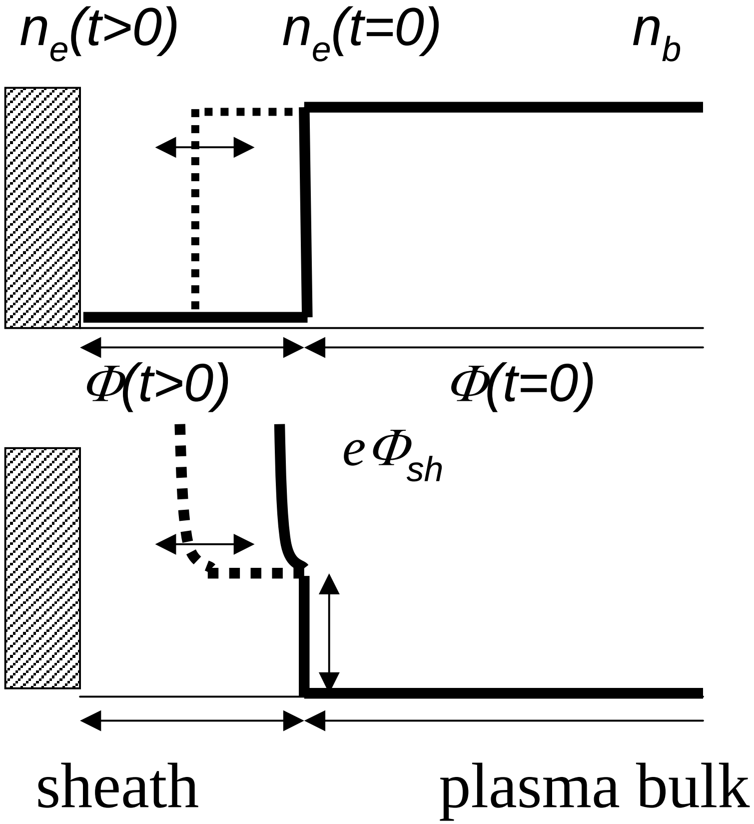
Anomalous Capacitive Sheath with Deep Radio Frequency Electric Field Penetration



Overview

- Quick overview of what this talk is all about
  - Influence of self-consistency and nonlocality on collisionless power deposition
  - What to expect: revision of previous test particle models

Schematic of the sheath

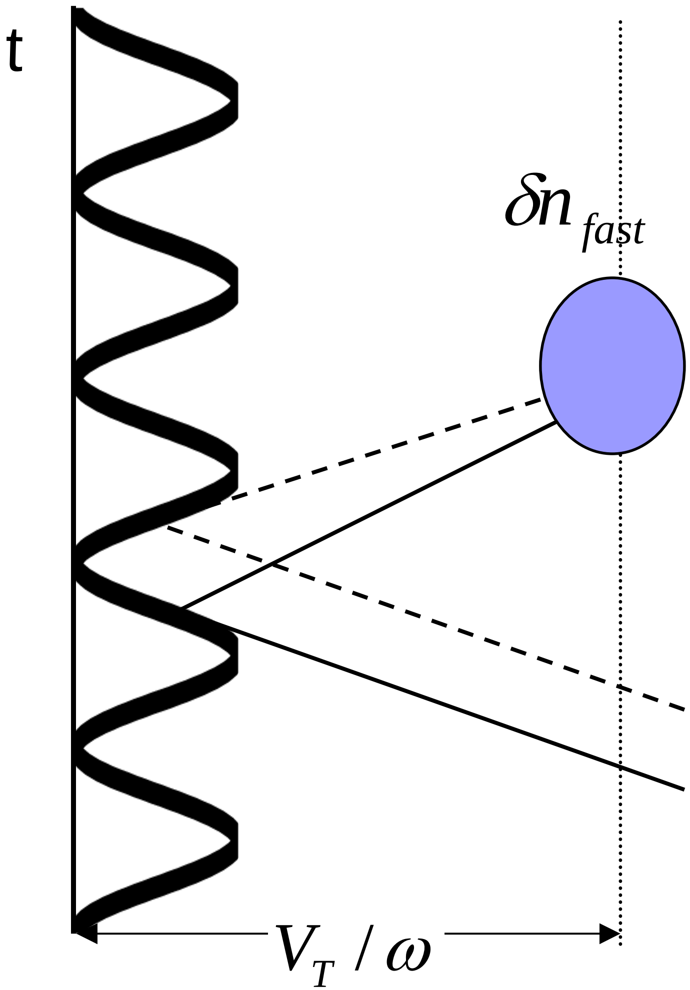


Schematic of a sheath. The negatively charged electrode pushes electrons away by different distances depending on the strength of the electric field at the electrode. Shown are the density and potential profiles at two different times. The solid line shows the maximum sheath expansion.

Objectives

- Design revised self-consistent kinetic theory of the capacitive sheath accounting for:
  - Perturbation of plasma near the sheath due to bunching in the sheath field.
  - Influence of the electric field in plasma on sheath heating

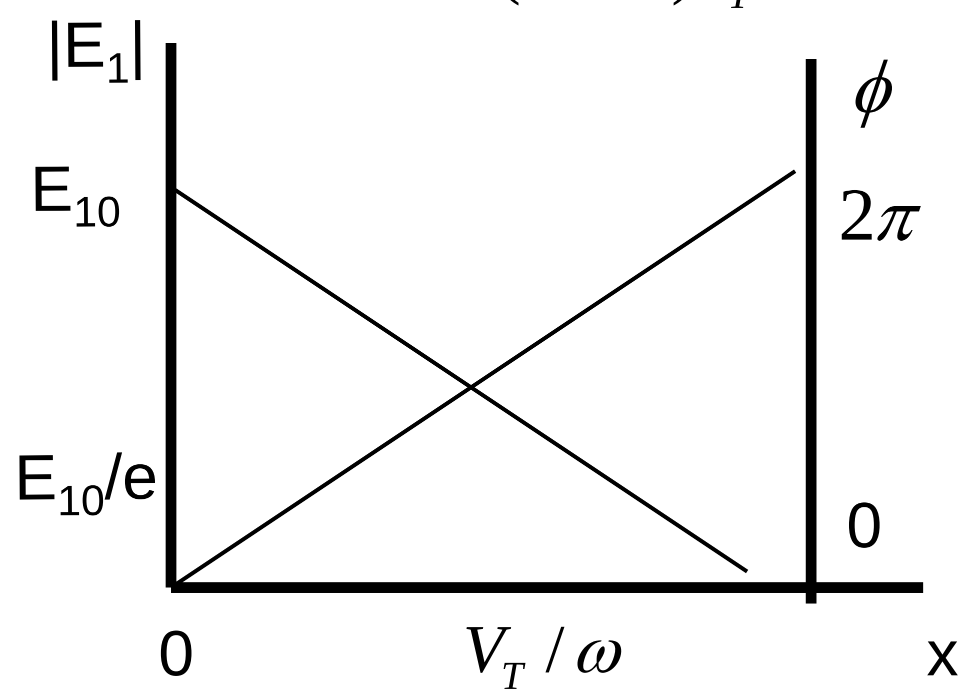
Electron density and electric field near the sheath



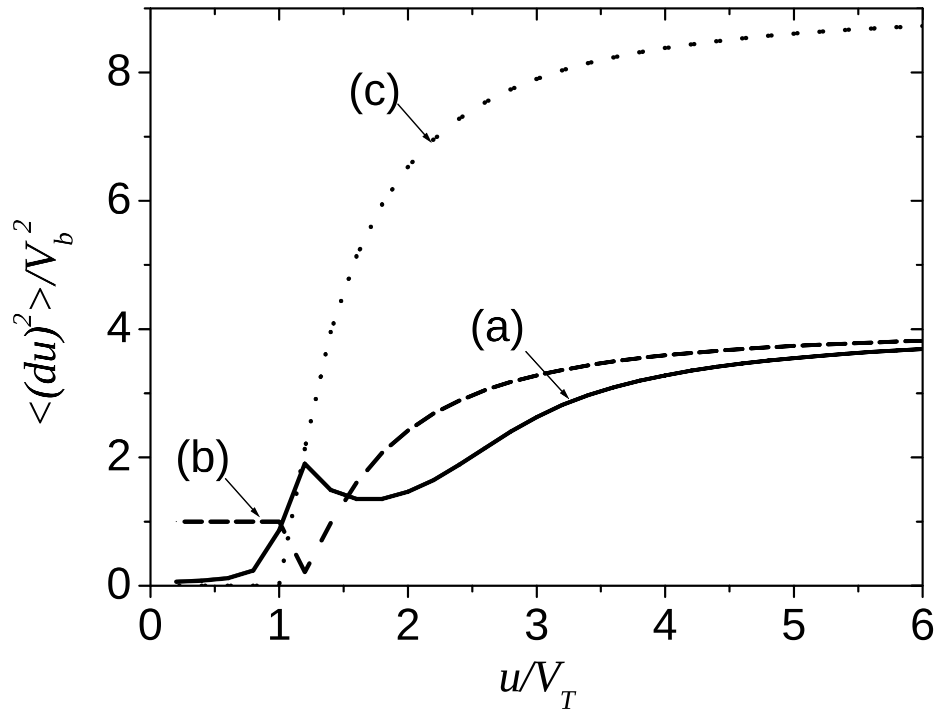
Fast electron bunches produce electric field E_1 on scale V_T / ω

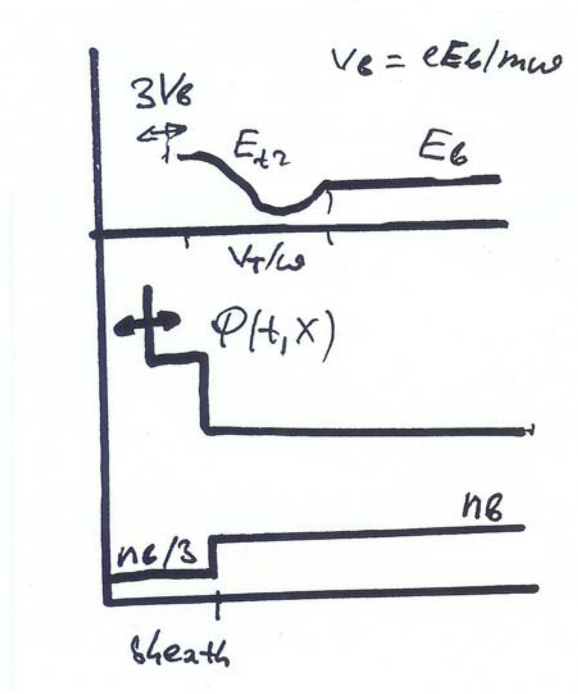
$$E_1 = E_{10} \exp(-x/\lambda)$$

$$\lambda = (a + ib) V_T / \omega$$



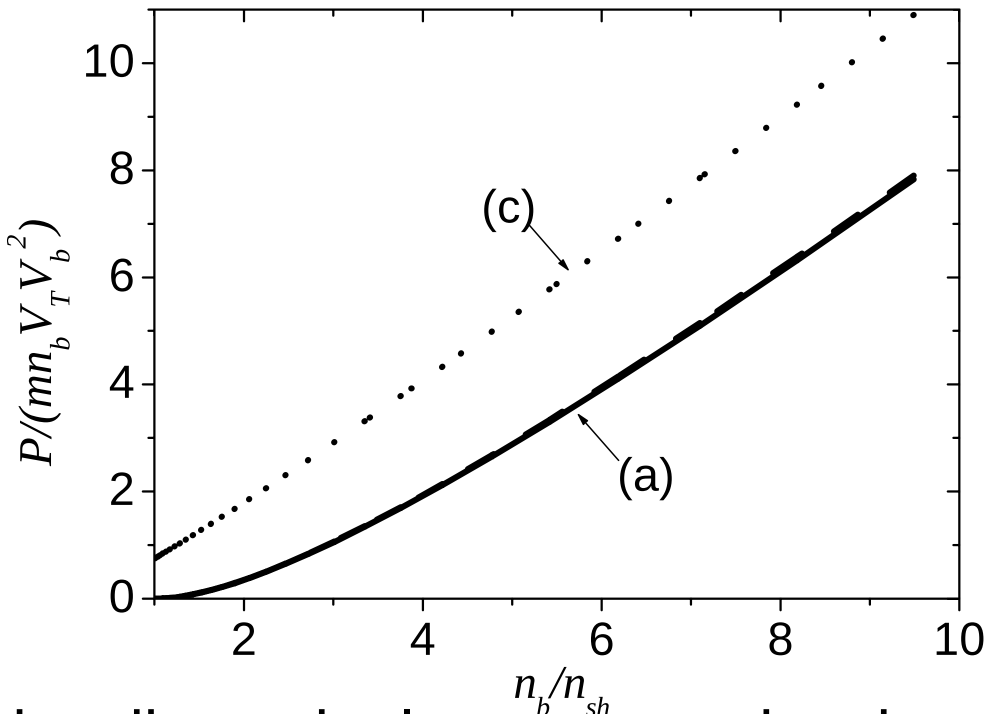
Change in velocity kick due electric field near the sheath





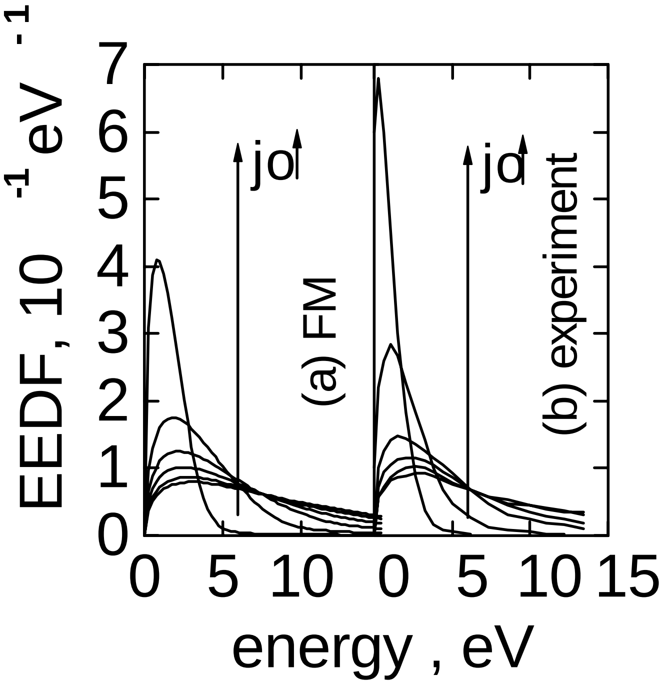
Plot of the average square of the dimensionless velocity kick as a function of the dimensionless velocity taking into account (a) both and - solid line; (b) only - dashed line; and (c) no electric field - dotted line.

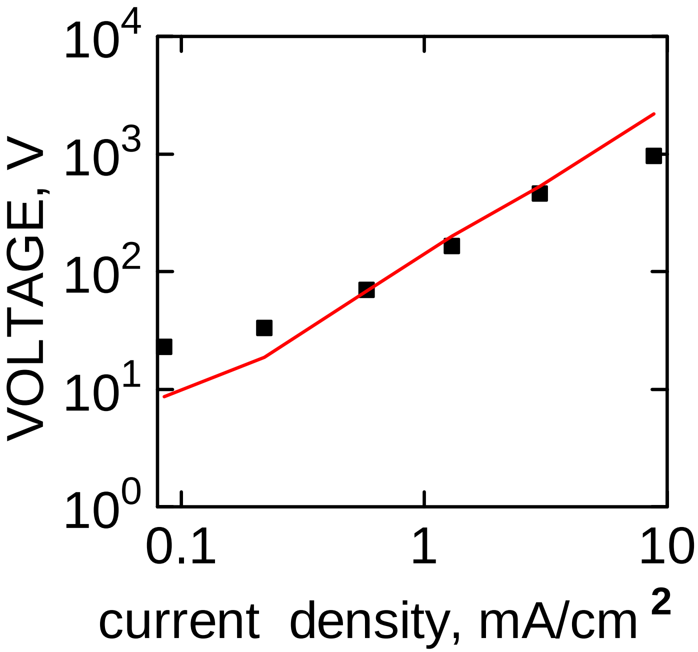
Effect of self consistency on power absorption



Plot of the dimensionless power density as a function of the ratio of the bulk plasma density to the sheath density, taking into account (a) self consistent treatment and (b) test particle model.

Comparison With Experiment for Capacitive Discharge





He, p=0.1 torr, ω=13.56 MHz, L=6.7 cm.
Experimental data - filled squares, the FM results - solid line j=0.085, 0.22, 0.58, 1.3, 6.0, 8.8ma/cm^2.

Conclusions

A novel nonlinear effect of anomalously deep penetration of an external radio frequency electric field into a plasma is described. A self-consistent kinetic treatment reveals a transition region between the sheath and the plasma. Because of the electron velocity modulation in the sheath, bunches in the energetic electron density are formed in the transition region adjacent to the sheath. The width of the region is of order V_T/ω, where V_T is the electron thermal velocity, and ω is frequency of the electric field. The presence of the electric field in the transition region results in a cooling of the energetic electrons and an additional heating of the cold electrons in comparison with the case when the transition region is neglected. Additional information on the subject is posted in

I. Kaganovich, PRL 2002

http://arxiv.org/abs/physics/0203042