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Drift compression and final focus options for heavy ion fusion

Hong Qin[a,*], Ronald C. Davidson[a], John J. Barnard[b], Edward P. Lee[c]

[a]Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543, USA
[b]Lawrence Livermore National Laboratory, Livermore, CA 94550, USA
[c]Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

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Abstract

A drift compression and final focus lattice for heavy ion beams should focus the entire beam pulse onto the same focal spot on the target. We show that this requirement implies that the drift compression design needs to satisfy a self-similar symmetry condition. For un-neutralized beams, the Lie symmetry group analysis is applied to the warm-fluid model to systematically derive the self-similar drift compression solutions. For neutralized beams, the 1D Vlasov equation is solved explicitly and families of self-similar drift compression solutions are constructed. To compensate for the deviation from the self-similar symmetry condition due to the transverse emittance, four time-dependent magnets are introduced in the upstream of the drift compression such that the entire beam pulse can be focused onto the same focal spot.

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1. Introduction

Recently, a warm fluid model has been developed to study the longitudinal dynamics of drift compression [1,2]. It was shown that a self-similar solution with parabolic density profile can be used for drift compression, and a pulse shaping technique has also been demonstrated so that any initial pulse shape can be shaped into a parabolic one which can then be self-similarly compressed. The effects of beam compression on the transverse dynamics were realized to be significant [2,3]. Because the space-charge force increases as the beam is compressed, a larger focusing force is needed to confine the beam in the transverse direction, and a non-periodic quadrupole lattice along the beam path is necessary. Another important issue is that the drift compression and final focus lattice should work for the entire beam pulse, which may have different perveance and emittance for different slices. Heavy

*Corresponding author.
E-mail address: hongqin@princeton.edu (H. Qin).

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ion fusion designs require that different slices are focused onto the same focal spot at the target. In this paper, we study these basic questions of drift compression and final focus [1–7]. In the transverse direction, a set of envelope equations is adopted. For the longitudinal dynamics, a set of warm fluid equations is used to model un-neutralized beam compression, and the 1D Vlasov equation is used for the neutralized beam compression.

This paper is organized as follows. In Section 2, starting from the basic equations and the requirement that the entire beam pulse is focused onto the same focal spot at the target, we show that it is necessary to develop a self-similar drift compression scheme in the longitudinal direction. The self-similar drift compression solutions are derived in Section 3 for un-neutralized beams by applying the Lie group symmetry analysis to the warm fluid equations. In Section 4, the 1D Vlasov equation is solved explicitly to construct self-similar drift compression solutions for neutralized beams. In Section 5, we demonstrate that the entire beam pulse can be compressed and focused onto the same focal spot at the target with the help of several time-dependent magnets at the upstream of drift compression when there is a small deviation from the perfect self-similar symmetry due to the transverse emittance.

2. Self-similar drift compression

For each slice in a bunched beam, the transverse dynamics in a quadrupole lattice is described approximately by the transverse envelope equations:

$$\frac{\partial^2 a}{\partial s^2} + \kappa_q a - \frac{2K}{a+b} - \frac{\varepsilon_x^2}{a^3} = 0 \tag{1}$$

$$\frac{\partial^2 b}{\partial s^2} - \kappa_q b - \frac{2K}{a+b} - \frac{\varepsilon_y^2}{b} = 0 \tag{2}$$

where a, b, K, ε_x, and ε_y are functions of s and Z. Here, Z is the longitudinal coordinate for different slices, and it enters the equations only parametrically. Note that Z is not in general the Cartesian coordinate in the z direction. A suitable choice for Z is the Lagrangian coordinate in the z direction. In the definition of $K(s,Z)$, the quantity $\lambda(s,Z)$ is the line density determined by the longitudinal dynamics, whose governing equations will be studied in Sections 3 and 4. It is not difficult to design a drift compression and final focus lattice to focus one slice of the beam onto the target. However, for different slices the line density and emittance may depend on s in different manners. A lattice design for one slice may not be able to transversely confine other beam slices and focus them onto the same focal spot at the target. Actually, most of the other slices cannot be focused at all due to the mismatch induced by the different s-dependences of the current and emittance. A fixed drift compression and final focus lattice will be able to focus the entire beam pulse onto the same focal spot only if the current and emittance of all the slices depend on s in the same manner. Using the language of symmetry group theory, this condition is equivalent to the statement that $a, b, \lambda, \varepsilon_x$, and ε_y for different Z are generated by the same solution of Eqs. (1) and (2) for one particular value of Z through a one-parameter group transformation admitted by Eqs. (1) and (2), i.e.,

$$\{a, b, \lambda, \varepsilon_x, \varepsilon_y\}[s, Z(\delta=0)] \longrightarrow \{a, b, \lambda, \varepsilon_x, \varepsilon_y\}[s, Z(\delta)]. \tag{3}$$

Here, δ is the parameter characterizing the 1D group transformation. For example, it is easy to check that the following scaling group

$$\begin{pmatrix} \{a,b\}[s, Z(\delta)] \\ \{\lambda, \varepsilon_x, \varepsilon_y\}[s, Z(\delta)] \end{pmatrix} = \begin{pmatrix} \delta\{a,b\}[s, Z(0)] \\ \delta^2\{\lambda, \varepsilon_x, \varepsilon_y\}[s, Z(0)] \end{pmatrix} \tag{4}$$

is a symmetry group of Eqs. (1) and (2). In other words, starting from a matched and focused solution for one slice, we obtain a family of matched and focused solutions for different slices if we scale up a and b by a factor of δ, and $\lambda, \varepsilon_x, \varepsilon_x$ by a factor of δ^2. We call such solutions self-similar because every field quantity for different slices has the same s-dependence. For example, the ratio of line density between different slices is s

independent,

$$\frac{\lambda[s, Z(\delta')]}{\lambda[s, Z(\delta)]} = \left(\frac{\delta'}{\delta}\right)^2. \tag{5}$$

Furthermore, because s is conserved by the one-parameter group transformation, it can be demonstrated [8] that the s-dependence and the Z-dependence of $\lambda(s, Z)$ are separable $\lambda(s, Z) = \lambda_b(s)h(Z)$. Since the line density during drift compression is determined by the longitudinal dynamics, the first step in designing a drift compression and final focus system is to find the self-similar drift compression solutions in the longitudinal direction. The functions $\lambda_b(s)$ and $h(Z)$will be determined from the symmetry groups of the governing equations for the longitudinal dynamics.

3. Self-similar drift compression for un-neutralized beams

We adopt a one-dimensional warm-fluid model [1,2] to describe the longitudinal dynamics of drift compression for un-neutralized beams. In the beam frame, the warm-fluid equations for the line density $\lambda(t, z)$, longitudinal flow velocity $u_z(t, z)$, and longitudinal pressure $p_z(t, z)$ are given by

$$\frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda u_z) = 0 \tag{6}$$

$$\frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial z} + \frac{e^2 g}{m\gamma^5} \frac{\partial \lambda}{\partial z} + \frac{\kappa_z z}{m\gamma^3} + \frac{r_b^2}{m\gamma^3 \lambda} \frac{\partial p_z}{\partial z} = 0 \tag{7}$$

$$\frac{\partial p_z}{\partial t} + u_z \frac{\partial p_z}{\partial z} + 3p_z \frac{\partial u_z}{\partial z} = 0 \tag{8}$$

where the g-factor model for the longitudinal electric field is used with $eE_z = -(ge^2/\gamma^2)\partial\lambda/\partial z$ and $g = 2\ln(r_w/r_b)$. Here, e is the charge, r_w is the wall radius, and r_b is the beam radius. We treat g and r_b as constants for present purposes. In the space-charge-dominated regime, the g-factor model adopted here is consistent with the result recently derived by Davidson and Startsev [9]. We also allow for an externally applied focusing force $F_z = -\kappa_z z$.

As discussed in Section 2, in order to focus the entire beam pulse onto the same focal spot on the target, it is necessary to find a self-similar drift compression solution admitted by the nonlinear hyperbolic partial differential equation (PDE) system (6)–(8). The systematic method for finding similarity solutions (group-invariant solutions) for PDEs is the Lie group symmetry analysis, which will be applied here. Two types of point symmetries can be used to generate similar solutions for PDEs. The first type is classical point symmetry, which transfers a solution of the PDEs into another solution. The second type is non-classical point symmetry, under which a solution is invariant. In general, the symmetry groups of both types are determined by the corresponding infinitesimal generators. The determining equations for the classical point symmetry are linear and algorithmically solvable, and the infinitesimal generators form a Lie algebra; on the other hand, the determining equations for the non-classical point symmetry are nonlinear and not algorithmically solvable, and the infinitesimal generators do not form a Lie algebra. Once the symmetry groups are found, the similarity solutions can be derived in a straightforward manner [8,10].

With the help of the symbolic computation tool Mathematica and the MathLie package [11], the infinitesimal generators of the classical point symmetry of the PDE system (6)–(8) are found to be a 4D Lie algebra

$$\begin{aligned}
&\frac{d\lambda}{d\delta} = 2k_2\lambda, \quad \frac{dp_z}{d\delta} = 4k_2 p_z, \quad \frac{dt}{d\delta} = k_1 \\
&\frac{du_z}{d\delta} = k_2 u_z + k_4 \cos(t\sqrt{\kappa}) + k_3 \sin(t\sqrt{\kappa}) \\
&\frac{dz}{d\delta} = k_2 z - k_3 \cos(t\sqrt{\kappa})/\sqrt{\kappa} + k_4 \sin(t\sqrt{\kappa})/\sqrt{\kappa}.
\end{aligned} \tag{9}$$

Here, $\kappa = \kappa_z/m\gamma^3$, δ is the parameter characterizing the Lie symmetry group, and k_i $(i = 1, 2, 3, 4)$ are arbitrary real constants. For every set of choices of k_i, the PDE system reduces to an ordinary differential equation (ODE) system, and there is a similarity solution. The self-similar solution defined in Section 2 requires that t is an invariant of the symmetry group transformation, and therefore corresponds to $k_1 = 0$. Thus, the self-similar solutions generated by the classical point symmetry form a 3D vector space. Some of the self-similar solutions are the desired self-similar drift compression solutions. As an example, let us consider the case of $k_1 = k_2 = 0, k_3 = \sin\alpha$, and $k_4 = \cos\alpha$. The reduced ODE system can be easily integrated, and the solutions are found to be

$$\begin{aligned}
\lambda(t,z) &= \lambda_0 \frac{\cos\alpha}{\cos(\alpha + t\sqrt{\kappa})}\\
u_z(t,z) &= -z\frac{z_b'(t)}{z_b(t)} = -z\sqrt{\kappa}\tan(\alpha + t\sqrt{\kappa})\\
p_z(t,z) &= p_{z0}\frac{\cos^3\alpha}{\cos^3(\alpha + t\sqrt{\kappa})}\\
z_b(t) &= z_{b0}\frac{\cos(\alpha + t\sqrt{\kappa})}{\cos\alpha}.
\end{aligned} \tag{10}$$

The velocity tilt is a linear function of z, and the density and pressure profiles are flat. The maximum compression ratio is

$$\frac{\lambda_f}{\lambda_0} = \frac{\cos\alpha}{\cos(\alpha + t_f\sqrt{\kappa})}. \tag{11}$$

For a given compression ratio and a maximum value of tolerable velocity tilt, we can always choose appropriate values for κ, α, and t_f to achieve the goal. In this case, the compression is achieved by both the external bunching force and the velocity tilt.

For the non-classical point symmetry group, the determining equations are nonlinear and difficult to solve for general solutions. Here, we list three non-classical infinite small generators without derivation.

Case (1) corresponds to a self-similar drift compression solution with flat-top density and pressure profiles and a linear velocity tilt. Its infinitesimal generator is

$$\frac{d}{d\delta}(\lambda, u_z, p_z, t, z) = \left(0, \frac{u_z}{z}, 0, 0, 1\right). \tag{12}$$

The self-similar solution is the same as Eq. (10), even though the symmetry group is not a classical one.

Case (2) corresponds to a self-similar drift compression solution with a flat-top density profile, a parabolic pressure profile, and a linear velocity tilt [1]. Its infinitesimal generator is

$$\frac{d}{d\delta}(\lambda, u_z, p_z, t, z) = \left(0, \frac{u_z}{z}, \frac{2p_z}{z}, 0, 1\right). \tag{13}$$

Case (3) corresponds to a self-similar drift compression solution with a parabolic density profile, double-parabolic pressure profile, and a linear velocity tilt [1]. Its infinitesimal generator is

$$\begin{aligned}
\frac{d}{d\delta}(\lambda, u_z, p_z, t, z) = \Bigg(&-\frac{2\lambda}{z_b^2(t) - z^2}, \frac{u_z}{z},\\
&-\frac{4p_z}{z_b^2(t) - z^2}, 0, 1\Bigg).
\end{aligned} \tag{14}$$

In the present analysis, we will focus on Case (3) only. It is easy to show that $t, \lambda/[1 - z^2/z_b^2(t)], u_z/z$, and $p_z/[1 - z^2/z_b^2(t)]^2$ are the invariants of the group transformation. It can then be demonstrated [8] that $\lambda_b(t) = \lambda/[1 - z^2/z_b^2(t)], v_b(t) = -u_z/z$, and $p_{zb}(t) = p_z/[1 - z^2/z_b^2(t)]^2$ are functions of t only. Substituting the definitions of $\lambda_b(t)$ and $p_{zb}(t)$ into Eqs. (6) and (8), we find that the z-dependence drops out, and

$$\frac{d\lambda_b}{dt} - \frac{v_{zb}}{z_b}\lambda_b = 0 \tag{15}$$

$$\frac{dp_{zb}}{dt} - 3\frac{v_{zb}}{z_b}p_{zb} = 0. \tag{16}$$

When the equation

$$\frac{dz_b}{dt} + v_{zb} = 0 \tag{17}$$

is satisfied, the z-dependence also drops out of the momentum equation (7), giving

$$-\frac{dv_{zb}}{dt} - \frac{e^2 g}{m\gamma^5}\frac{2\lambda_b}{z_b} + \frac{\kappa_z z_b}{m\gamma^3} - \frac{4r_b^2 p_{zb}}{m\gamma^3\lambda_b z_b} = 0. \tag{18}$$

Eqs. (15)–(18) form a coupled ODE system. Remarkably, these equations recover the longitudinal envelope equation

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2c^2}z_b - \frac{K_l}{z_b^2} - \frac{\varepsilon_l^2}{z_b^3} = 0 \tag{19}$$

where $s = \beta ct, K_l \equiv 3N_b e^2 g/2m\gamma^5\beta^2c^2$ is the effective longitudinal self-field perveance, and $\varepsilon_l \equiv (4r_b^2 z_b^3 p_{zb}/m\gamma^3\beta^2c^2N_b)^{1/2}$ is the longitudinal emittance for the parabolic self-similar drift compression solution. The solutions for $\lambda_b(t)$, $z_b(t)$, and $p_{zb}(t)$ can be obtained by solving the longitudinal envelope equation (19) numerically.

4. Self-similar drift compression for neutralized beams

In this section, self-similar drift compression for neutralized beams is studied using the 1D Vlasov equation

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = 0 \tag{20}$$

where we have neglected the external bunching force, and the space-charge force is assumed to be completely neutralized by the background plasma. Of course, the general solution of Eq. (20) is a function of two trivial invariants—the Lagrangian coordinate $z - v_z t$ and the velocity v_z, i.e.,

$$f(t, z, v_z) = f(0, z - v_z t, v_z). \tag{21}$$

However, a class of self-similar drift compression solutions can be more easily constructed using another familiar conserved quantity [12]

$$\chi = \frac{z^2}{z_b^2(t)} + \frac{z_b^2(t)}{z_{b0}^2 v_{T0}^2}\left[v_z - z_b'(t)\frac{z}{z_b(t)}\right]^2 \tag{22}$$

where $z_b(t)$ satisfies the envelope equation

$$\frac{d^2 z_b(t)}{dt^2} = \frac{z_{b0}^2 v_{T0}^2}{z_b^3(t)}. \tag{23}$$

The solution to Eq. (23) is

$$z_b^2(t) = (z_{b0} + z_{b0}' t)^2 + v_{T0}^2 t^2 \tag{24}$$

where $z_{b0}' = (dz_b/dt)_{t=0}$ and v_{T0} is an effective thermal speed. Let us consider the class of distribution $f(\chi)$. The line density is

$$\lambda = \int dv_z f(\chi) = \frac{z_{b0} v_{T0}}{z_b(t)} \int dV f[Z^2 + (V - \alpha Z)^2] \tag{25}$$

where $Z = z/z_b(t)$, $V = z_b v_z/(z_{b0} v_{T0})$, and $\alpha = z_b z_b'/(z_{b0} v_{T0})$. Defining

$$\lambda_b(t) = \frac{z_{b0} v_{T0}}{z_b(t)} f_{b0}, \quad f_{b0} = \int dV f(V^2)$$
$$h(Z^2) = \frac{1}{f_{b0}} \int dV f[Z^2 + (V - \alpha Z)^2] \tag{26}$$

we can cast $\lambda(t, z)$ into the self-similar form

$$\lambda(t, z) = \lambda_b(t) h(Z^2). \tag{27}$$

A simple calculation shows that the velocity profile is linear,

$$u_z = \frac{1}{\lambda} \int dv_z\, v_z f(\chi) = -z_b'(t) Z. \tag{28}$$

To design a drift compression scheme, we would like to know which kind of distribution function can generate the desired line density profile. This question is answered by the following inversion theorem. For a given self-similar line density profile in Eq. (27), the corresponding distribution function is

$$f(\chi) = -\frac{1}{\pi} \frac{\lambda_b(t) z_b(t)}{z_{b0} v_{T0}} \int_\chi^\infty \frac{\partial h(Z^2)}{\partial Z^2} \frac{dZ^2}{\sqrt{Z^2 - \chi}}. \tag{29}$$

As an application of the inversion theorem, we consider the family of self-similar line density profiles

$$\lambda(t, z) = \lambda_b(t) h(Z^2) = \begin{cases} \lambda_b(t)(1 - Z^2)^n, & Z \leqslant 1 \\ 0, & Z > 1. \end{cases} \tag{30}$$

A straightforward calculation shows that

$$f(\chi) = \begin{cases} -\frac{1}{\sqrt{\pi}} \frac{\lambda_b(t) z_b(t)}{z_{b0} v_{T0}} (1 - \chi)^{n-1/2} \frac{\Gamma(n)}{\Gamma(n+1/2)}, & \chi \leqslant 1 \\ 0, & \chi > 1, \end{cases} \tag{31}$$

where $\Gamma(n)$ is the gamma function. For $n = 1$ and $\lambda \sim 1 - Z^2$, the distribution function $f \sim \sqrt{1 - \chi}$

when $\chi \leqslant 1$. For $n = 1/2$ and $\lambda \sim \sqrt{1 - Z^2}$, f is a flat-top function of χ. For $n < 1/2$, the distribution function diverges near $\chi = 1$. Another family of self-similar line density profiles that is useful for drift compression design is

$$\lambda(t, z) = \lambda_{\rm b}(t) h(Z^2) = \begin{cases} \lambda_{\rm b}(t)(1 - Z^{2n}), & Z \leqslant 1 \\ 0, & Z > 1. \end{cases} \tag{32}$$

The inversion theorem gives

$$f(\chi) = \begin{cases} -\frac{1}{\pi} \frac{\lambda_{\rm b}(t) z_{\rm b}(t)}{z_{\rm b0} v_{\rm T0}} \Big[\sqrt{\pi} n \chi^{2n-1/2} \frac{\Gamma(1/2-2n)}{\Gamma(1-2n)} \\ \quad + \frac{4n}{4n-1} F\left(\frac{1}{2}, \frac{1}{2} - 2n; \frac{3}{2} - 2n; \chi\right) \Big], & \chi \leqslant 1 \\ 0, & \chi > 1 \end{cases} \tag{33}$$

where $F(\frac{1}{2}, \frac{1}{2} - 2n; \frac{3}{2} - 2n; \chi)$ is the hypergeometric function. The distribution function $f(\chi)$ in Eq. (33) is well-defined for arbitrary n, and this family of solution allows for arbitrarily flat line density profiles when $2n \gg 1$.

5. Transverse dynamics

For the entire beam pulse to be focused onto the same focal spot on the target, the self-similar symmetry condition (4) needs to be satisfied. In Sections 3 and 4, we have constructed longitudinal compression solutions where the symmetry condition is satisfied for the line density. It is difficult to guarantee the symmetry condition for the transverse emittance, due to the complex dynamical behavior of the transverse emittance when the beam is longitudinally compressed and transversely subject to a non-periodic focusing lattice and final focus magnets. However, in most heavy ion fusion systems, the transverse emittance is small. The deviation from the self-similar symmetry condition due to the transverse emittance can be treated as a perturbation. We can then deliberately impose another perturbation to the system to cancel out the perturbation due to the un-symmetric transverse emittance. The perturbation introduced to cancel out the un-symmetric emittance effect will be four time-dependent magnets. First, a drift compression and final focus lattice is designed for the central slice ($Z = 0$), and then four quadrupole magnets at the beginning of the drift compression are replaced by four time-dependent magnets whose strength varies around the design value for the central slice. The time-dependent magnets essentially provide a slightly different focusing lattice for the different slices. For example, for the line density profile

$$\lambda(s, Z) = \lambda_{\rm b}(s)[1 - z^2/z_{\rm b}^2(s)] \tag{34}$$

the self-similar symmetry condition implies that the solution to the transverse envelope equations for all of the slices can be scaled down from that of the central slice according to

$$\left\{ a, b, \frac{\partial a}{\partial s}, \frac{\partial b}{\partial s} \right\} [s, Z] = \sqrt{1 - z^2/z_{\rm b}^2(s)} \left\{ a, b, \frac{\partial a}{\partial s}, \frac{\partial b}{\partial s} \right\} [s, 0] \tag{35}$$

provided the emittance is negligible or scales with the perveance according to $(\varepsilon_x, \varepsilon_y) \propto 1 - z^2/z_{\rm b}^2(s)$. However, the emittance in general is small but not negligible, and does not scale with the perveance. In fact, during adiabatic drift compression or pulse shaping for an initially isothermal beam, the emittance scales with the beam size, i.e., $\varepsilon_x \propto a$ and $\varepsilon_y \propto b$. Our solution to this difficulty is to vary the strength of the four magnets in the very beginning of the drift compression phase for different z such that the desired scaling holds at the end of the last magnet. In a recently published paper [7], we demonstrated this technique using the parabolic longitudinal drift compression scheme for a typical un-neutralized heavy ion fusion beam. We considered a Cs^+ beam with 2.43 GeV kinetic energy, and 5.85 m initial beam half length. The beam was compressed by a factor of 21.8 to reach 2254 A average final current, and then the entire beam pulse was focused onto a focal point 1.2 mm in radius at the target.

6. Conclusion

We have studied the drift compression and final focus options for heavy ion fusion. Two of the most important requirements of the drift compression and final focus systems were considered. First of all, a large compression ratio needs to be achieved. Equally important, the entire beam pulse needs to be focused onto the same focal spot at the target. These two requirements determine many of the basic features of such systems. We demonstrated that it is necessary to use a self-similar drift compression scheme. For un-neutralized beams, the Lie symmetry group analysis was applied to the warm-fluid model to systematically derive the self-similar drift compression solutions. For neutralized beams, the 1D Vlasov equation was solved explicitly and families of self-similar drift compression solutions were constructed. To compensate for the deviation from the self-similar symmetry condition due to the transverse emittance, four time-dependent magnets were introduced in the upstream of the drift compression region such that the entire beam pulse can be focused onto the same focal spot. The self-similar longitudinal drift compression scheme, combined with the non-periodic, time-dependent lattice design, provides the essential elements of a leading-order drift compression method. The next-step investigation will be focused on second-order effects, such as emittance growth during drift compression, and the two-way coupling between the longitudinal and transverse dynamics.

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