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# Ion beam pulse neutralization by a background plasma in a solenoidal magnetic field

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#### Abstract

Ion beam pulse propagation through a background plasma in a solenoidal magnetic field has been studied analytically. The neutralization of the ion beam current by the plasma has been calculated using a fluid description for the electrons. This study is an extension of our previous studies of beam neutralization without an applied magnetic field. The high solenoidal magnetic field inhibits radial electron transport, and the electrons move primarily along the magnetic field lines. For high-intensity ion beam pulses propagating through a background plasma with pulse duration much longer than the electron plasma period, the quasi-neutrality condition holds,  $n_e = n_b + n_p$ , where  $n_e$  is the electron density,  $n_b$  is the density of the ion beam pulse, and  $n_p$  is the density of the background plasma ions (assumed unperturbed by the beam). For one-dimensional electron motion, the charge density continuity equation  $\partial \rho / \partial t + \nabla \cdot \mathbf{j} =$ 0 combined with the quasi-neutrality condition  $[\rho = e(n_b + n_p - n_e) = 0]$  yields  $\mathbf{j} = 0$ . Therefore, in the limit of a strong solenoidal magnetic field, the beam current is completely neutralized. Analytical studies show that the solenoidal magnetic field starts to influence the radial electron motion if  $\omega_{ce} \ge \omega_{pe}\beta$  (where  $\omega_{ce} = eB/mc$  is the electron gyrofrequency,  $\omega_{pe}$  is the electron plasma frequency, and  $\beta = V_b/c$  is the ion beam velocity relative to the speed of light). This condition holds for relatively small magnetic fields. For example, for a 100 MeV, 1 kA Ne<sup>+</sup> ion beam  $(\beta = 0.1)$  and a plasma density of  $10^{11}$  cm<sup>-3</sup>, *B* corresponds to a magnetic field of 100 G.  $\mathbb{C}$  2005 Elsevier B.V. All rights reserved.

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## 1. Ion beam neutralization in a background plasma without external magnetic field

\*Corresponding author. Tel.: + 609 243 3277. *E-mail address:* ikaganov@pppl.gov (I.D. Kaganovich). Ion beam pulses are used in many applications, including heavy ion inertial fusion [1–3] high-density laser-produced proton beams for fast

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ignition of inertial confinement fusion targets [4], positron beams for electron-positron colliders [5], etc. To neutralize a large repulsive spacecharge force of the ion beam charge, the ion beam pulses can be transported through a background plasma. The plasma electrons can effectively neutralize the ion beam charge, and the background plasma can provide an ideal medium for ion beam transport and focusing. Because the detailed parameter values for heavy ion fusion drivers are not well prescribed at the present time, an extensive study is necessary for a wide range of beam and plasma parameters to determine the conditions for optimum beam propagation.

The electron response time to an external charge perturbation is determined by the electron plasma frequency,  $\omega_{pe} = (4\pi n_p e^2/m_e)^{1/2}$ , where  $n_p$  is the background plasma density. Therefore, as the beam pulse enters the background plasma, the plasma electrons tend to neutralize the ion beam on a time scale of order  $\tau_{pe} \equiv 1/\omega_{pe}$ . Typically, the ion beam pulse propagation duration through the background plasma is long compared with  $\tau_{pe}$ . As a result, after the beam pulse passes through a short transition region, the plasma disturbances are stationary in the beam frame. We have developed reduced nonlinear models, which describe the stationary plasma disturbance (in the beam frame) excited by the intense ion beam pulse [6]. In recent calculations [7,8], we have studied the nonlinear quasi-equilibrium properties of an intense, long ion beam pulse propagating through a cold, background plasma, assuming that the beam pulse duration  $\tau_b$  is much longer than  $\tau_{pe}$ , i.e.,  $\omega_{\rm pe}\tau_{\rm b} \ge 1$ . In the study reported in Ref. [9], we extended the previous results to general values of the parameter  $\omega_{pe}\tau_{b}$ . The key assumption in these papers is that the electron thermal velocity  $(V_{Te})$ can be neglected, because it is much smaller than the ion beam velocity  $(V_b)$ . The typical electron temperature of the background plasma is a few eV, and the condition  $V_{\text{Te}} \ll V_{\text{b}}$  is satisfied for sufficiently fast ion beams with velocity  $V_{\rm b} \ge 0.01c$ . This assumption allowed us to use the fluid approximation and obtain an analytical solution for the self-electric and self-magnetic fields of the ion beam pulse. Theoretical predictions agree well with the results of calculations utilizing a particle-in-cell (PIC) code [7–9].

The model predicts very good charge neutralization during quasi-steady-state propagation, provided the beam is nonrelativistic and the beam pulse duration  $\tau_b$  is much longer than the electron plasma period  $2\pi/\omega_{pe}$ , independent of the ion beam current. However, the degree of beam current neutralization depends on the background plasma density and the ion beam current. The ion beam current is neutralized by the electron current, provided the beam radius is large compared with the electron skin depth  $c/\omega_{pe}$ . This condition can be written as

$$I_{\rm b} > 4.25\beta n_{\rm b}/n_{\rm p}kA.$$
 (1)

For the parameters characteristic of heavy ion fusion drivers, the condition in Eq. (1) holds [3,6], whereas for present scaled experiments condition (1) does not hold [2]. In Ref. [10], it was proposed that the unneutralized electrostatic electric field is reduced by the background plasma to potential values of order

$$\phi_0 = m_{\rm e} V_{\rm b}^2 / 2. \tag{2}$$

This potential accelerates the plasma electrons up to the ion beam velocity. Analytical and numerical studies [7,9,11] show that the potential in Eq. (2) emerges at the end boundary of a neutralization section as the beam exits this section. The neutralization section may consist of an electronemitting electrode, a biased foil, or a short plasma plug without any background plasma during further ion beam propagation.

The estimate in Eq. (2) does not pertain to neutralization by an extended background plasma when the beam pulse is immersed inside the plasma. In the latter case, the longitudinal electric field is predominantly inductive, and in the laboratory frame it is described by the vector potential  $\mathbf{A} = A_z \mathbf{e}_z$  rather than the electrostatic potential. The radial electric field is determined from the equivalent electrostatic potential

$$\phi = m_{\rm e} V_{\rm e}^2 / 2, \tag{3}$$

where  $V_e = eA_z/cm_e$ , and  $A_z$  is determined from Maxwell's equations [7,8].

For sufficiently large beam currents,

$$I_{\rm b} \gg 4.25\beta n_{\rm b}/n_{\rm p}kA,\tag{4}$$

the beam radius  $r_{\rm b}$  is large compared with the electron skin depth ( $\delta = c/\omega_{\rm pe}$ ), because  $r_{\rm b}/\delta_{\rm p} = (I_{\rm b}n_{\rm p}/4.25n_{\rm b}\beta kA)^{1/2}$ . The beam current density is neutralized by the electrons everywhere ( $n_{\rm b}V_{\rm b} = n_{\rm e}V_{\rm e}$ ) [7], which gives

$$V_{\rm e} = V_{\rm b} \, \frac{n_{\rm b}}{n_{\rm b} + n_{\rm p}}.\tag{5}$$

If the plasma density is much larger than the beam density  $n_p \ge n_b$ , then it follows that  $V_e \ll V_b$  and  $\phi \ll \phi_0$ .

In the opposite limit of sufficiently small beam currents,

$$I_{\rm b} \ll 4.25\beta n_{\rm b}/n_{\rm p}kA,\tag{6}$$

the beam radius is small compared with the electron skin depth. To estimate the electron velocity in this limit, we can utilize the fact that magnetic field of the ion beam is totally screened by the plasma far away from the beam. (This follows from the conservation of general vorticity [7]: far away from the beam the electron flow velocity is equal to zero; therefore the vector potential should also be equal to zero  $V_e = eA_z/mc = 0.$ )

Because the magnetic field is equal to zero, the total current

$$\pi e \int_0^\infty (n_b V_b - n_e V_e) r \, \mathrm{d}r = 0 \tag{7}$$

consisting of the beam and the electron current must be zero, i.e., the ion beam current is neutralized globally, not locally. The electron current is spread over distances of order the skin depth  $\delta$  around the ion beam pulse. As a result, it follows from Eq. (7) that

$$V_{\rm e} \sim V_{\rm b} n_{\rm b} r_{\rm b} / n_{\rm p} \delta \ll V_{\rm b}. \tag{8}$$

Correspondingly, the electrostatic potential is much smaller than the estimate in Eq. (3), i.e.,  $\phi \leq \phi_0$ . To demonstrate the difference between the two theoretical predictions, we have performed simulations of beam neutralization by a plasma under conditions corresponding to  $I_b =$  4.25 $\beta n_{\rm b}/n_{\rm p}kA$ , and the ion beam unneutralized space-charge potential ( $\phi_{\rm b} = 2\pi e^2 n_{\rm b} r_{\rm b}^2$ ) is so small that the beam cannot create the potential  $\phi_0$ , i.e.,  $\phi_{\rm b} \ll \phi_0$ .

The theory in Ref. [10] predicts no charge neutralization for the case  $\phi_b \ll \phi_0$ , whereas the theory in Ref. [7] predicts neutralization depending on the pulse duration. Fig. 1 shows the results of EDPIC simulations for an ion beam pulse with  $\phi_b/\phi_0 = \omega_e^2 r_b^2/c^2 \beta^2 = 0.04$ . It is evident that long ion beam pulses with  $\omega_{pe}\tau_b \ge 1$  propagating in volumetric plasma are well neutralized, in contrast to the plasma plug case considered in Ref. [10]. Note that plasma waves are excited for the case of an ion beam pulse with sharp edges, if  $\omega_{pe}\tau_b^e < 2\pi$ , where  $\tau_b^e$  is the time scale of the beam head rise.

The neutralization of an intense fast ion beam pulse by a background plasma is a complex process. For example, Fig. 1 shows the formation of electron holes near the non-emitting surfaces. These holes slowly lag behind the beam, as described in Refs. [9,11]. The visualization of the numerical simulation data shows complex collective phenomena during beam entry and exit from the plasma [9,11].

### 2. Ion beam neutralization in a background plasma with external magnetic field

If the neutralization of the intense fast ion beam pulse by a background plasma is already a complex process, adding an external magnetic field makes the process even more complicated. Analytical studies show that the solenoidal magnetic field starts to influence the radial electron motion if  $\omega_{ce} \ge \omega_{pe}\beta$  [12]. The condition  $\omega_{ce} \ge \omega_{pe}\beta$ already holds for relatively small magnetic fields; for example, for a 100 MeV, 1 kA Ne<sup>+</sup> ion beam ( $\beta = 0.1$ ) and plasma density of 10<sup>11</sup> cm<sup>-3</sup>, and the condition  $\omega_{ce} = \omega_{pe}\beta$  corresponds to a magnetic field of only 100 G.

If  $\omega_{ce} \ll \omega_{pe}\beta$ , the external magnetic field does not influence the charge and current neutralization relative to the unmagnetized case. In the case where  $\omega_{ce} \gg \omega_{pe}\beta$ , the electron current completely neutralizes the ion beam current. A small unneutralized current *j* is associated with the remnant



Fig. 1. Neutralization of an ion beam pulse during steady-state propagation of the beam pulse through a cold, uniform, background plasma in planar geometry calculated using the EDPIC code [11]. The beam propagates in the *y*-direction. The beam density has an elliptical, flat-top density profile with smooth edges of 0.8 (left) and 0.2 (right) fraction of the full ion beam pulse size. Shown in the figure are color plots of the normalized beam density  $(n_b/n_p)$  (top) and the normalized electron density  $(n_e/n_p)$  (middle and bottom) for particle-in-cell simulations in  $(x\omega_p/c, y\omega_p/c)$  space. The beam velocity is  $V_b = 0.5c$ , the beam density is  $n_b = 0.5n_p$ , and the ion beam charge state is  $Z_b = 1$ . The beam dimensions correspond to a beam radius  $r_b = 0.1c/\omega_p$ , and pulse duration (middle)  $\tau_b\omega_p = 60$  and (bottom)  $\tau_b\omega_p = 6$ .

radial electron transport across the magnetic field and is proportional to  $(\omega_{ce}/\omega_{pe}\beta)^{-2}$ . In the limit of very large magnetic field  $\omega_{ce} \gg \omega_{pe}\beta$ , the electron inertia terms may be neglected compared with the  $\mathbf{E} \times \mathbf{B}$  electron drift, and the corresponding theory was developed in Ref. [13]. In this case, the magnetic field lines are attached to the electron fluid and the beam self-magnetic field is due to the ion beam rotation. If there is no ion beam rotation, there is no net current and the beam self- magnetic field vanishes. However, the selfmagnetic field of the ion beam pulse can be generated due to the ion beam rotation by the dynamo effect. Electrons follow the ion rotation and they drag and twist the solenoidal magnetic field. As a result, a poloidal magnetic field emerges due to the twisting of the external solenoidal magnetic field. This theory was developed for ion

rings, where high-intensity ion beams are injected into a strong magnetic field of a tokamak-like configuration. However, the ion beams for heavy ion fusion drivers have very small rotation and the results of the theory in Ref. [13] do not apply, because the assumption that the electron inertia terms may be neglected compared with the  $\mathbf{E} \times \mathbf{B}$ electron drift is not valid for heavy ion fusion driver parameters.

Note that for the theory of ion beam-plasma interactions, we extensively use the slice approximation, which describes the self-electric and self-magnetic fields of the ion beam pulse propagating in a background plasma for long beam pulses  $(l_b \gg r_b)$ . In this approximation, the magnetic field is a function of only the beam and plasma density in the local cross section of the beam pulse. Fig. 2 shows the self-magnetic field structure of the ion



Fig. 2. The charge and current neutralization of the ion beam pulse is calculated in two-dimensional slab geometry using the LSP code [16] for various magnetic field strengths corresponding to  $\omega_{ce}/\omega_{pe} = 0, 0.7, 1.4$ . The background plasma density is  $n_p = 10^{11} \text{ cm}^{-3}$ . The beam velocity is  $V_b = 0.2c$ , the beam current is 1.2 kA (48.0 A/cm<sup>2</sup>), which corresponds to ion beam density  $n_b = 0.5n_p$ , and the ion beam charge state is  $Z_b = 1$ . The beam dimensions ( $r_b = 2.85$  cm and  $\tau_b = 1.9$  ns) correspond to a beam radius  $r_b = 1.5c/\omega_p$ , and pulse duration  $\tau_b \omega_p = 75$ . The solenoidal magnetic field 1014G corresponds to  $\omega_{ce} = \omega_{pe}$ .

beam pulse for the case where  $\omega_{ce} > \omega_{pe}\beta$ , where the slice approximation is inapplicable, even though the beam pulse is long,  $l_b \gg r_b$ . Additional visualization images are available in Refs. [14,15].

### 3. Conclusions

Analytical studies show that the solenoid magnetic field starts to influence the radial electron motion when  $\omega_{ce} \ge \omega_{pe}\beta$  [12]. In the case where  $\omega_{ce} \gg \omega_{pe}\beta$ , the electron current completely neutralizes the ion beam current. If  $\omega_{ce} = \omega_{pe}\beta$ , the slice approximation can be used to describe the self-electric and self-magnetic fields of the beam pulse propagating in a background plasma for long beam pulses where the beam length is much larger than the beam radius  $(l_{\rm b} \ge r_{\rm b})$ . In this approximation, the magnetic field is a function only of the beam density and plasma density in the local cross section of the beam. In the opposite case where  $\omega_{ce} \ge \omega_{pe}\beta$ , the simulations show that the slice approximation does not apply, as the magnetic field structure is essentially two-dimensional and non-stationary. Plasma waves generated by the beam head are greatly modified when  $\omega_{\rm ce} > \omega_{\rm pe} \beta$ . Application of the external solenoidal magnetic field clearly makes collective processes of ion beam-plasma interaction more versatile and interesting than in the absence of solenoidal field.

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