Excitation of Transverse Dipole and Quadrupole Modes in a Pure Ion Plasma in a Linear Paul Trap to Study Collective Processes in Intense Beams

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Overview

Developing an improved understanding of intense beam propagation in high energy accelerators is essential for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.

Critical issues for accelerators:

- Generally, long time, long distance propagation of intense beam bunches.
- Specifically, stability against lattice noise, and,
- stability against coherent periodic perturbations.

The experimental results demonstrate that the external perturbations act on the charge bunch through their interactions with the collective modes of the charge bunch.

Outline of this presentation:

- Introduction to accelerators, Paul traps, and the analogy between them.
- Controlled excitation and observation of dipole and quadrupole modes.
- Coherent periodic dipole and quadrupole errors in multi-turn rings. Interaction of errors with beam modes. Excitation of large-amplitude, nonlinear modes.
- Random lattice dipole and quadrupole errors and their interaction with beam modes. Suppression of emittance growth by filtering noise spectrum.

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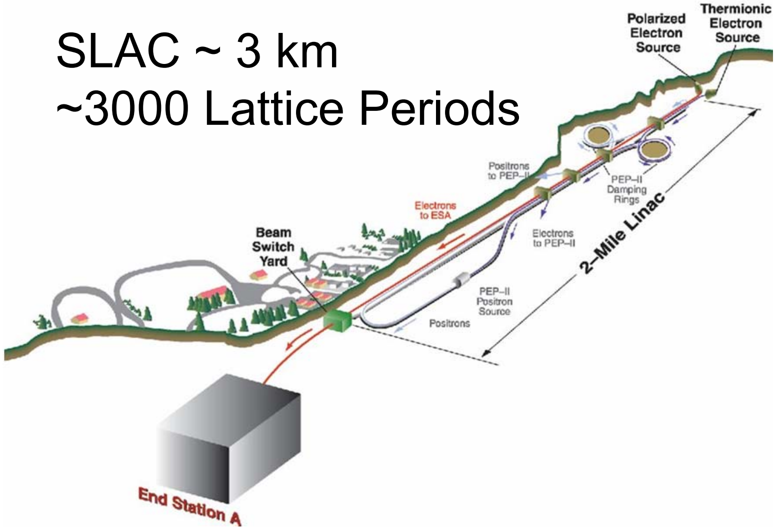
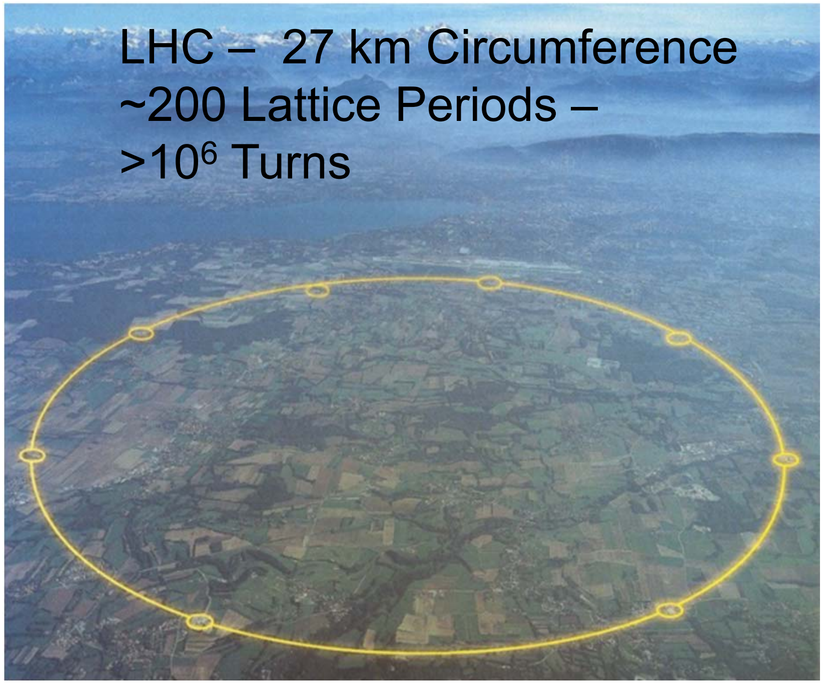
Accelerators use Periodic Lattices of Quadrupole Magnets for Transverse Confinement – Noise and Periodic Perturbations Can Limit Beam Lifetime

SLAC ~ 3 km
~3000 Lattice Periods



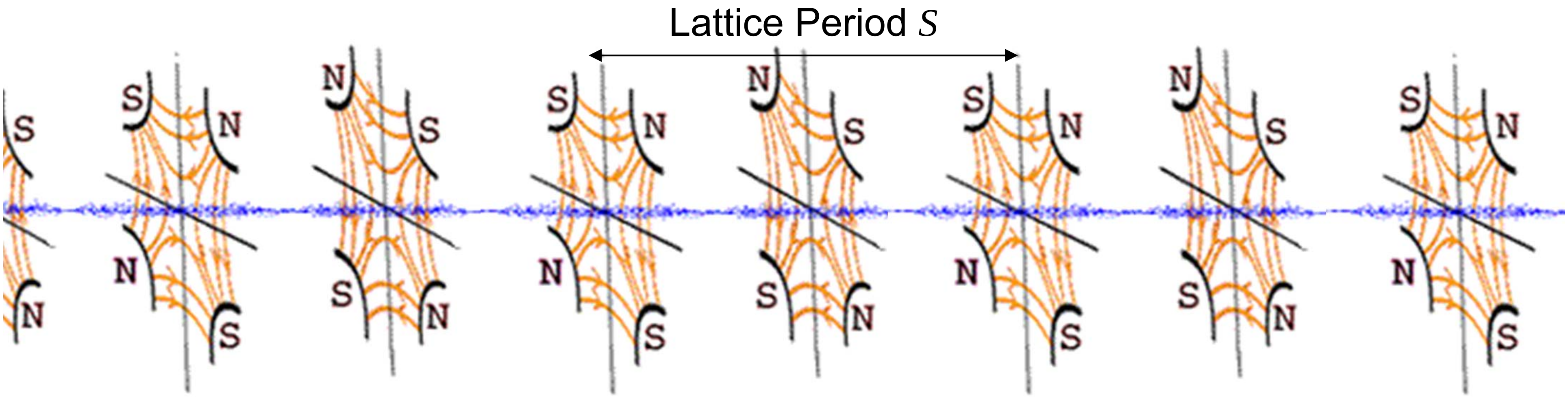
SNS Ring – 248 m Circumference
~24 Lattice Periods – 1000 Turns

FAIR at GSI
1 km Circumference
~80 Lattice Periods



LHC – 27 km Circumference
~200 Lattice Periods –
>10^6 Turns

Alternating-Gradient Transport Systems Use a Spatially Periodic Lattice of Quadrupole Magnets for Transverse Dynamic Stability



Focusing-Off-Defocusing-Off (FODO) Lattice

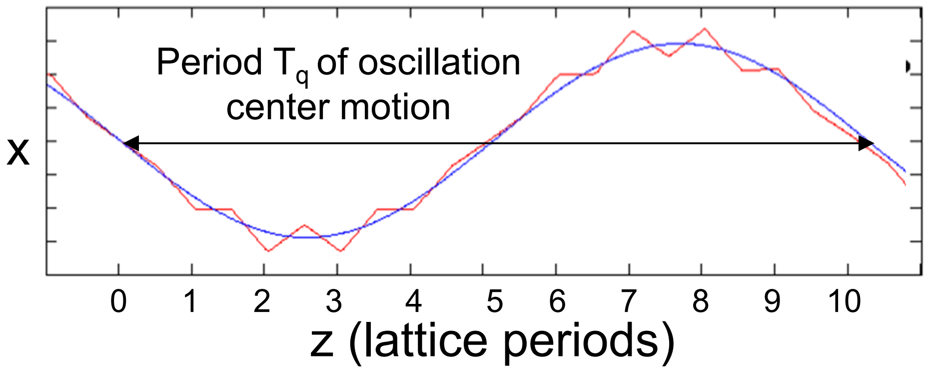
$$\boxed{\begin{aligned}\boldsymbol{B}_q^{foc}(\boldsymbol{x}) &= B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)\\ \boldsymbol{F}_{foc}(\boldsymbol{x}) &= -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)\end{aligned}}$$

$$\boxed{\kappa_q(z) \equiv \frac{ZeB_q'(z)}{\gamma m\beta c^2}}$$

In its frame of reference, a transverse slice of a long thin beam experiences time-dependent oscillating forces that stretch or compress it in the transverse plane.

There can be errors in the magnet strength, the magnet spacing along the beamline, or the transverse alignment that can affect beam transport and quality.

Average Transverse Focusing Frequency, Phase Advance, Emittance, and Line Charge Characterize the Beam



$\omega_q = 2\pi/T_q$ is the average transverse focusing frequency.

$\sigma_v = \omega_q/f$ is the phase advance. Here, σ_v, is 36°.

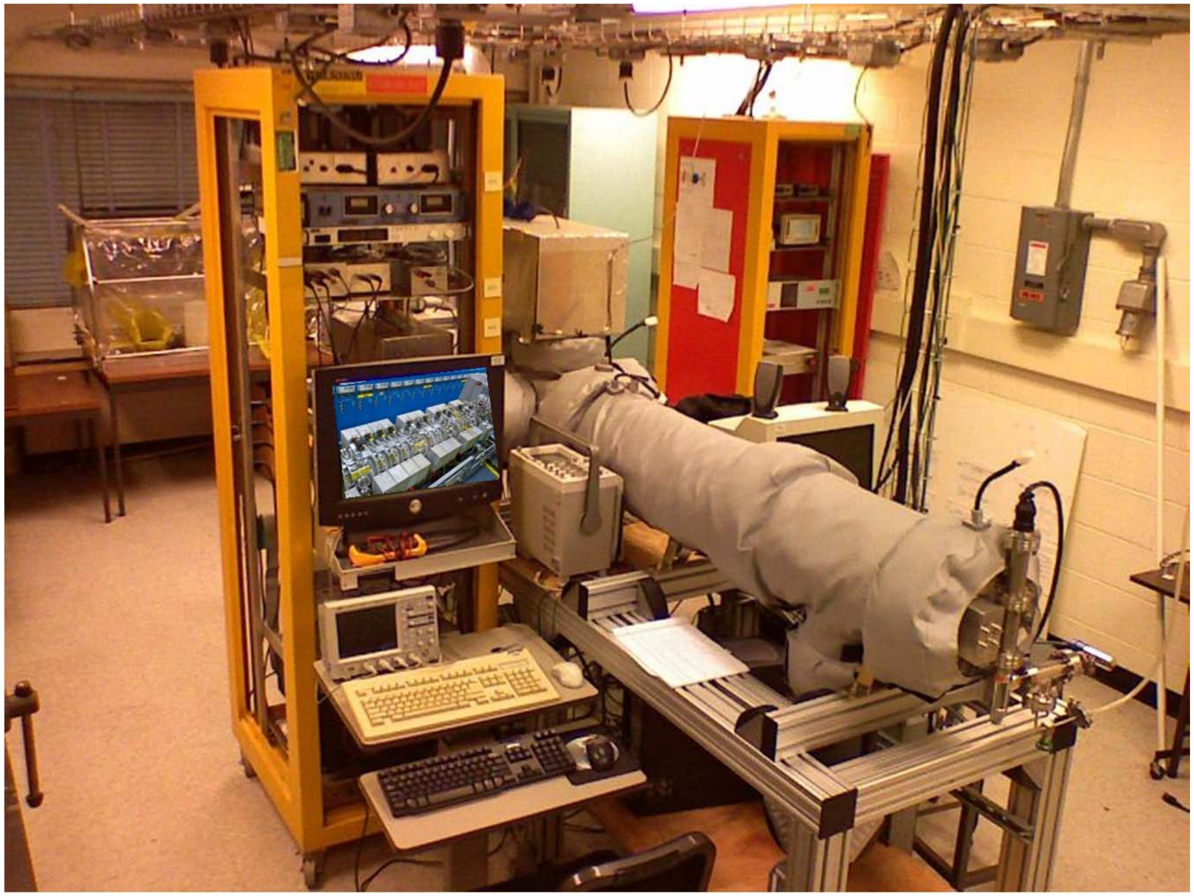
The areas of the distribution in (x,v_x) and (y,v_y) phase space are the emittances ε_x and ε_y and therefore scale as $R_b\,(kT)^{1/2}$

The Kapchinskij-Vladimirskij (KV) distribution corresponds to a uniform charge density, and if the beam envelope is an ellipse with radii a and b, the envelope equations can be written as (N_b is the line charge):

$$a'' + \kappa_x(z)a - \frac{2N_b}{a+b} - \frac{\varepsilon_x^{\,2}}{a^3} = 0 \qquad b'' + \kappa_y(z)b - \frac{2N_b}{a+b} - \frac{\varepsilon_y^{\,2}}{b^3} = 0$$

The Paul Trap Simulator Experiment (PTSX) Simulates Transverse Beam Dynamics by Placing Us in the Beam's Frame of Reference

The oscillating quadrupole electric field and trapped plasma self-field are the Lorentz transforms of the fields in the accelerator system. The whole trapped plasma column simulates the dynamics of one beam slice.



See also:
R. C. Davidson, H. Qin, and G. Shvets, Phys. Plasmas **7**, 1020 (2000).
N. Kjærgaard and M. Drewsen, Phys. Plasmas **8**, 1371 (2001).
H. Okamoto and H. Tanaka, Nucl. Instrum. Methods Phys. Res. A **437**, 178 (1999).

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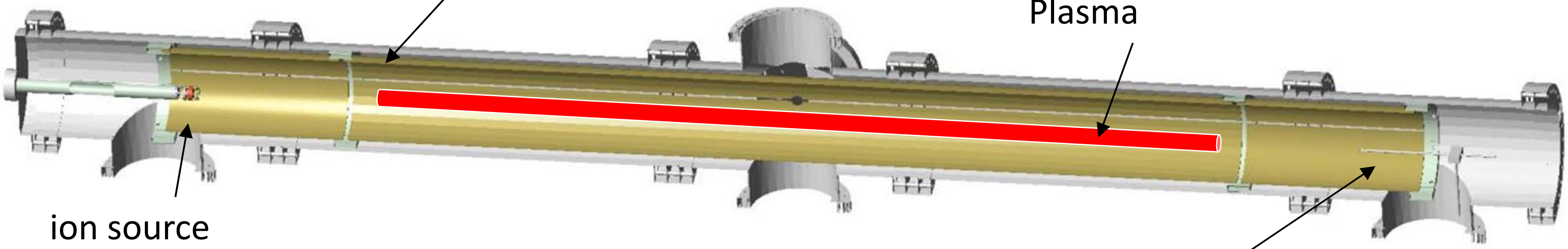
PTSX Apparatus



Paul trap electrodes

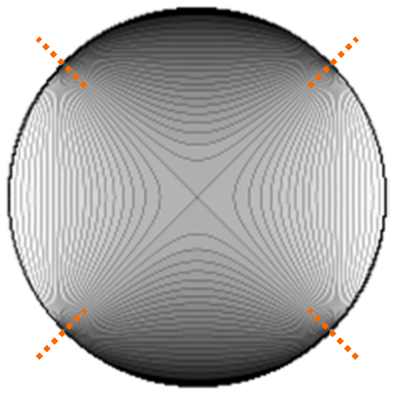
Plasma

ion source
(aluminosilicate cesium emitter)

Collector
(5 mm diameter moveable
copper disk)

$$e\phi_{ap}(x,y,t)=\frac{1}{2}\kappa_q'(t)(x^2-y^2)$$

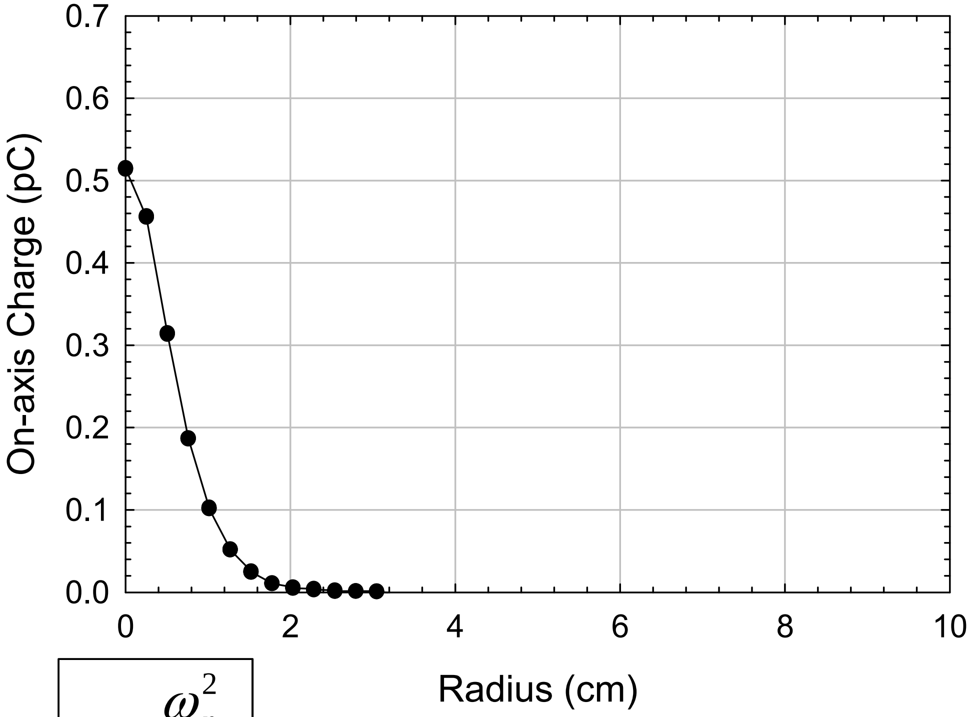
$$\kappa_q'(t)=\frac{8eV_0(t)}{m\pi\, r_w^2}$$



$$\omega_q=\frac{8e_bV_{0\,\max}}{m_b\pi\, r_w^2\, f}\xi \qquad \hat{s}=\frac{\omega_p^2}{2\omega_q^2}$$

Plasma length	2 m	Wall voltage	140 V
Wall radius	10 cm	End electrode voltage	20 V
Plasma radius	~ 1 cm	Frequency	60 kHz
Cesium ion mass	133 amu	Pressure	5 x 10^{-10} Torr
Ion source grid voltages	< 10 V	Trapping time	100 ms

Results From a Typical PTSX Experiment



Trapping time consists of:

equilibrate 20 ms
perturb 30 ms
re-equilibrate 50 ms

Experimental data include:

On-axis charge: Q = 515 fC
On-axis number density: n = 10^5 cm^{-3}
Line charge: $N_b = 2.0\ 10^7\ \text{m}^{-1}$
RMS radius: $R_b = 0.9$ cm
Effective temperature: kT = 0.15 eV
Normalized intensity: **s = 0.22**
(Tevatron injector s~0.1, SNS Ring s~0.3, HIF s~0.99)

$$\boxed{\hat{s} = \frac{\omega_p^2}{2\omega_q^2}}$$

Effective transverse temperature is computing using the equation for local force balance on a fluid element and integrating over the transverse distribution to obtain:

$$\boxed{m\omega_q^2 R_b^2 = \frac{N_b q^2}{4\pi\varepsilon_o} + 2kT}$$

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Studies of Collective Beam Modes Begin with Models for the Modes

Transverse plasma modes can be thought of in the contexts of different models:

- KV Vlasov-Poisson smooth focusing model:

$$D_\ell(\omega)=1-\frac{\omega_p^2}{2^{\ell+1}\ell\upsilon^2}\left[1-\left(\frac{r_b}{r_w}\right)^{2\ell}\right]\sum_{m=0}^{\ell}\frac{\ell!}{m!(\ell-m)!}\frac{(\ell-2m)\upsilon}{[\omega-(\ell-2m)\upsilon]}=0$$

- KV envelope smooth focusing model:

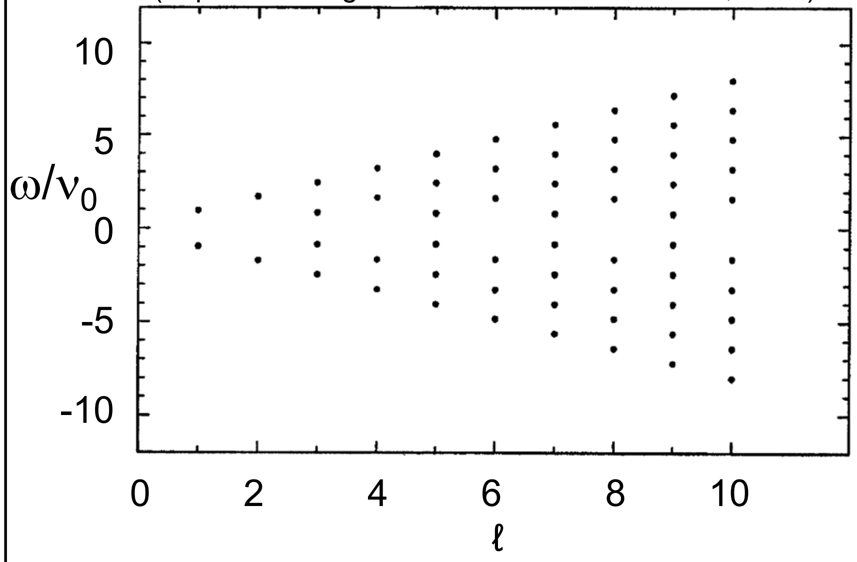
$$\ddot{a}+\omega_q^2 a-\frac{2N}{a+b}-\frac{\varepsilon^2}{a^3}=0$$

- KV envelope time-dependent-lattice model:

$$\ddot{a}+\kappa_q(t)a-\frac{2N}{a+b}-\frac{\varepsilon^2}{a^3}=0$$

- PIC simulation

Davidson & Qin, Physics of Intense Charged Particle Beams in High Energy Accelerators (Imperial College Press and World Scientific, 2001).

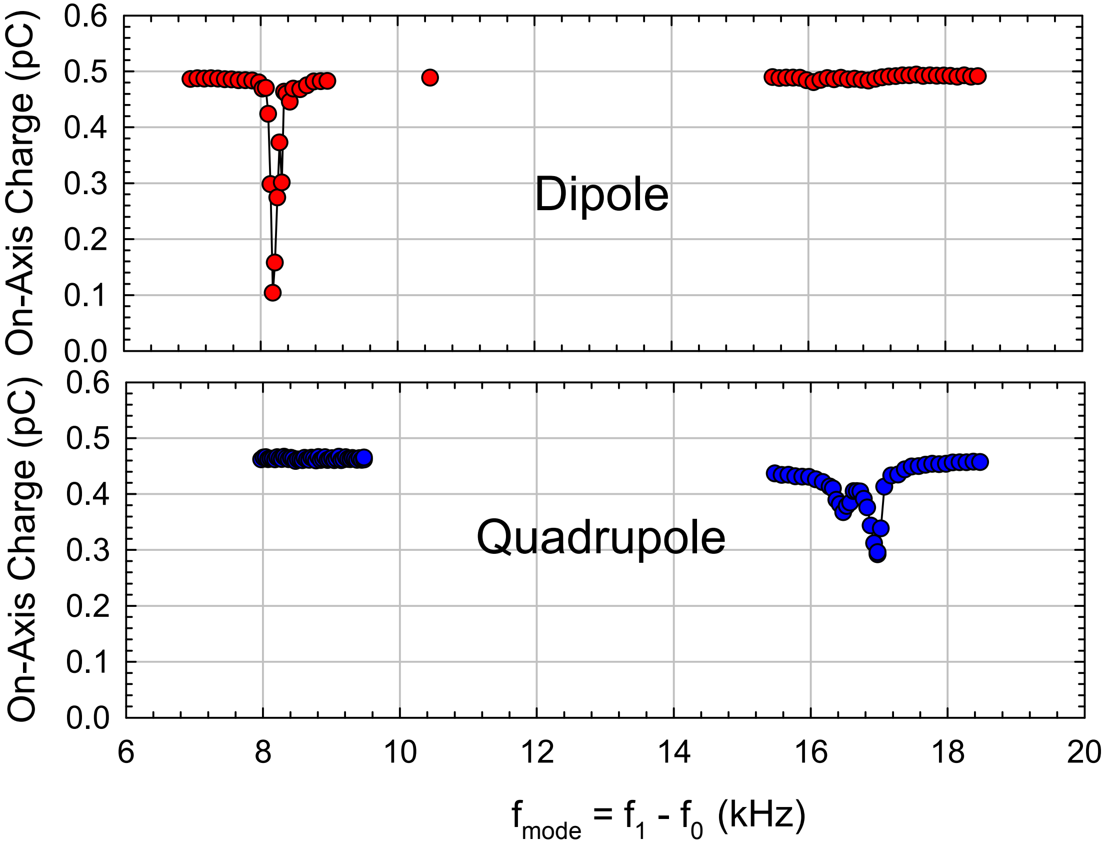


Dipole mode: An ℓ = 1 surface mode where the entire plasma column moves back and forth. $f_{dipole} = f_q = \omega_q/2\pi$

Quadrupole mode: An ℓ = 2 surface mode where the beam perturbation has an elliptical shape. $f_{Quadrupole} \sim 2f_q$

Both the Dipole Mode and the Quadrupole Mode are Observed Near Their Expected Frequencies

$$V(t) = V_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t)$$



$\delta V/V_0 = 0.01$

Electrode voltage: $V_0 = 140$ V

Electrode frequency: $f_0 = 60$ kHz

Phase advance: $\sigma_v = 49$ deg.

$$f_{dipole} = f_q = \frac{\omega_q}{2\pi}$$

$$f_{Quadrupole} \sim 2 f_q$$

Both the frequency and the spatial structure of the perturbation must be appropriate to excite the mode. Driving all four PTSX wall electrodes with properly phased V(t) excites the quadrupole mode, while driving a single PTSX wall electrode excites the dipole mode.

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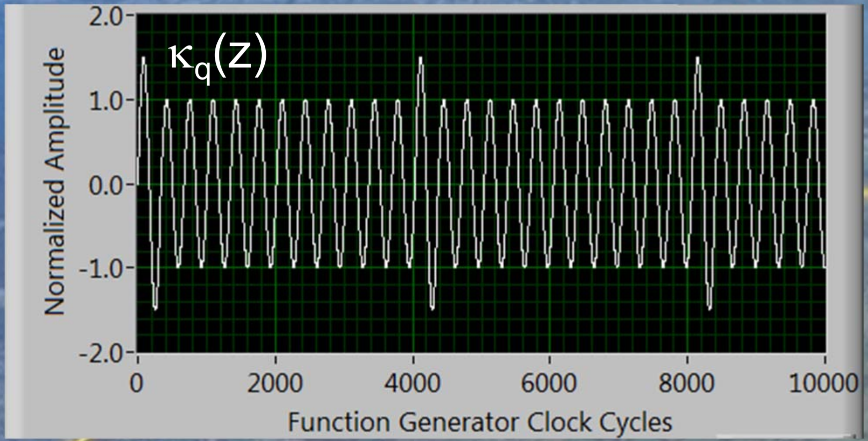
Resonance Between Beam Modes and Coherent Periodic Lattice Errors in a Ring

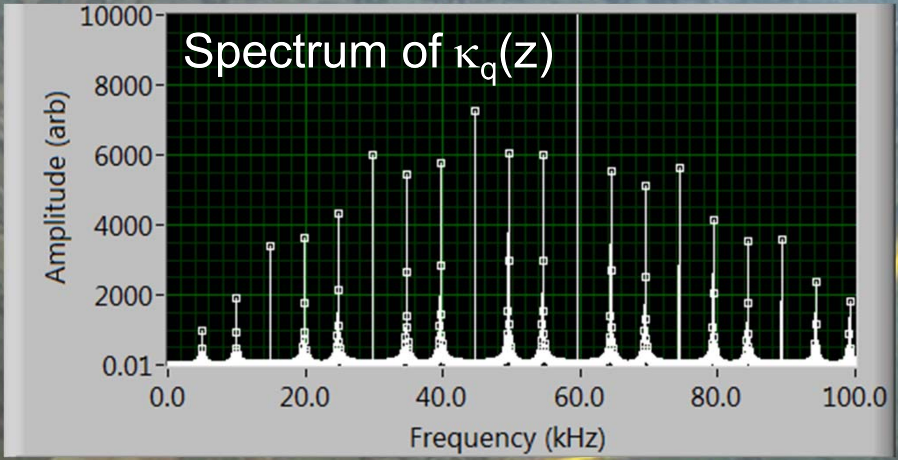
Example: ring period N = 12



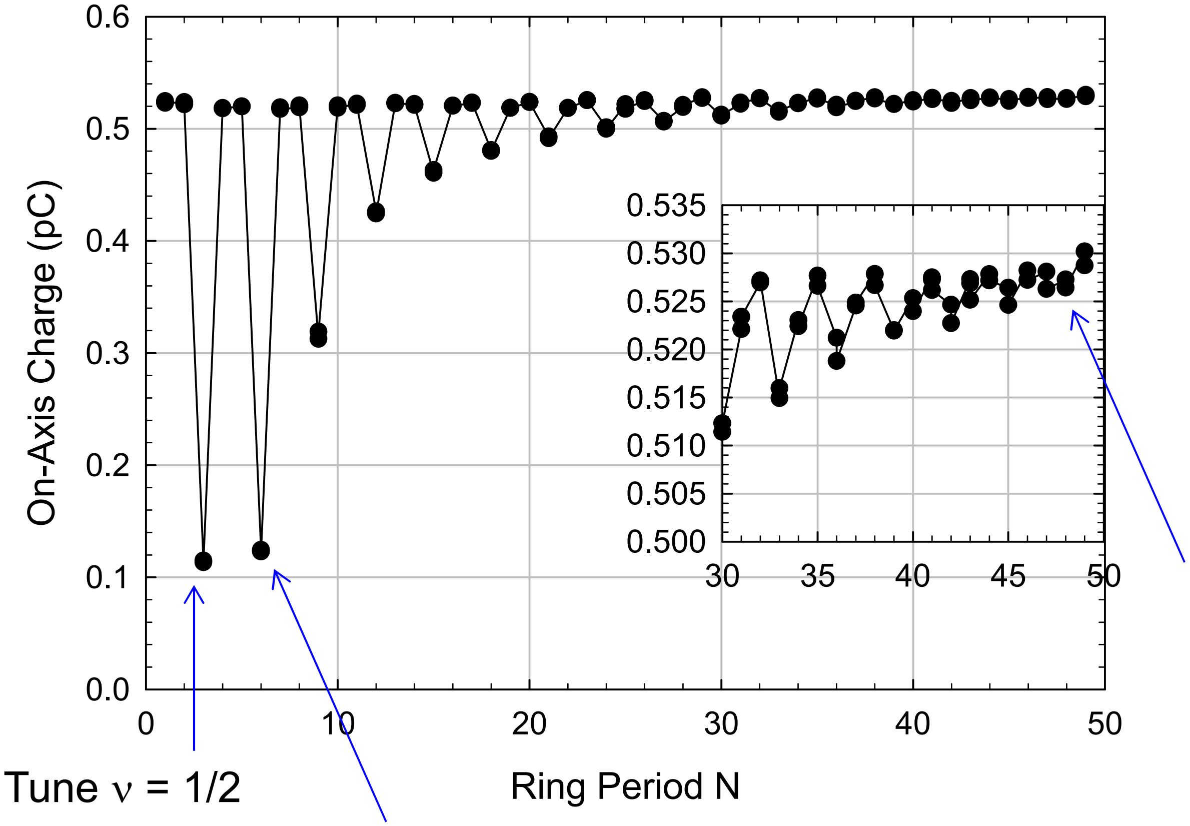
Tune $\nu \equiv f_q/f_{ring}$

Tune is the number of betatron periods (dipole mode periods) per trip around the ring.

First 100 kHz of spectrum. Components at multiples of 60/N kHz = 5 kHz



For a 60° Phase Advance the Period of the Quadrupole Mode is $3/f_0$ and There is a Resonance When N is a Multiple of 3


f_0 = 60 kHz, 1800 Lattice periods
2% Amplitude

Tune ν = 1/2

Tune ν = 1

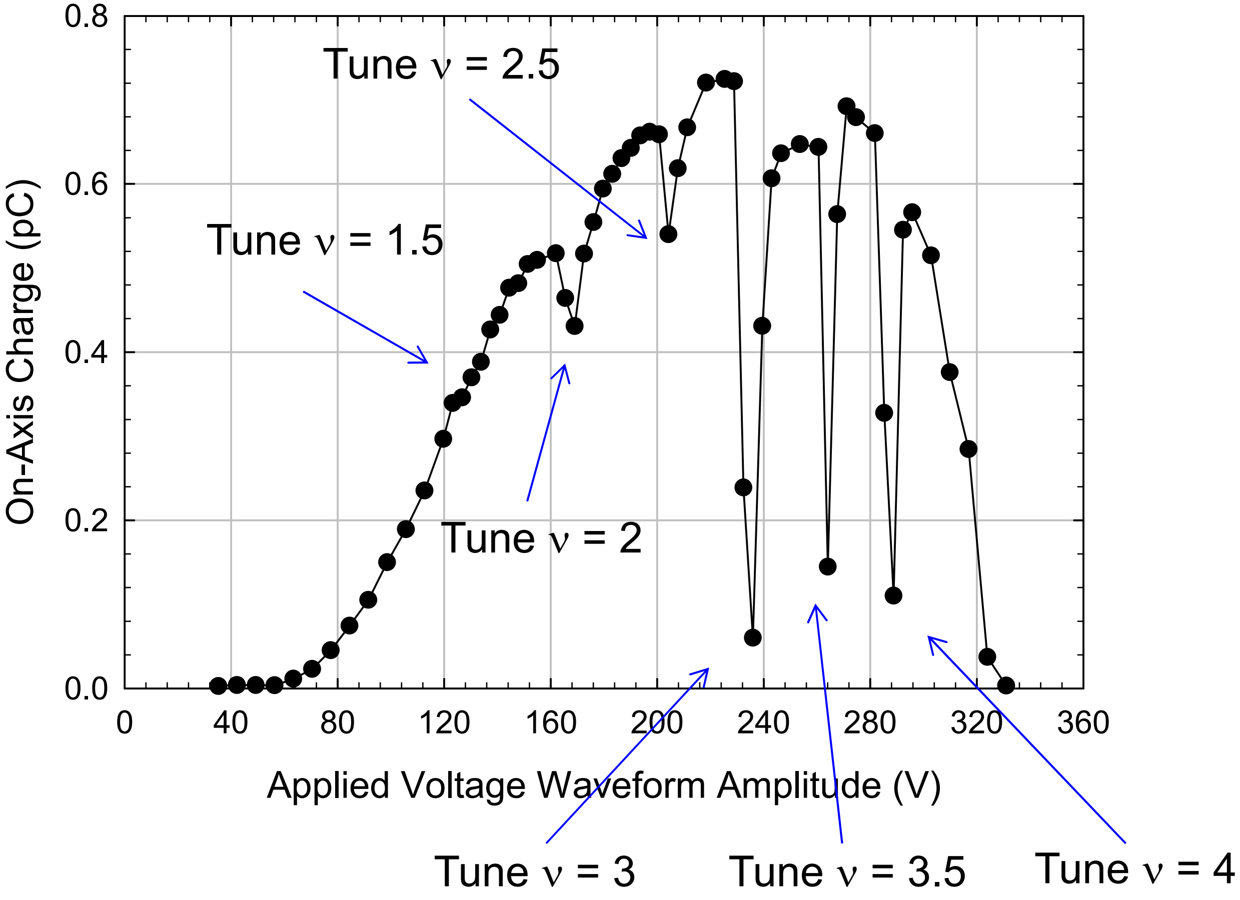
Tune ν = 8

The Tune Also Can be Changed for Fixed Ring Periodicity N by Changing V_0 to Change the Mode Frequency

N = 12

$$\omega_q = \frac{8 e_b V_{0\,\max}}{m_b \pi r_w^2 f}\xi$$

See also:
Ohtsubo et al., “Experimental Study of Coherent Betatron Resonances with a Paul Trap”, Phys. Rev. ST Accel. Beams, **13**, 044201 (2010).

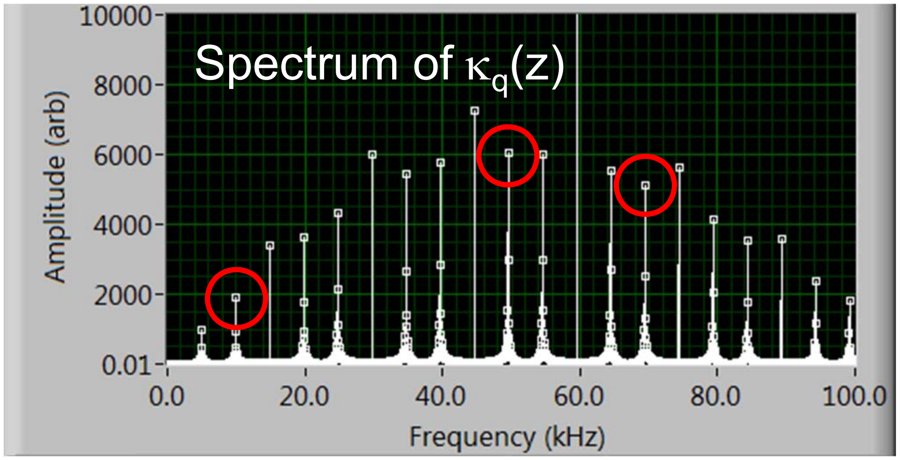


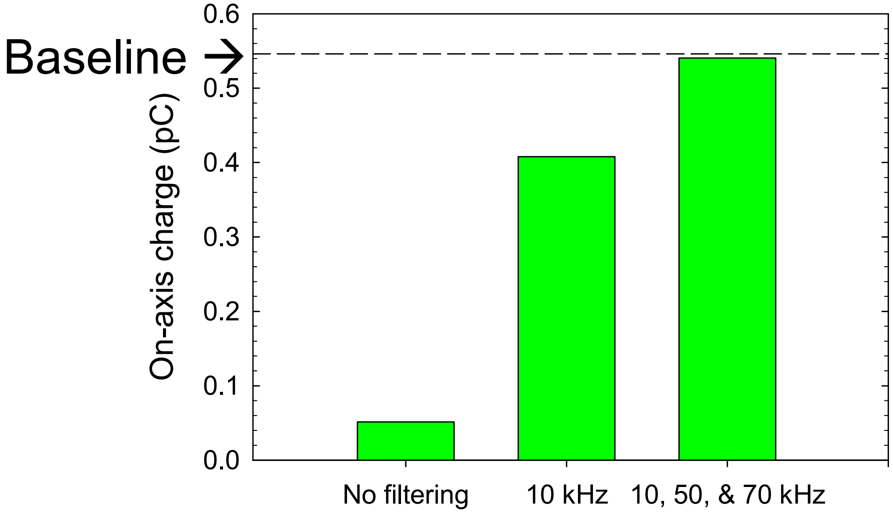
Dipole errors lead to resonances at integer tunes because their frequency is f_q.
Quadrupole errors lead to half-integer resonances because their frequency is near $2f_q$

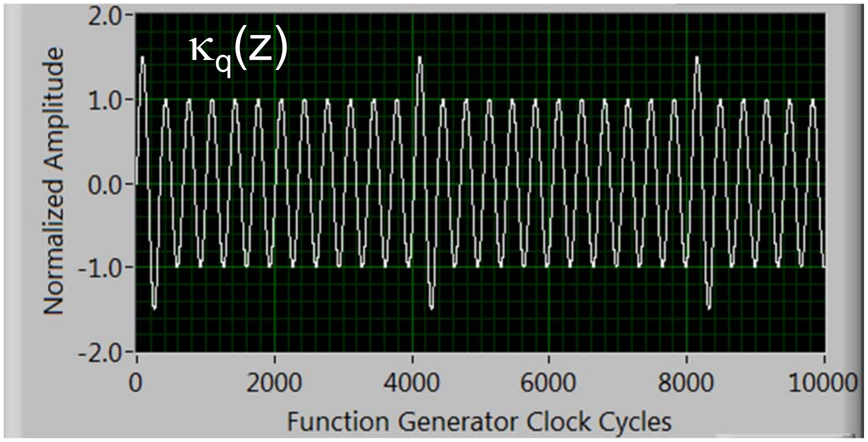
The Deleterious Effects are Eliminated When Frequency Components that Drive the Mode are Removed

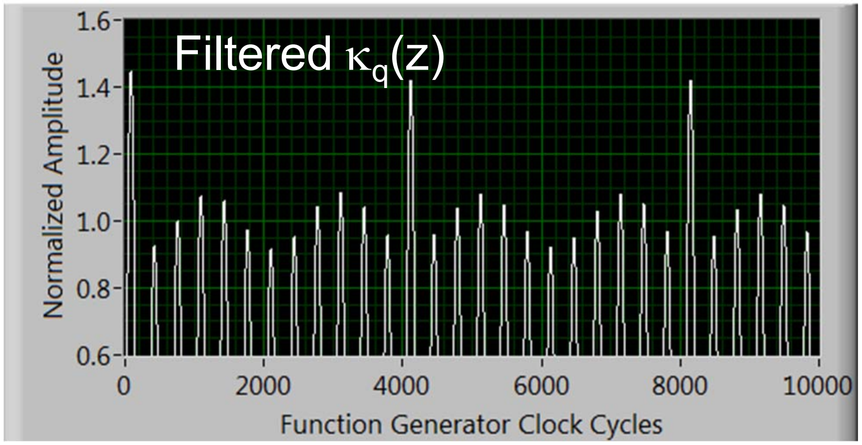
Ring Periodicity N = 12

2% amplitude dipole, $\sigma_v = 60°$, tune = 2



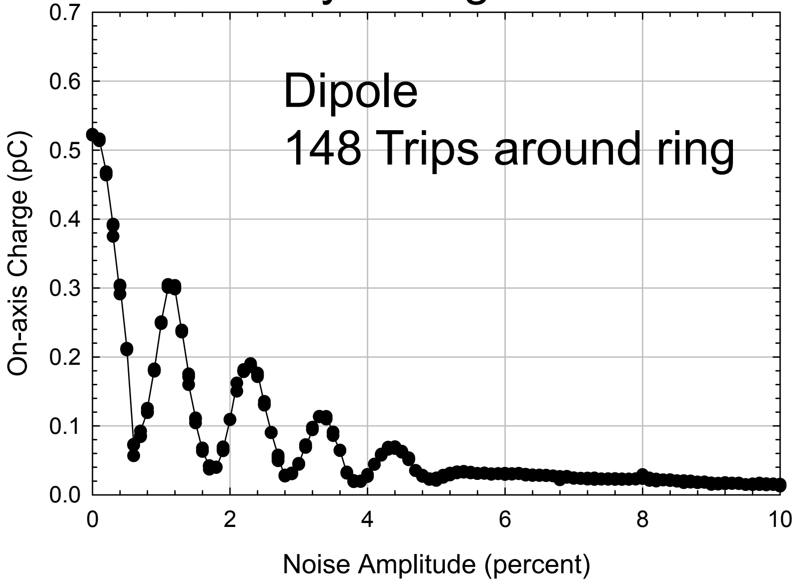


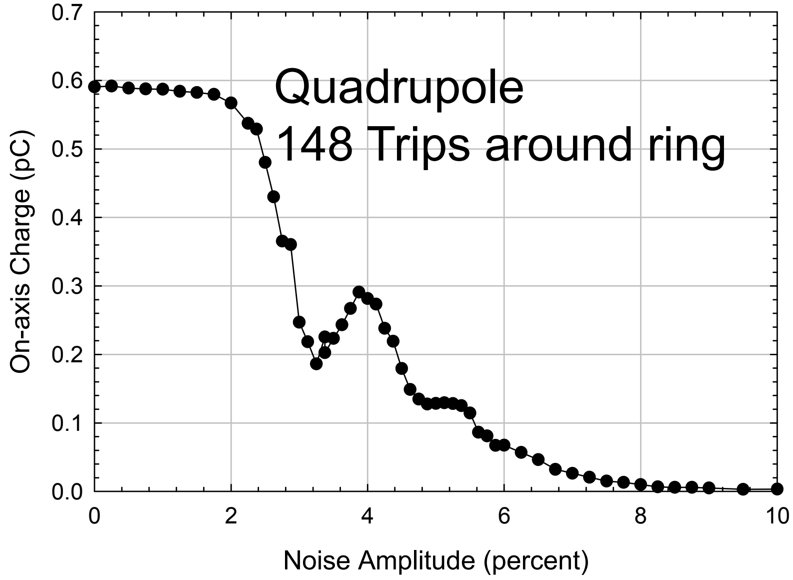


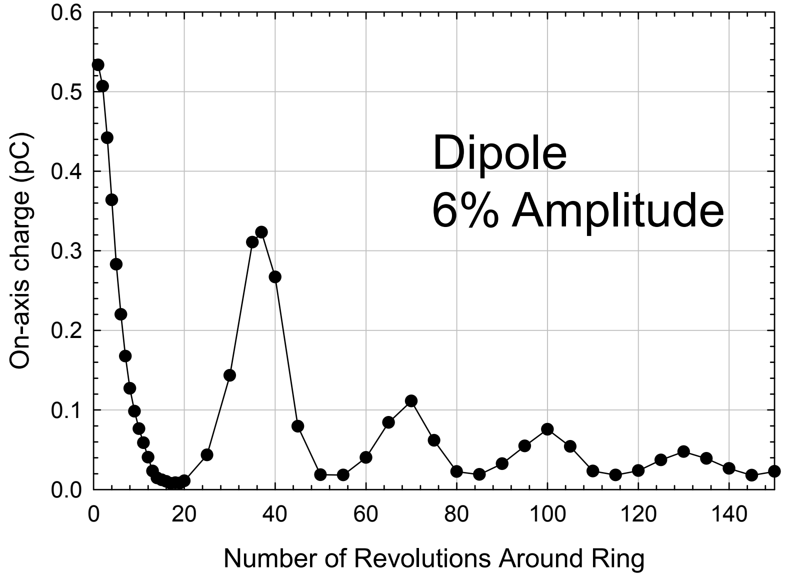


Observations Suggest that the Modes are Being Driven Into Nonlinear Regimes

Periodicity of ring N = 12







Driving a nonlinear oscillator at the linear mode frequency causes the mode amplitude to periodically increase and decrease.

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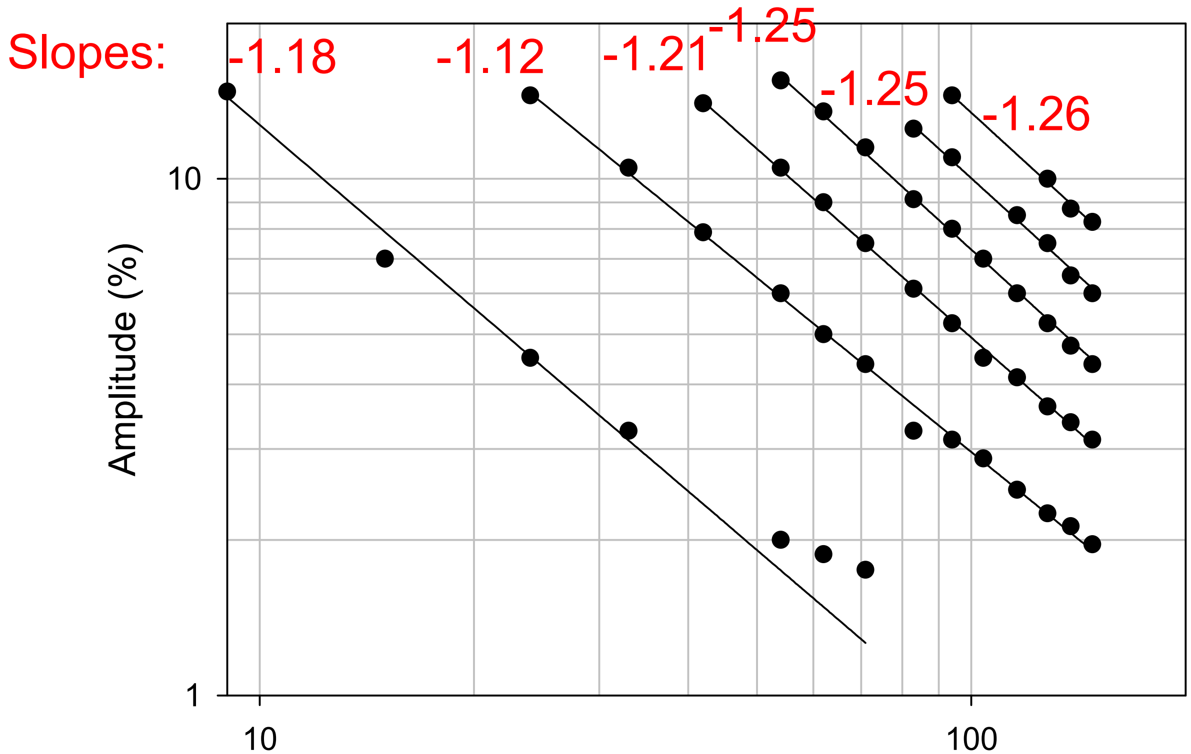
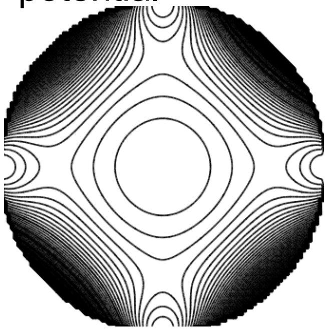
Dipole Perturbation Scaling of “Valley” Location ~ (Amplitude x Duration)$^{-5/4}$



Scaling is consistent with nonlinearity from 12-pole contribution to ponderomotive force.

Image charge effect would have r^4 term and -3/2 scaling instead.

Ponderomotive potential



$$V(r,\theta,t) \sim A\sin(2\pi f_0 t)\left[\left(\frac{r}{r_w^2}\right)^2\cos(2\theta)-\frac{1}{3}\left(\frac{r}{r_w^2}\right)^6\cos(6\theta)+\ldots\right]$$

The r^n term gives scaling as -(n-1)/(n-2)

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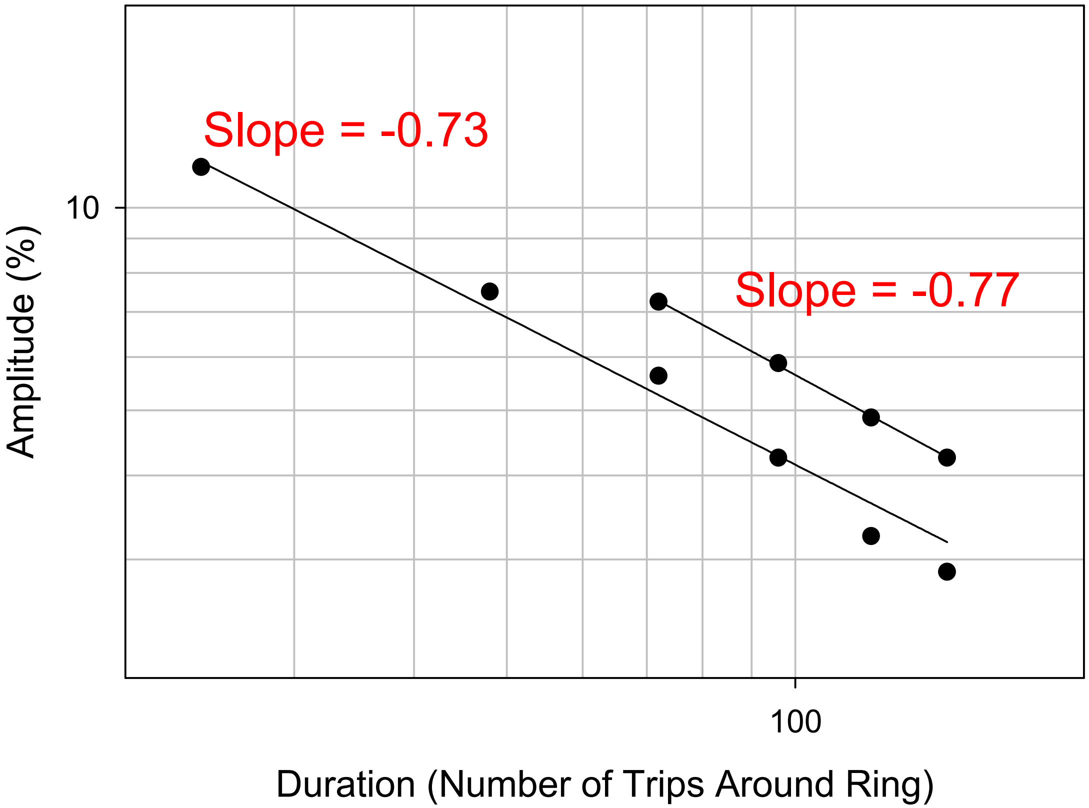
Quadrupole Perturbation Scaling of “Valley” Location ~ (Amplitude x Duration)$^{-3/4}$



The nonlinearity in the quadrupole perturbation case is likely from the finite space-charge of the charge bunch. The scaling is consistent neither with image charges nor trap nonlinearities.

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Previous Results on PTSX Demonstrated the Adverse Effect of Quadrupole Noise as a Function of Amplitude and Duration

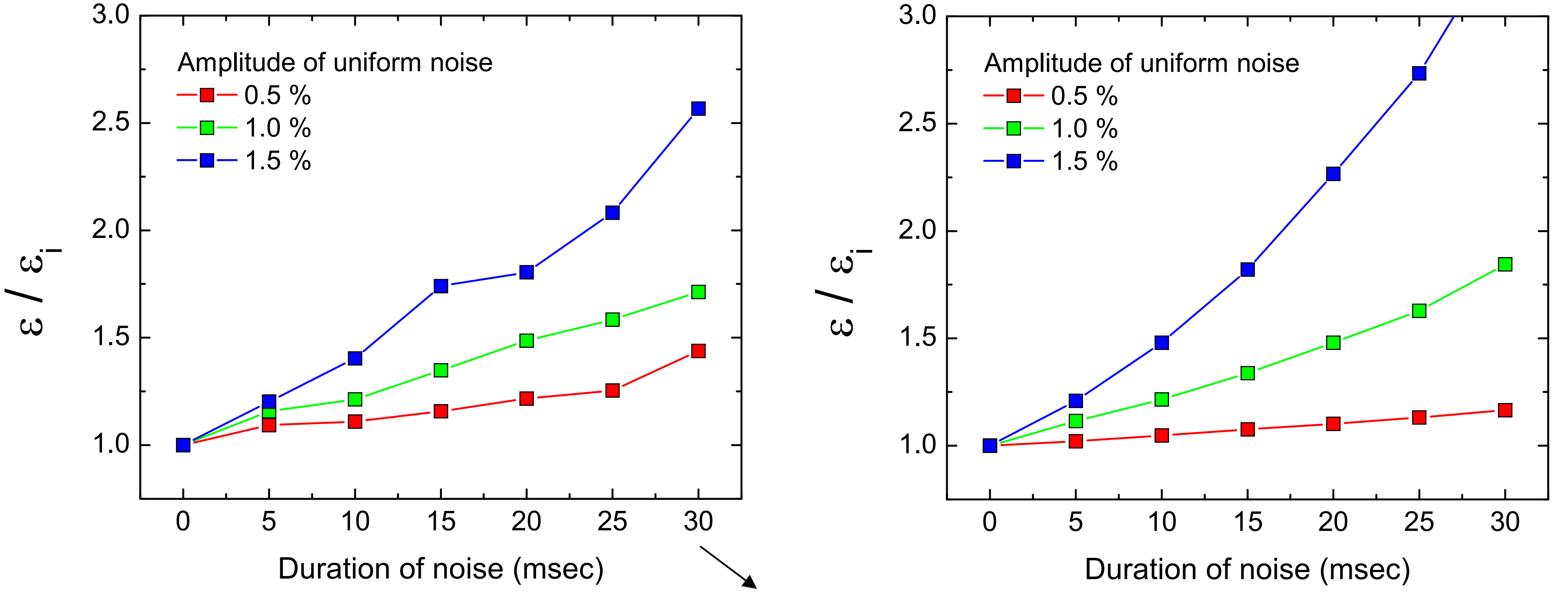
δ_n (n = 1 to 3600) are random numbers generated from a given distribution.

$$V(t) = A\sin(2\pi t/T) \qquad A = V_0(1+\delta_n) \quad \text{for} \quad (n-1)\frac{T}{2} < t < n\frac{T}{2}$$

$$\varepsilon \propto R_b\sqrt{kT}$$

Experiments

WARP 2D PIC Simulations

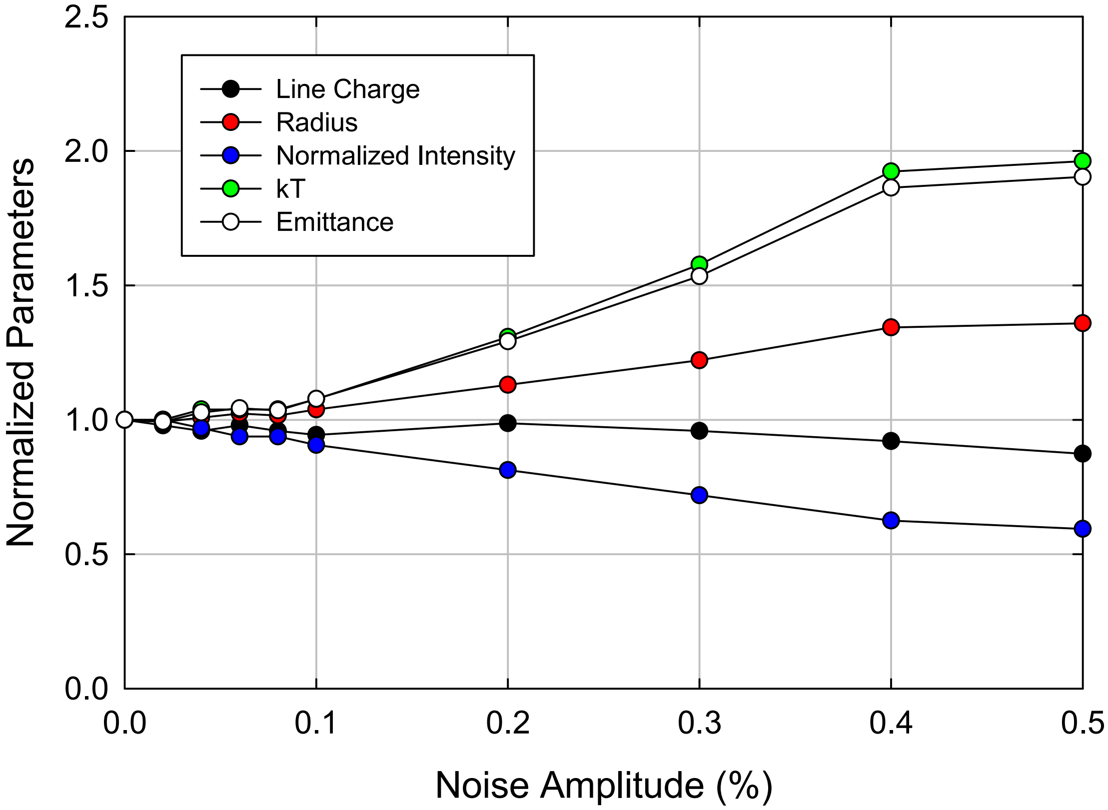


1800 lattice periods

Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

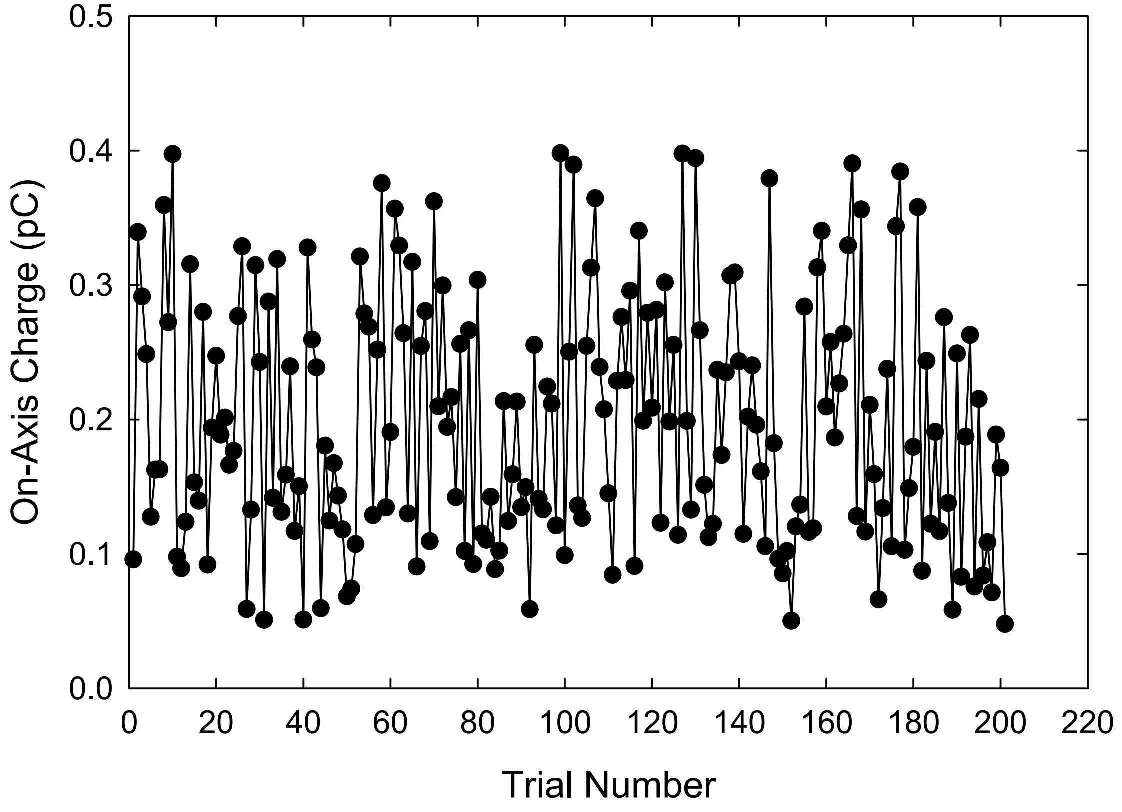
New Results on PTSX Demonstrate the Adverse Effect of Dipole Noise – A Stronger Effect Since the Dipole Field is Nonzero On-Axis

Dipole noise could originate from transverse misalignment of magnet sets.



Time Series of On-axis Charge Measurements for 200 Sets of Random Numbers Shows Large Variation

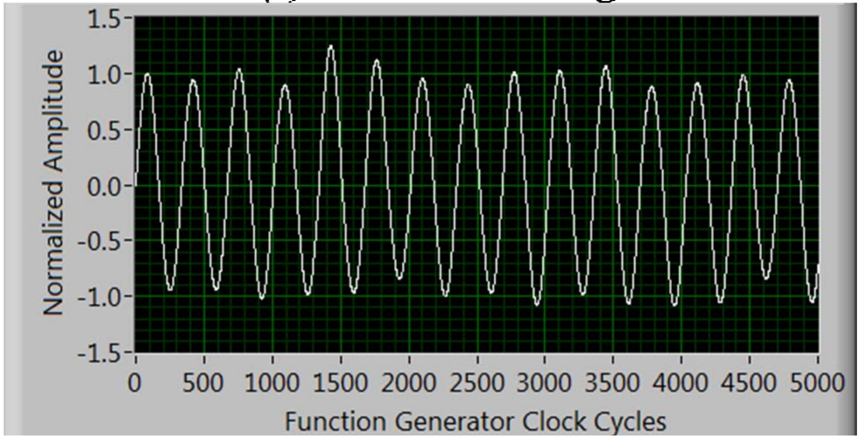
Simulating 200 Different Realizations of Accelerators with 0.5% Dipole Noise



In some cases, the trapped plasma is nearly destroyed.
In some cases, the trapped plasma is nearly unaffected.

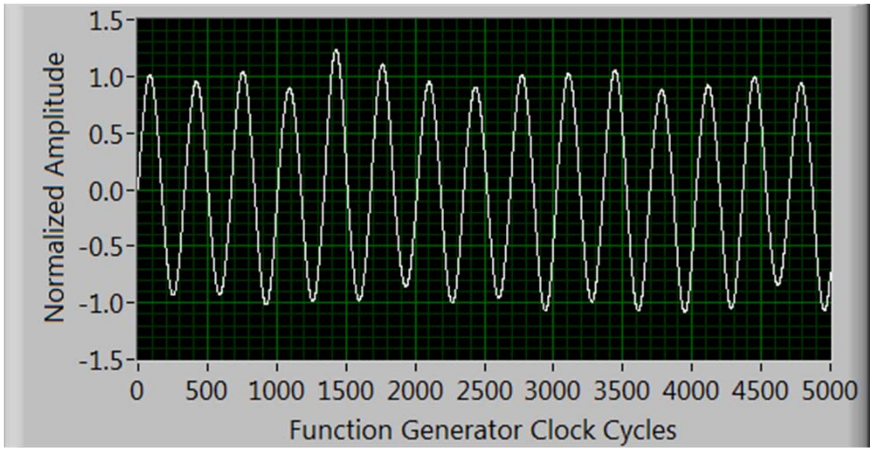
The Waveform Can be Filtered to Understand How the Noise Couples to the Modes

$\kappa(t)$ before filtering



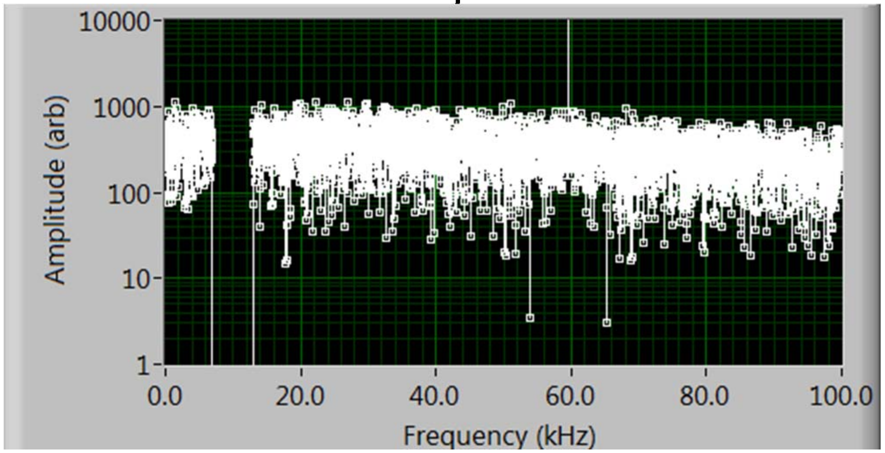
First 15 periods

$\kappa(t)$ after filtering



First 15 periods

Filtered spectrum



First 100 kHz of the FFT of the Waveform

Example with 10% noise to make it easy to see.

1786 lattice periods = 30 ms → 33.3 Hz resolution
20 MHz clock → 10 MHz maximum frequency

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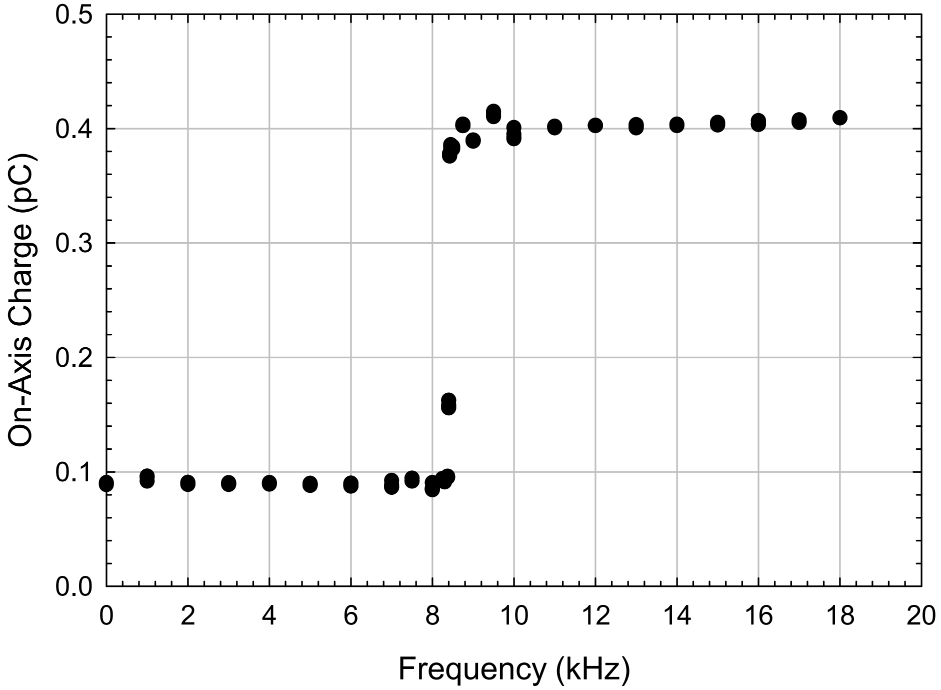
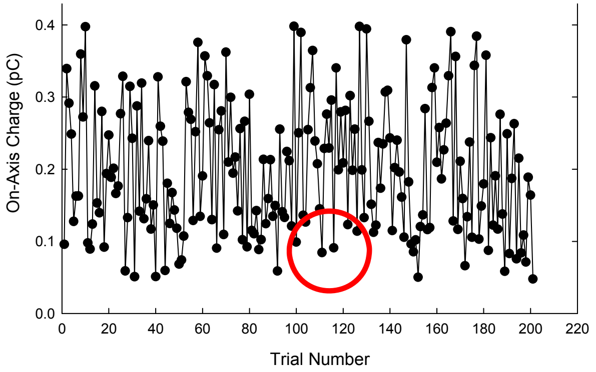
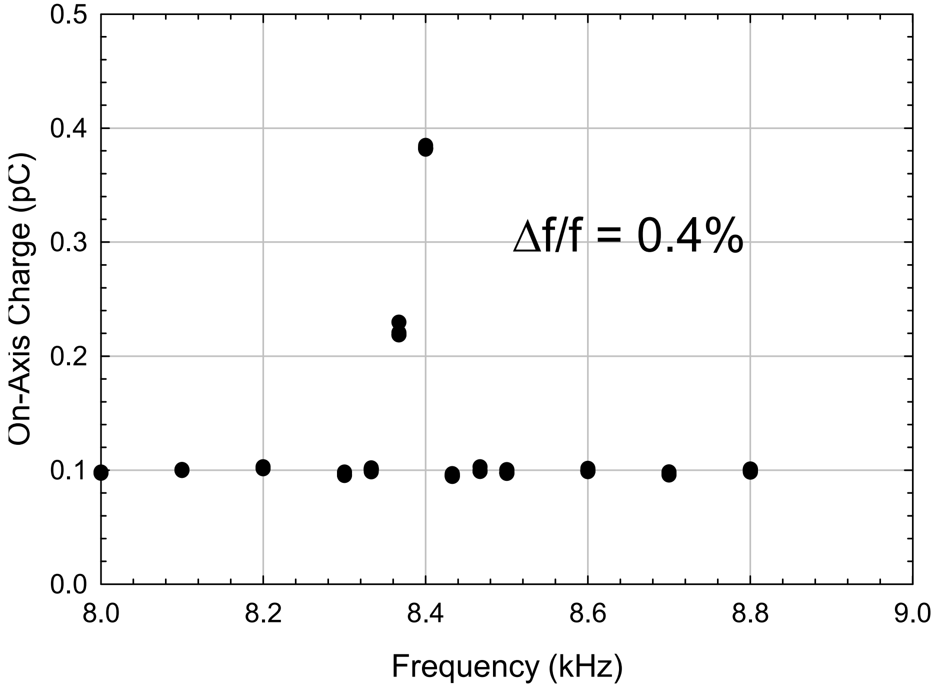
Noise Applied to One Electrode Damages the Beam Through Its Interaction with the ℓ = 1 Dipole Mode

One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes Frequencies from Zero to f kHz





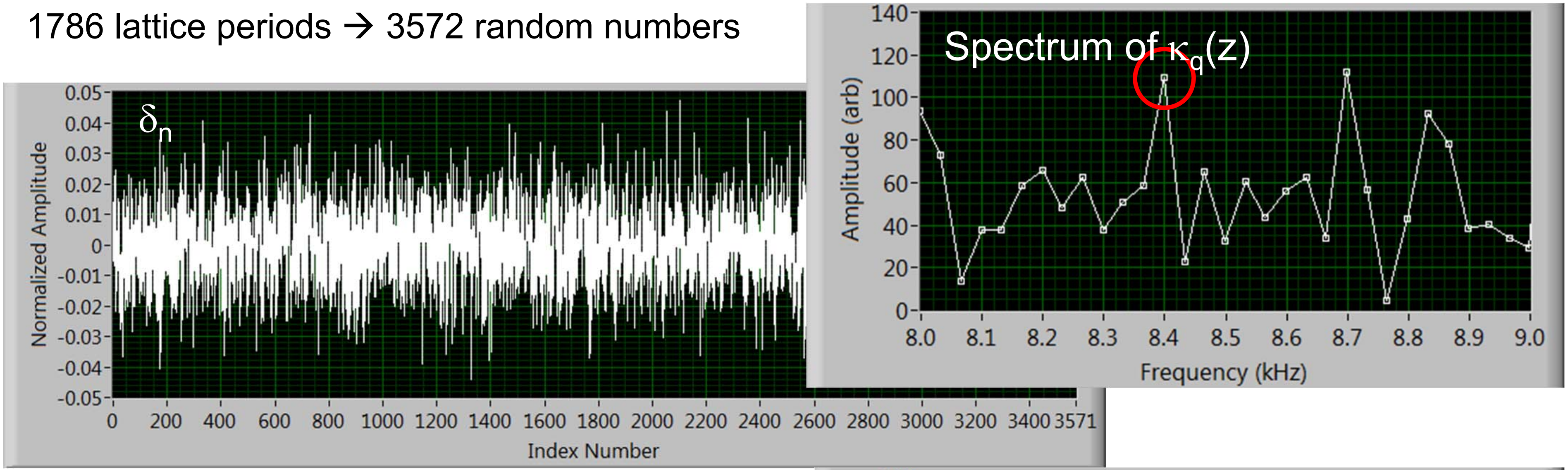
One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes One Frequency Component

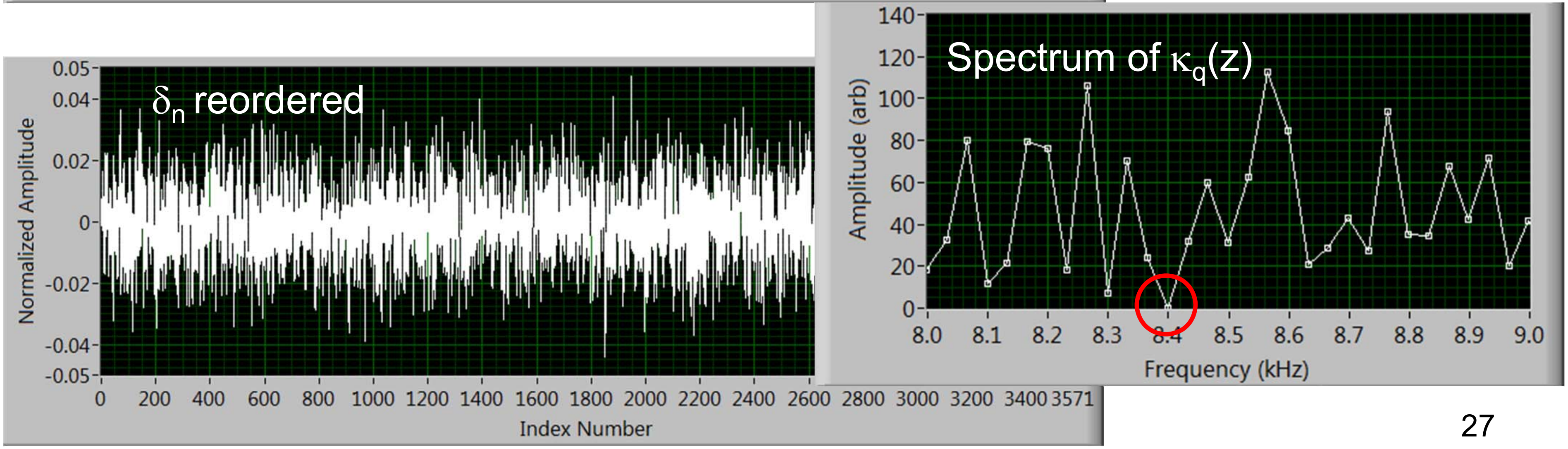


Similar data for ℓ = 2 quadrupole experiment shows the phenomenon occurring at ~16 kHz, but with a width of ~ 200 Hz.

The Lattice Can Be Reordered to Remove the Component at the Mode Frequency

1786 lattice periods → 3572 random numbers





Summary

- Experimental results demonstrate that the external perturbations act on the charge bunch through their interactions with the collective modes of the charge bunch.
- PTSX can be used to study ring machines and coherent periodic perturbations that can lead to beam loss. Experiments on PTSX have begun to study the detailed effects of the spectral content of the perturbation on the charge bunch at resonant tunes.
- The deleterious effects of lattice noise can be reduced by filtering or manipulation of the noise spectrum. In principle, magnet tolerances could then be relaxed to lower costs.
- In the future, further segmenting the PTSX electrodes will allow for the controlled application of higher-order multipole fields to study the interaction of noise with higher-order beam modes and investigate nonlinear accelerator lattices.